



Top Quark Mass Measurement in the 2.8 fb^{-1} Tight Lepton and Isolated Track Sample using Neutrino ϕ Weighting Method

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URL <http://www-cdf.fnal.gov>
(Dated: July 28, 2008)

We report on a measurement of the top quark mass in the lepton + track sample of $t\bar{t}$ events from $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV. This selection applied to dilepton channel of $t\bar{t}$ decays for increasing the number of selecting events due to relaxing the cuts for one lepton. The unconstrained system of dilepton events is solved using the hypothesis about neutrino ϕ values, and the top quark mass is reconstructed for each event. The integrated luminosity of the data sample is 2.8 fb^{-1} . 330 candidate events were reconstructed according to the $t\bar{t}$ hypothesis and fitted as a superposition of the signal and background. Using the background constrained fit (with 139.1 ± 17.0 events expected from background) we measure $M_{top} = 165.1 \pm 3.3$ (stat) GeV/c^2 . The estimate of systematic error is ± 3.1 GeV/c^2 .

I. INTRODUCTION

One of the main physics goals of CDF [1] in Run II is the study of top quark properties. First observed by the CDF and D0 collaborations in 1995 [2], the top quark is very massive, more than 35 times heavier than b quark. The top mass is one of the fundamental parameters of the Standard Model (SM). Within the SM its precise measurement together with W-mass gives a constraint on the Higgs boson mass.

In the CDF Run II we study proton-antiproton collisions at a center-of-mass energy 1.96 TeV. Top quarks are mostly produced in pairs ($t\bar{t}$) from quark-antiquark annihilations ($\sim 90\%$) or from gluon-gluon fusion. According to the SM, both top quarks decay almost exclusively as $t \rightarrow Wb$. The channels of $t(\bar{t})$ -decay are classified according to the decay modes of the W boson. The *dilepton* channel, when both W decay to leptons (e, μ) gets only 5% of decays, but has the best signal-to-background ratio (S/B). Near 30% of decays go to the *lepton + jets* channel, with one W producing an electron or a muon, and the other decaying into a quark pair and producing jets. The *all-hadronic* decay channel collects 44% of events, but has a large QCD background.

In this note we report a measurement of the top quark mass in the dilepton channel using the lepton + track event selection to collect more events due to the relaxed cuts for one of the leptons.

II. DATA SAMPLE & EVENT SELECTION

In our analysis we used data collected between March 2002 and April 2008, corresponding to a total integrated luminosity of 2.8 fb^{-1} . The data are collected with an inclusive lepton trigger that requires an electron with $E_T > 18 \text{ GeV}$ or a muon with $P_T > 18 \text{ GeV}/c$. After full event reconstruction we select events with a tight electron $E_T > 20 \text{ GeV}$ or muon with $P_T > 20 \text{ GeV}/c$, an isolated high- p_t track $P_T > 20 \text{ GeV}/c$ ("track lepton" or "tl"), two or more jets $E_T > 20 \text{ GeV}$, and significant $\cancel{E}_T > 25 \text{ GeV}$.

Tight electron candidates have a well-measured track pointing at an energy deposition in the calorimeter. In addition, the candidate's electromagnetic shower profile must be consistent with that expected for electrons. Tight muon candidates must have a well-measured track linked to hits in the muon chambers and energy deposition in the calorimeters consistent with that expected for muons. Tight lepton have to be isolated that means the total transverse energy within cone $\Delta R \equiv \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} < 0.4$, minus the candidate lepton E_T , is less than 10% of the candidate lepton E_T .

To count as the second lepton (track lepton) for the our analysis a well-measured track must have $P_T > 20 \text{ GeV}/c$, and pass a track isolation requirement. The track isolation is defined as the ratio of the transverse momentum of the candidate track to the sum of the transverse momenta of all tracks in a cone of $\Delta R \equiv \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} < 0.4$ around it, including the candidate track itself. The track isolation value should be > 0.9 .

The tight lepton and the track lepton have to be oppositely charged.

Two (or more) jets with corrected $E_T > 20 \text{ GeV}$ and $|\eta| < 2.0$ are also required.

If $\cancel{E}_T < 50 \text{ GeV}$ we additionally require that the angle between $\vec{\cancel{E}}_T$ and the nearest jet is $\Delta \phi > 25^\circ$.

