Combination of DØ and CDF Results on $\Delta \Gamma_s$ and the CP-Violating Phase $\beta_{s}^{J/\psi \phi}$

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A procedure is described for the combination of DØ and CDF measurements of the $B_0^s$ width difference, $\Delta \Gamma_s$, and the CP-violating phase, $\phi_{s}^{J/\psi \phi} = -2\beta_{s}^{J/\psi \phi}$, between the $B_0^s$ mixing and decay amplitudes determined via the time-dependent angular analysis of flavor-tagged $B_0^s \rightarrow J/\psi \phi$ decays. Results are presented based on the latest DØ and CDF experimental measurements, each using an integrated luminosity of 2.8 fb$^{-1}$. 

Preliminary Results for Summer 2009 Conferences
I. THEORY AND NOMENCLATURE

For the $B_s^0$ system, we have the matrix time evolution equation:

$$\frac{d}{dt}\begin{pmatrix} \left|B_s^0\rightangle \\ \left|\bar{B}_s^0\rightangle \end{pmatrix} = \begin{pmatrix} M - \frac{\Gamma_s}{2} & M_{12} - \frac{\Gamma_{12}}{2} \\ M_{12}^* & M - \frac{\Gamma_s}{2} \end{pmatrix} \begin{pmatrix} \left|B_s^0\rightangle \\ \left|\bar{B}_s^0\rightangle \end{pmatrix}.$$  \hspace{1cm} (1)

In the Standard Model, $B_s^0$-$\bar{B}_s^0$ oscillations are caused by flavor-changing weak interaction box diagrams that induce non-zero off-diagonal elements in the above. The mass eigenstates, defined as the eigenvectors of the above matrix, are different from the flavor eigenstates, with a heavy $(H)$ and light $(L)$ mass eigenstate, respectively:

$$|B_{sH}\rangle = p|B_s^0\rangle - q|\bar{B}_s^0\rangle; \quad |B_{sL}\rangle = p|B_s^0\rangle + q|\bar{B}_s^0\rangle,$$  \hspace{1cm} (2)

with $|p|^2 + |q|^2 = 1$. If $CP$ is conserved in mixing in the $B_s^0$ system, then $q = p$, and

$$|B_{sH}\rangle = |B_s^{CP-\text{odd}}\rangle; \quad |B_{sL}\rangle = |B_s^{CP-\text{even}}\rangle.$$  \hspace{1cm} (3)

Matrix elements can be extracted experimentally by measuring a mass and width difference between mass eigenstates:

$$\Delta m_s = M_L - M_H \approx 2|M_{12}|,$$

$$\Delta \Gamma_s = \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}| \cos \phi_s,$$  \hspace{1cm} (4)

where $\phi_s$ is defined below. Note the sign convention for $\Delta \Gamma_s$ compared to $\Delta m_s$. In this convention, the Standard Model (SM) prediction for $\Delta \Gamma_s$ is positive. The current theoretical expectation in the SM is $\Delta \Gamma_s^{\text{SM}} = 2|\Gamma_{12}| = 0.096 \pm 0.039$ ps$^{-1}$ [1].

The parameter $\Gamma_{12}$ is dominated by the quark transition $b \rightarrow c\bar{c}s$ in decays into final states common to both $B_s^0$ ($\bar{b}s$) and $\bar{B}_s^0$ ($b\bar{s}$). Examples of such decays are $B_s^0 \rightarrow J/\psi\phi$ and $B_s^0 \rightarrow D_{s}^{(*)}D_{s}^{(*)}$, as shown in Fig. 1.

![FIG. 1: Example $B_s^0$ decays giving rise to a non-zero $\Gamma_{12}$.](image)

The analogous decay diagram for a width difference in the $B_d^0$ system substitutes the $s$ quark for a $d$ quark. This decay is Cabibbo suppressed, hence $\Delta \Gamma_d$ is negligible. In the case of $\Delta \Gamma_s$, decays into $CP$-even final states increase the value of $\Delta \Gamma_s$, while decays into $CP$-odd final states decrease it.

An average width is defined as $\Gamma_s = (\Gamma_L + \Gamma_H)/2$. The measured lifetime of the $B_s^0$ will depend on the mix of $CP$ eigenstates involved in its decay. A more fundamental lifetime based on the average width is defined as $\bar{\tau}_s = 1/\Gamma_s$.