Events with cosmic ray, conversion or Z are eliminated.

After these selection cuts 330 events were left, which were reconstructed according to the $t\bar{t}$ hypothesis. The same cuts were applied to the Monte Carlo generated signal or background events.

III. TOP MASS RECONSTRUCTION

A. Brief Description of The Method

The estimated top mass value for each event is returned from a kinematic event reconstruction procedure [3]. In brief, event reconstruction is the result of minimization of the chisquare functional (χ^2) by the MINUIT routines. This chisquare functional has resolution terms related to the measured physical variables and constrained terms to take into account the kinematic equations.

For the dilepton case due to the existence of two neutrinos we have a non-constrained kinematics. The number of independent variables is one more than the number of kinematic constraints ($-1C$ kinematics). Obviously, it is impossible to pick up directly only one solution per event. We must assume some of the event parameters (\vec{R}) as known in order to constrain the kinematics and then vary the \vec{R} to determine a set of solutions. In addition, we attach a χ^2 -dependent weight to each solution.

The minimal requirement in the case of $-1C$ kinematics to perform a minimization is to use a two dimensional vector as \vec{R} . For our analysis we choose the azimuthal angles of the neutrino momenta $\vec{R} = (\phi_{\nu 1}, \phi_{\nu 2})$ and we

create a net of solutions in the $(\phi_{\nu 1}, \phi_{\nu 2})$ plane. We need to cover full $(0 < \phi_{\nu 1} < 2\pi, 0 < \phi_{\nu 2} < 2\pi)$ plane by the net. Actually, taking into account the symmetry of the solutions for $\phi'_{\nu 1, \nu 2} = \phi_{\nu 1, \nu 2} + \pi$ it is enough to find the solutions in quadrant $(0 < \phi_{\nu 1} < \pi, 0 < \phi_{\nu 2} < \pi)$ and expand them over the whole $(0 < \phi_{\nu 1} < 2\pi, 0 < \phi_{\nu 2} < 2\pi)$ plane.

The formula for χ^2 we use is:

$$\chi^2 \equiv -2\ln(\mathcal{P}(\mathbf{x})) \quad (1)$$

where \mathbf{x} is a general notation to indicate a variable and \mathcal{P} its probability density distribution. The expanded formula of χ^2 is :

$$\begin{aligned} \chi^2 &= \chi_{reso}^2 + \chi_{constr}^2 \\ \chi_{reso}^2 &= \sum_{l=1}^2 \frac{(P_T^l - \tilde{P}_T^l)^2}{\sigma_{P_T}^l} + \sum_{j=1}^2 [-2\ln(\mathcal{P}_{tf}(\tilde{P}_T^j | P_T^j))] + \sum_{i=x,y} \frac{(UE^i - \tilde{UE}^i)^2}{\sigma_{UE}^i} \\ \chi_{constr}^2 &= -2\ln(\mathcal{P}_{BW}(m_{inv}^{l_1, \nu_1} | M_W, \Gamma_{M_W})) - 2\ln(\mathcal{P}_{BW}(m_{inv}^{l_2, \nu_2} | M_W, \Gamma_{M_W})) + \\ &\quad - 2\ln(\mathcal{P}_{BW}(m_{inv}^{l_1, \nu_1, j_1} | \tilde{M}_t, \Gamma_{\tilde{M}_t})) - 2\ln(\mathcal{P}_{BW}(m_{inv}^{l_2, \nu_2, j_2} | \tilde{M}_t, \Gamma_{\tilde{M}_t})) \end{aligned} \quad (2)$$

The variables with a tilde sign refer to the output of the minimization procedure, whereas P_T and UE (unclustered energy) represent measured values corrected for known detector and physics effects. \tilde{M}_t is the fit parameter giving the reconstructed top mass. BW and tf are for the relativistic Breit-Wigner and transfer function respectively. Notice that we splitted the χ^2 into two parts: the first one, χ_{reso}^2 , takes into account the detector uncertainties, whereas the second one deals with the known mass constraints.

The first sum of χ_{reso}^2 runs over the primary lepton (tight lepton) and the track lepton. The second sum is over the two leading jets. The third sum is over the two transverse components of the unclustered energy UE which is defined as the sum of all the calorimeter towers energy which are not associated with any of the objects previously considered in the χ_{reso}^2 formula (tight lepton, track-lepton, two leading jets). The other term in the formula (2), χ_{constr}^2 , refers to the invariant masses of the couples lepton-neutrino and of the lepton-neutrino-leading jet system. We set $M_W = 80.41 GeV/c^2$, $\Gamma_{M_W} = 2.06 GeV/c^2$, and we insert the function Γ_{M_t} according to the standard model [4]. We also suppose that the $t\bar{t}$ quarks have equal masses: $m_{inv}^{l_1, \nu_1, j_1} = m_{inv}^{l_2, \nu_2, j_2}$.

For every point of the $(\phi_{\nu 1}, \phi_{\nu 2})$ plane we have 8 solutions. Double ambiguity corresponds to the two way of associating the two charged leptons to the two leading jets (which are supposed to be b-jets). The four solutions are generated from the possibility for every neutrino to have two p_z momenta satisfying the $t\bar{t}$ kinematics. We selected the one solution out of the 8 with minimal χ^2 for every point of the $(\phi_{\nu 1}, \phi_{\nu 2})$ plane.

Our investigation shows that the distribution of reconstructed mass has the smaller error and the more accurate mean value if we apply for the equations (2) renormalized Breit-Wigner formula:

$$BW(m_{inv}; m, \Gamma) = \frac{\Gamma \cdot m^2}{(m_{inv}^2 - m^2)^2 + m^2 \Gamma^2} \quad (3)$$

As a result of Monte Carlo optimization [3] we choose to split the quadrant of $(0 < \phi_{\nu 1} < \pi, 0 < \phi_{\nu 2} < \pi)$ into 12x12 points.

The overall normalization of the weight distribution is chosen to be one. The expression for the weight is:

$$w_{ij} = \frac{\exp(-\chi_{ij}^2/2)}{\sum_{i=1}^{12} \sum_{j=1}^{12} \exp(-\chi_{ij}^2/2)}. \quad (4)$$

The output from this procedure is an array of 144 χ_{ij}^2 and $m_{inv}^{l_1, \nu_1, j_1} = m_{inv}^{l_2, \nu_2, j_2}$ ($i, j = 1, 12$).

The distribution of masses taking into account the appropriate weights is built for every event. Then we perform the averaging of this mass distribution with content above 30% of the maximum value. This cut was chosen [3] to minimize the expected statistical errors. At the end we get one mass per event.

The final extraction of the top quark mass from a sample of lepton + track candidates is provided by the likelihood fit. The expected signal and background distributions (templates) are obtained using Monte Carlo samples with full detector simulation.

B. Monte Carlo Signal Templates

The official MC samples were used. The signal templates for input top masses in the 155÷195 GeV range were created with 2 GeV steps. Then the obtained set of templates was parametrized by one Landau and two Gaussian functions

$$f_s(M_t^{reco}|M_{top}) = p_7(p_6 \frac{1}{\sqrt{2\pi}p_2} e^{-0.5(\frac{M_t^{reco}-p_1}{p_2} + e^{-\frac{M_t^{reco}-p_1}{p_2}})} + (1-p_6) \frac{1}{\sqrt{2\pi}p_5} e^{-0.5(\frac{M_t^{reco}-p_4}{p_5})^2}) + (1-p_7) \frac{1}{\sqrt{2\pi}p_3} e^{-0.5(\frac{M_t^{reco}-p_8}{p_3})^2} \quad (5)$$

Notice that this parametrizing function is strongly dependent from the the input top mass M_{top} , or it is better to say that its parameters p_1, \dots, p_8 , are M_{top} -dependent:

$$p_k = \alpha_k + \alpha_{k+8} \cdot M_{top} \quad (6)$$

The examples of our templates are presented in Fig. 1.