A. Weak Phase in $B_s^0$ Mixing

In general there will be a $CP$-violating weak phase difference:

$$\phi_s = \arg \left[-M_{12}/\Gamma_{12}\right],$$  \hspace{1cm} (5)

between the $B_s^0$-$\bar{B}_s^0$ amplitude and the amplitudes of the subsequent $B_s^0$ and $\bar{B}_s^0$ decay to a common final state. In this convention, $\phi_s$ is defined to fall in the range $[-\pi/2, \pi/2]$. This can affect the observed $\Delta \Gamma_s$ as given above. The SM prediction for this phase is tiny, $\phi_s^{\text{SM}} = 0.004$ [1]; however, new physics in $B_s^0$ mixing could change this observed phase to

$$\phi_s = \phi_s^{\text{SM}} + \phi_s^{\text{NP}}.$$  \hspace{1cm} (6)

The relative phase between the $B_s^0$ mixing amplitude and that of specific $b \rightarrow c\bar{c}s$ quark transitions such as for $B_s^0$ or $\bar{B}_s^0 \rightarrow J/\psi\phi$ in the SM is $[1, 2]$: \[2\beta_s^{\text{SM}} = 2 \arg [-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*] \approx 0.04.\hspace{1cm} (7)\]
This angle is analogous to the $\beta$ angle in the usual CKM unitarity triangle replacing $d \rightarrow s$ aside from the negative sign (resulting in a positive angle in the SM). The same additional contribution $\phi_s^{NP}$ due to new physics would show up in this observed phase [1], i.e.:

$$2\beta_s = 2\beta_s^{SM} - \phi_s^{NP}. \quad (8)$$

The current experimental precision does not allow these small $CP$-violating phases $\phi_s^{SM}$ and $\beta_s^{SM}$ to be resolved, and for large new physics effect, we can approximate $\phi_s \approx -2\beta_s \approx \phi_s^{NP}$, i.e., a significantly large observed phase would indicate new physics.

II. COMBINING EXPERIMENTAL MEASUREMENTS

The most direct and precise experimental results on $\Delta \Gamma_s$ and $\phi_s$ come from the Tevatron, where reconstructed decays $B_s^0 \rightarrow J/\psi \phi$ are separated into $CP$-even and $CP$-odd components from fits to angular distributions of $J/\psi$ and $\phi$ decay products as a function of proper decay time. Including information on the $B_s^0$ flavor (i.e., $B_s^{0}$ or $\bar{B}_s^{0}$) at production time via flavor tagging improves precision and also resolves the sign ambiguity on the weak phase for a given $\Delta \Gamma_s$. DØ [3] has published such an analysis based on $2.8$ fb$^{-1}$ of data, and CDF has a preliminary update [4], also based on $2.8$ fb$^{-1}$, of their previously published result [5]. Both experiments measure the same observed phase angle and report two-dimensional profile likelihoods and confidence-level ($CL$) contours in the $\phi_s^{J/\psi \phi}$ vs. $\Delta \Gamma_s$ (or $\beta_s^{J/\psi \phi}$ vs. $\Delta \Gamma_s$ where $\phi_s^{J/\psi \phi} = -2\beta_s^{J/\psi \phi}$) plane. Details of the analyses and likelihood fits can be found in the indicated references. The DØ published result [3] imposed weak constraints on the strong phases $\delta_i$, i.e., the phases between polarization amplitudes in the decays, while the CDF analysis [4] allowed these $\delta_i$ to float freely in the fit as additional nuisance parameters. In order to produce a coherent constraint-free combination, the DØ likelihood fit was redone by allowing the strong phases to float freely; tables collating the two-dimensional likelihood profile for this case can be found in Ref. [6]. Also, the CL contours of both experiments now explicitly account for the non-Gaussian behavior of the uncertainties (see the following section). Previous HFAG combinations [7] did not include systematic uncertainties in the two-dimensional countours, and also did not correct for coverage using results of toy MC simulations. Our combination of CDF and DØ results is based on the idea of adding the log-likelihoods of independent measurements to produce a combined log-likelihood function. This idea has been extended to the use of profile-likelihood rather than simple likelihoods [7]. These distribution actually depend on systematic parameters, and we account for their presence by picking up the worst-case distribution in each case. All of this is in accordance with the procedure followed by the individual experiments to obtain their results with full account for systematic uncertainties.