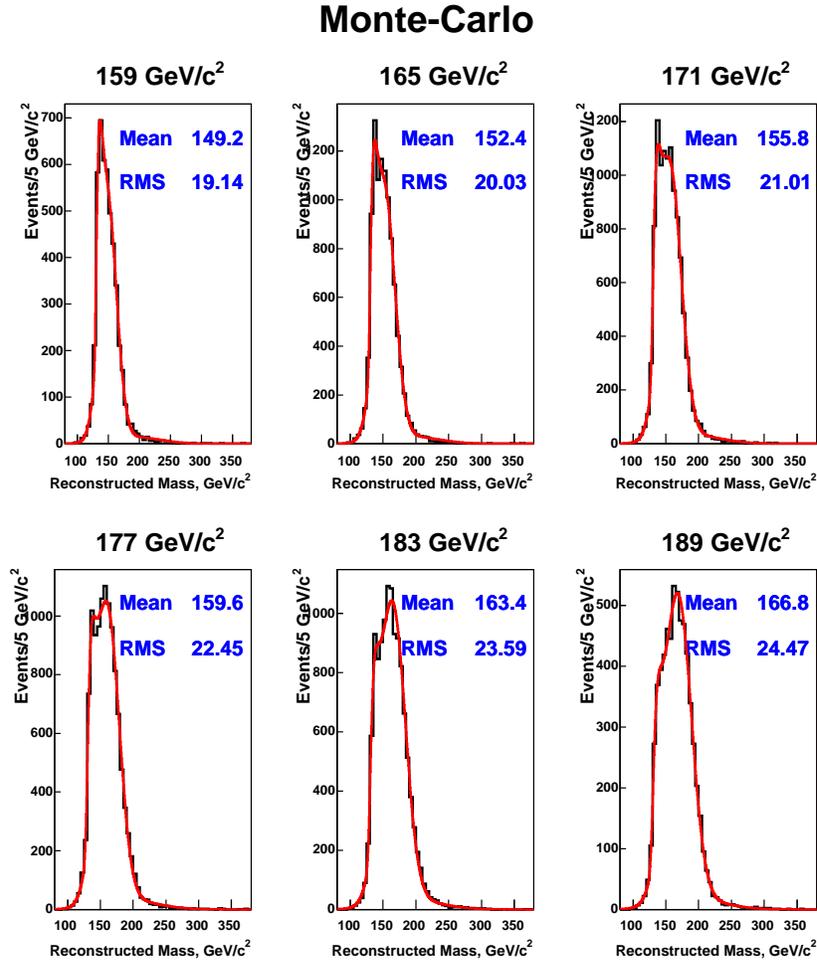


FIG. 1: The examples of the signal templates

C. Background template

The dominant background in the dilepton channel comes from the ($WZ \rightarrow ll$, $WW \rightarrow ll$, $ZZ \rightarrow ll$, Drel-Yan, $Z \rightarrow \tau\tau$) processes. To take into account the contribution from background events we create background

templates for each of the above mentioned sources. In order to build general template for Drell-Yan events the templates for each sub-process were combined using their cross-sections and acceptances.

Template for fake events was obtained by weighting the fakeable events from W+jets data sample according to the fake rate probability matrix.

The obtained templates (Fig. 2) for these processes were combined together according to the expected number of events, as derived by the $t\bar{t}$ cross section group.

The result for the combined background template is shown in Fig. 2, right lower plot.

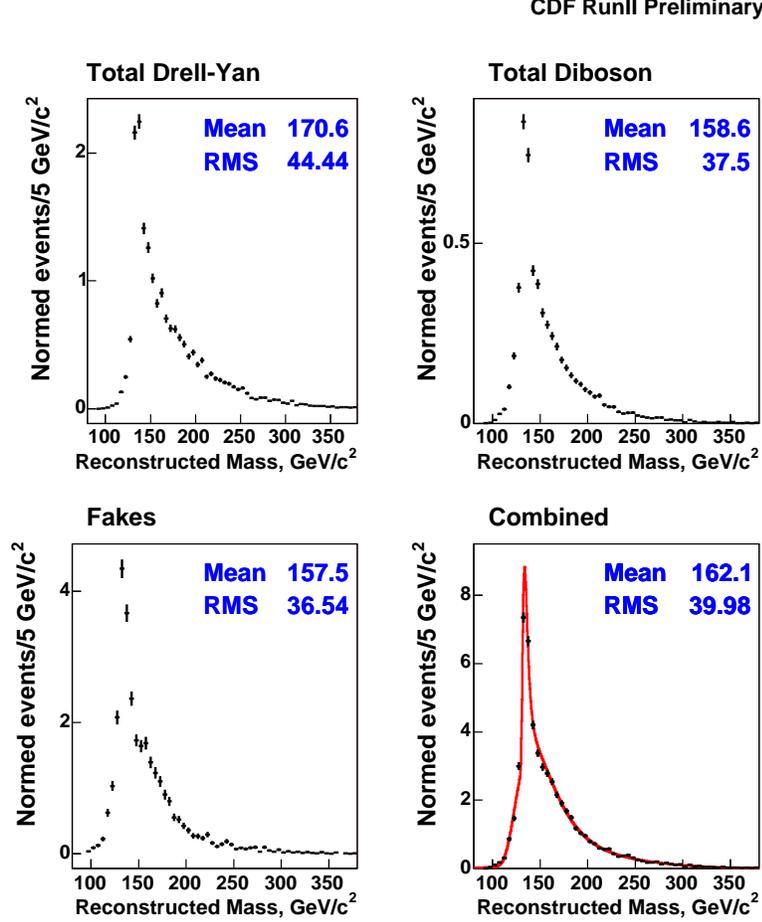


FIG. 2: Templates of background processes Drell-Yan, Diboson, “fake” events. Lower right plot shows the combined background.

The fitting function, f_b , is a slightly bit different from the one used for signal templates, as you can see from the formula (7).

$$\begin{aligned}
 f_b(M_t^{reco}) = & q_7(q_6 \frac{1}{\sqrt{2\pi}q_2} e^{-0.5(\frac{M_t^{reco}-q_1}{q_2} + e^{-\frac{M_t^{reco}-q_1}}{q_2}})} + \\
 & + (1-q_6) \frac{1}{\sqrt{2\pi}q_5} e^{-0.5(\frac{M_t^{reco}-q_4}{q_5})^2}) + (1-q_7) \frac{1}{\sqrt{2\pi}q_3} e^{-0.5(\frac{M_t^{reco}-q_8}{q_3} + e^{-\frac{M_t^{reco}-q_8}}{q_3}})}
 \end{aligned} \tag{7}$$

However, the main difference from f_s is: the f_b parameters q_1, \dots, q_8 are not depend from the top mass:

$$q_k = \beta_k \tag{8}$$

D. Likelihood

The likelihood function finds the probability that our data candidates are described by an appropriate admixture of background events and dilepton $t\bar{t}$ decays with a certain top quark mass. The likelihood function has the following form:

$$\mathcal{L} = \mathcal{L}_{shape} \cdot \mathcal{L}_{backgr} \cdot \mathcal{L}_{param}; \quad (9)$$

where,

$$\mathcal{L}_{shape} = \frac{e^{-(n_s+n_b)} \cdot (n_s+n_b)^N}{N!} \cdot \prod_{n=1}^N \frac{n_s \cdot f_s(m_n|M_{top}) + n_b \cdot f_b(m_n)}{n_s+n_b} \quad (10)$$

where parameters n_s and n_b are the expected signal and background numbers in the lepton + track data sample. N is the total number of events observed in the data. Also the additional terms were added to constrain number of the background events and to constrain $\vec{\alpha}$, $\vec{\beta}$ parameters, obtained from the signal and background template parametrization:

$$\mathcal{L}_{backgr} = \exp\left(\frac{-(n_b - n_b^{exp})^2}{2\sigma_{n_b}^2}\right) \quad (11)$$

$$\mathcal{L}_{param} = \exp\{-0.5[(\vec{\alpha} - \vec{\alpha}_0)^T U^{-1}(\vec{\alpha} - \vec{\alpha}_0) + (\vec{\beta} - \vec{\beta}_0)^T V^{-1}(\vec{\beta} - \vec{\beta}_0)]\}. \quad (12)$$

Here U and V are the covariance matrices for the parameters $\vec{\alpha}_0$ and $\vec{\beta}_0$ respectively. The likelihood maximization procedure (we usually minimize $-\ln(\mathcal{L})$) returns a true top quark mass estimator M_{top} .