A. Correcting for Non-Gaussian Uncertainties

The non-Gaussian behavior of the uncertainties on the fit parameters of the DØ and CDF analyses are different. To combine the two-dimensional likelihood scans in $\beta_s^{J/\psi \phi}$ vs. $\Delta \Gamma_s$ space, the likelihood value at each point needs to be adjusted to put the two scans on the same footing, i.e., to make sure that a particular value of the profile-likelihood ratio in each case has a matching tail probability. In the case of CDF, as indicated in Ref. [4], the two-dimensional profile likelihood with all parameters floating [8] is adjusted to produce a new variable having the chi-squared distribution, since the original variable significantly deviates from the ideal asymptotic behavior. The real profile-likelihood distribution was calculated using 10,000 default Monte Carlo (MC) pseudo-experiments, each having the same sample size as the data, and generated assuming the Standard Model values of $\beta_s^{J/\psi \phi} = 0.02$ and $\Delta \Gamma_s = 0.096$ ps$^{-1}$. For CDF, the resulting CL contours for the two-dimensional profile likelihood for a given likelihood ratio are shown in Fig. 2(a).

Similarly, in the case of DØ, 2,000 MC pseudo-experiments, generated with the same statistics as for the DØ analysis at the same Standard Model values are used to find the coverage, i.e., the CL value that corresponds to a given profile-likelihood ratio value in the two-dimensional likelihood scans. This curve is shown in Fig. 2(b). While the distribution could in principle depend on the assumed values of $\beta_s$ and $\Delta \Gamma_s$, extensive MC tests have not shown any such dependency.

B. Including Systematic Uncertainties

A large number of different systematic effects influence the results of these measurements. In order to ensure that the quoted contours actually have the confidence level that is nominally assigned to them (that is, that they have
the correct coverage), both experiments include systematics by taking a projection on the physics parameters of the many-dimensional confidence region that includes all possible systematic parameters, that is, they use the worst-case p-value. The use of the profile likelihood as an ordering function in this multidimensional space has almost-optimal efficiency properties and allows to use quite conservative uncertainties on the systematic parameters without weakening significantly the resolution of the measurements [9].

In the case of CDF, as described in Ref. [4], systematic uncertainties are treated by the description of signal models, background models, Δms, etc., as nuisance parameters in the fit. CDF generates 5,300 pseudo-experiments conducted in sixteen “alternative universes”, in which the nominal values of all nuisance parameters have been picked at random within ±5σ flat range. The adjustment curves for the profile likelihood for the different cases are shown in Fig. 2(a). The most conservative value of the (1−CL) value (i.e., largest value) for each likelihood ratio value is then selected. Detailed descriptions of systematic uncertainties and their numerical values on one-dimensional estimates of the extracted parameters are found in Ref. [4].

Dominant systematic uncertainties for the DØ analysis are also included as nuisance parameters. The largest effect is the inclusion of the uncertainty on Δms = 17.77 ± 0.12 ps⁻¹, which is allowed to vary within a Gaussian constraint of 0.12 ps⁻¹. Parameters in the signal and background models and their systematic uncertainties are also treated as nuisance parameters in the fit. Explicit variations to take into account other dominant systematic uncertainties are also used to generate curves of (1−CL) versus likelihood ratio in “alternative universes”. As above, the most conservative value is taken, i.e., the largest value of (1−CL) for a given likelihood ratio. 2,000 pseudo-experiments are generated for each of four different acceptance parameterizations, and for an alternative parameterization of the function used to estimate dilution of the flavor tag. Fig. 2(b) shows the resulting adjustment curve including systematic uncertainties.

C. Adjusted Two-Dimensional Profile Likelihoods

Using the curves above, the likelihood values in the βJ/ψφ vs. ΔΓs scans from CDF and DØ are adjusted to correspond to those expected for Gaussian errors corresponding to a given CL. An example is shown in Fig. 3 where ensemble coverage tests for CDF indicate that a value of 2Δlog L = 8.42 corresponds to 95% CL. In the two-dimensional likelihood scans, a value of 8.42 is then replaced with 5.99, the value of 2Δlog L expected for Gaussian errors (i.e., χ² with two degrees of freedom). Although CDF and DØ start with different coverage, the use of the different curves ensures that, after adjustment, a Δlog L value in each case has the same CL value.