IV. RESULTS FROM PSEUDO-EXPERIMENTS

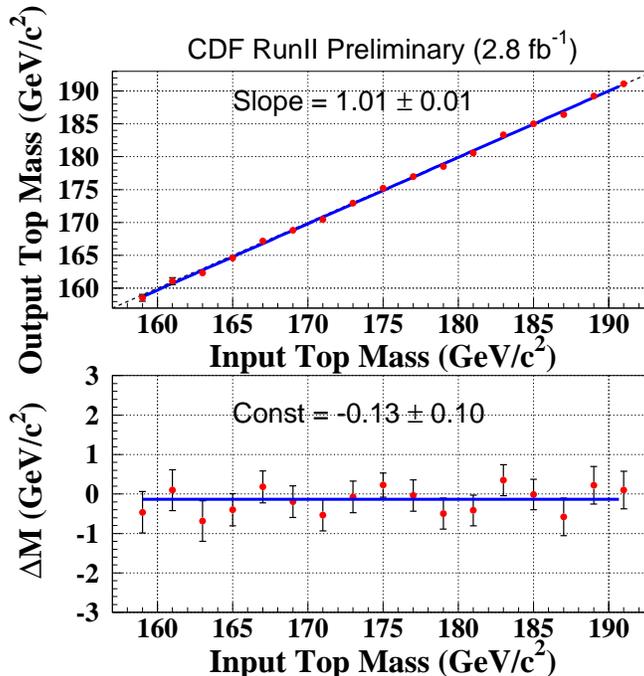


FIG. 3: The extracted top mass as a function of input mass. The result of a linear fit is also shown. The lower plot shows the residuals (reconstructed - input top mass).

We checked whether the fit with likelihood form (9) was able to return the correct mass by performing the “sanity check” pseudo-experiments (PE) for different input top mass values.

The numbers of signal and background events in PE's were Poisson distributed with mean values as their expected numbers. We took as expected numbers 155.9 ± 4.8 and 139.1 ± 17.0 for signal and background respectively.

The output M_{top} (median of distribution) vs. input M_{top} is shown in Fig. 3. A linear fit yielded a slope of 1.01 ± 0.01 .

The correction for the top mass mean value is $0.13 \pm 0.10 \text{ GeV}/c^2$. It is obtained from the fit of the distribution: residual versus top mass (see Fig. 3 (bottom)).

The mean and width of the pull distributions as a function of input top mass are shown in Fig. 4.

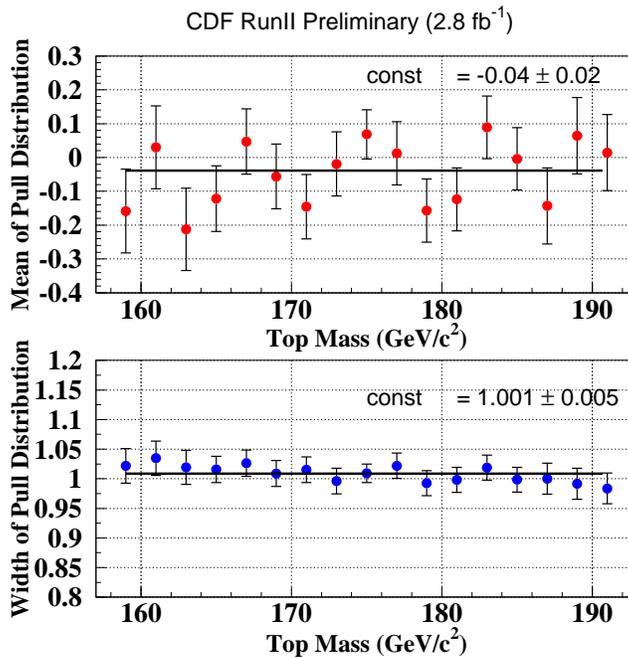


FIG. 4: Mean (above) and σ (below) of pull distributions determined from the pseudo-experiments as a function of input top mass.

From the pull width distribution we understand that we are underestimating our statistical errors by 0.9%. We take into account this effect by scaling the returned errors by 1.009.

V. SYSTEMATIC UNCERTAINTIES

We have considered the following sources of systematic uncertainties on the fitted mass value: a) jet energy scale, b) discrepancy between data and simulation luminosity profile (pileup), c) amount of initial and final state radiation, d) shape of the background template, e) parton distribution functions, and f) approximations made by Monte Carlo generators, g) b-jet energy scale and lepton energy scale. The magnitudes of these uncertainties were estimated using large Monte Carlo samples generated only for the systematics study.

The procedure for estimating the systematic uncertainty is similar for all sources. For each source we varied the input value as appropriate (by 1σ , or changing PDF, etc) and evaluated the impact on the returned top mass. This was done by simulating a large number (usually 10000 or more) of pseudo-experiments (PE) with the nominal assumption and with the alternate assumption. The reconstructed mass distribution from each PE was fitted with the same likelihood procedure as for the data. The obtained mass value was entered into an ensemble of results of simulated experiments. The systematic uncertainty assigned to our measurement is the difference in the average of these result distributions for the nominal and shifted ensembles or half the difference between results obtained with $+\sigma$ and $-\sigma$ of the corresponding parameter change.

The largest contribution comes from the uncertainty in the jet energy measurement, which includes jet energy corrections for different calorimeter response (as a function of η), the absolute hadron energy scale, and jet fragmentation. Discrepancy in the data and MC luminosity profile is estimated by rescaling the top mass dependence on the number of interactions in the event by the difference in the number of interactions between

data and MC. The initial and final state radiation (IFSR) uncertainties are estimated using the Pythia [5] Monte Carlo samples, in which QCD parameters for parton shower evolution in the initial and final states are varied simultaneously. The amount of variation is based on the CDF studies of Drell-Yan data. For the parton distribution functions (PDF) we considered two different group of PDF (CTEQ and MRST), two sets of MRST for different Λ_{QCD} values, and 20 pairs of CTEQ6M uncertainty sets. The effect of using different top Monte Carlo generators was checked by comparing the nominal Pythia [5] with alternate Herwig [6] samples.

In order to estimate effect on top mass from the uncertainty in background composition we varied the contribution in combined background template of main sources (Diboson, Drell-Yan and "fakes") by $\pm\sigma$. Contribution from another subsamples was corrected to maintain the total expected number of background events. We also studied the effect from changing the shape of the main background contributors: Drell-Yan and "fakes" according to the CDF official procedure.

Also the additional uncertainty for the b-jet scale due to the heavy quark fragmentation, semileptonic b-jet branching ratio, and b-jet calorimeter response was taken into account.

The effect on the top mass from the uncertainty on lepton energy scale was studied by applying $\pm 1\%$ shifts for lepton p_T .

The systematic uncertainties are summarized in Table I. The total systematic uncertainty is estimated to be $3.1 \text{ GeV}/c^2$.

Source	Uncertainty (GeV/c^2)
Jet Energy Scale	2.9
b-Jet Energy Scale	0.4
Pileup	0.2
Initial and Final State Radiations	0.3
Parton Distribution Functions	0.3
Monte Carlo Generators	0.2
Background composition	0.5
Fakes shape	0.4
DY shape	0.3
Lepton Energy Scale	0.3
Total	3.1

TABLE I: Summary of systematic uncertainties

VI. RESULTS

The two-component background-constrained fit (with $N_b=139.1\pm 17.0$ expected background events) for the obtained 330 lepton+track candidates returns $M_{top} = 164.97 \pm_{3.18}^{3.29} \text{ GeV}/c^2$, with $182.7 \pm_{21.6}^{22.1}$ signal events and $147.1 \pm_{15.4}^{15.5}$ background events.

The left plot in Fig. 5 shows the fitted mass distribution. The insert shows the mass dependence of the negative log-likelihood function. The right plot is the expected statistical errors from Monte Carlo sample, where the arrows indicate the errors returned by the fit to the data. The probability to have better accuracy than our one from data is 79%.

After the correction of the mean value increased on $0.13 \text{ GeV}/c^2$ and statistical errors multiplying by factor of 1.009 (see section IV), our preliminary result on the CDF data sample with the integrated luminosity of 2.8 fb^{-1} is:

$$M_{top} = 165.1 \pm_{3.2}^{3.3} \text{ (stat)} \pm 3.1 \text{ (syst)} \text{ GeV}/c^2$$

We also performed a fit when the number of the background events was unconstrained. This fit returns $M_{top}=164.96 \pm_{3.22}^{3.31} \text{ GeV}/c^2$, with $181.3 \pm_{31.4}^{32.2}$ signal events and $148.7 \pm_{29.7}^{31.8}$ background events.