We have used a parameterization of Fig. 2(a) to adjust the CDF likelihood scans to give the CL contours shown in Fig. 4. The comparison of this result with the original calculation, shown in Fig. 11 of Ref. [4] shows good agreement.
Δ\log(L) (1 - CL)

Gaussian errors, χ² distr. 2 dof

Coverage from ensemble tests 5.99 8.42

5.99 8.42

FIG. 3: An example of adjusting the likelihood ratio value in each scan to correspond to expected Gaussian uncertainties according to relevant coverage.

and gives confidence in the correctness of the present procedure.

FIG. 4: Adjusted two-dimensional profile likelihood as confidence contours of \( \beta_{1/\psi} \) and \( \Delta \Gamma_s \) for CDF’s analysis using 2.8 fb\(^{-1}\) of data (to be compared to Fig. 11 of Ref. [4]). The Standard Model expectation and uncertainty is indicated by the black line. The region allowed in new physics models given by \( \Delta \Gamma_s = 2|\Gamma_{12}| \cos \phi_s \) (i.e., CP violation in the interference between mixing and decay amplitudes) is also shown (light green band).

Figure 5 shows the same adjusted CL contours for DØ, both with and without systematic uncertainties included, as described above. Note that these results allow the strong phases \( \delta_i \) to float, and are hence different from those reported in the DØ publication [3] where weak constraints were imposed on \( \delta_i \).

D. Combined Result

The CDF and DØ two-dimensional adjusted log likelihoods are then added for a combined result shown in Fig. 6.

For this combination, the p-value at the Standard Model central point is 3.4% or 2.12\( \sigma \) (this takes as reference the central predicted value of \( \Delta \Gamma_s^{\text{SM}} \), without accounting for the associated theoretical uncertainty. If the lower bound given by the theoretical uncertainty of \( \Delta \Gamma_s = 0.096 \pm 0.039 \) is instead taken, the p-value is found to be 4.2% or 2.03\( \sigma \).

The likelihood profile for \( \beta_{1/\psi} \) alone where \( \Delta \Gamma_s \) is allowed to float is shown in Fig. 7. From this, the 68% CL interval for \( \beta_{1/\psi} \) is \([0.27, 0.59] \cup [0.97, 1.30]\) and the 95% CL interval is \([0.10, 1.42]\). In this projection, the p-value for the Standard Model point is 2.0% or 2.33\( \sigma \). It is worth noting that, although a correct estimate of the combined result,
FIG. 5: Adjusted two-dimensional profile likelihood as confidence contours of $\beta_s^{J/\psi \phi}$ and $\Delta \Gamma_s$ for DØ’s published analysis using 2.8 fb$^{-1}$ of data [3], but allowing strong phases, $\delta_i$ to float when systematic uncertainties are (a) not included, and (b) included. (Note: systematic uncertainties on the two-dimensional profile likelihood implemented after publication, hence are preliminary.) The Standard Model expectation and uncertainty is indicated by the black line.

This is not the most complete and optimal way to combine CDF and DØ data, being a combination of two-dimensional slices of a much higher dimensional likelihood function. An example is the fitted value of $\bar{\tau}_s$, that will not necessarily be the same for CDF and DØ for any given $(\beta_s^{J/\psi \phi}, \Delta \Gamma_s)$ point. Work is currently ongoing towards implementing a combined fit to the CDF and DØ data sets in all dimensions, effectively providing a combined analysis of both data samples, thus yielding the maximum achievable sensitivity.

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FIG. 6: Combined two-dimensional profile likelihood as confidence contours of $\beta_s^{J/\psi\phi}$ and $\Delta\Gamma_s$ for DØ’s published analysis using 2.8 fb$^{-1}$ of data [3] and CDF’s preliminary analysis also using 2.8 fb$^{-1}$ of data [4]. The Standard Model expectation and uncertainty is indicated by the black line. The region allowed in new physics models given by $\Delta\Gamma_s = 2|\Gamma_{12}| \cos \phi_s$ (i.e., $CP$ violation in the interference between mixing and decay amplitudes) is also shown (light green band).

FIG. 7: One-dimensional likelihood profile for $\beta_s^{J/\psi\phi}$ for DØ's published analysis using 2.8 fb$^{-1}$ of data [3] and CDF’s preliminary analysis also using 2.8 fb$^{-1}$ of data [4].