VII. CONCLUSION

We applied the neutrino ϕ weighting method to solve a non-constrained kinematics of the top quark decay in dilepton mode.

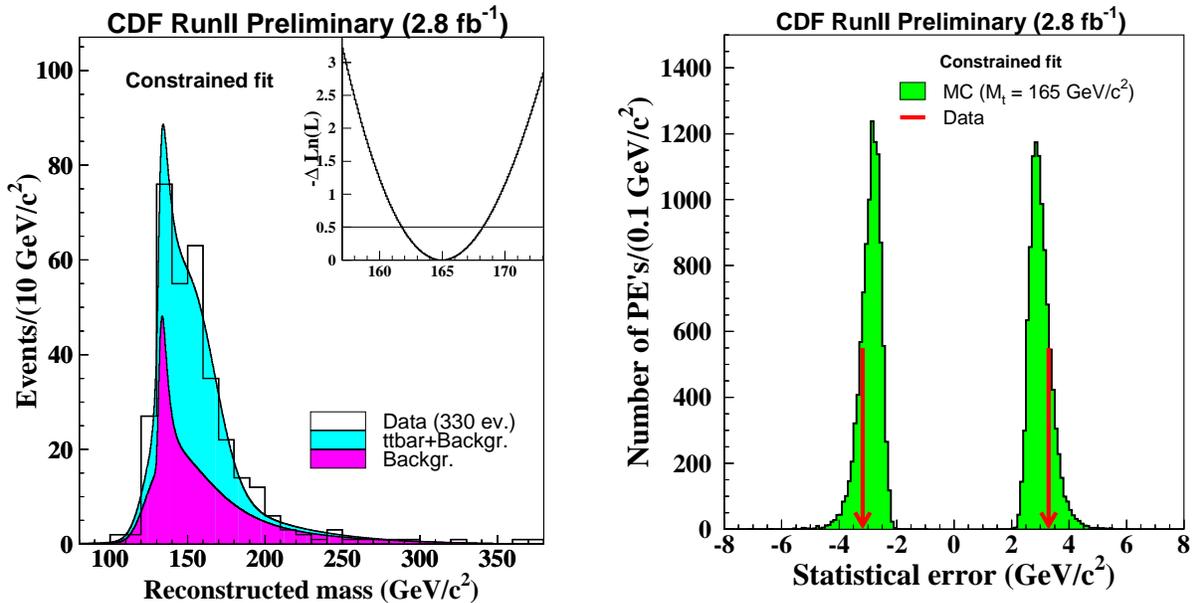


FIG. 5: Left: two-component background-constrained fit to the lepton + track sample. The pink area corresponds to the background returned by the fit and the blue area is the sum of background and signal events. The insert shows the mass-dependent negative log-likelihood used in the fit. Right: left/right error distributions returned by the PE's. The arrows indicate the errors returned by the fit to the data.

330 candidate events were selected from the CDF data sample with integrated luminosity of 2.8 fb^{-1} . Our preliminary measurement of the top quark mass in the lepton+track sample is: $M_{top} = 165.1 \pm_{3.2}^{3.3} \text{ (stat)} \pm 3.1 \text{ (syst)} \text{ GeV}/c^2$.

Acknowledgments

We thank the Fermilab staff and the technical staffs of the participating institutions for their vital contributions. This work was supported by the U.S. Department of Energy and National Science Foundation; the Italian Istituto Nazionale di Fisica Nucleare; the Ministry of Education, Culture, Sports, Science and Technology of Japan; the Natural Sciences and Engineering Research Council of Canada; the National Science Council of the Republic of China; the Swiss National Science Foundation; the A.P. Sloan Foundation; the Bundesministerium für Bildung und Forschung, Germany; the Korean Science and Engineering Foundation and the Korean Research Foundation; the Science and Technology Facilities Council and the Royal Society, UK; the Institut National de Physique Nucleaire et Physique des Particules/CNRS; the Russian Foundation for Basic Research; the Comisión Interministerial de Ciencia y Tecnología, Spain; the European Community's Human Potential Programme; the Slovak R&D Agency; and the Academy of Finland.

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