CDF/PHYS/BOTTOM/CDFR/9787
DØ Note 5928-CONF
Version 1.1
July 22, 2009

Combination of DØ and CDF Results on $\Delta\Gamma_s$ and the CP-Violating Phase $\beta_s^{J/\psi\phi}$

CDF/DØ $\Delta\Gamma_s, \beta_s$ Combination Working Group

(Dated: July 22, 2009)

A procedure is described for the combination of DØ and CDF measurements of the B_s^0 width difference, $\Delta\Gamma_s$, and the CP-violating phase, $\phi_s^{J/\psi\phi} = -2\beta_s^{J/\psi\phi}$, between the B_s^0 mixing and decay amplitudes determined via the time-dependent angular analysis of flavor-tagged $B_s^0 \to J/\psi\phi$ decays. Results are presented based on the latest DØ and CDF experimental measurements, each using an integrated luminosity of 2.8 fb^{-1}.

Preliminary Results for Summer 2009 Conferences

I. THEORY AND NOMENCLATURE

For the B_s^0 system, we have the matrix time evolution equation:

$$i\frac{d}{dt}\begin{pmatrix}|B_s^0\rangle\\|\bar{B}_s^0\rangle\end{pmatrix}=\begin{pmatrix}M-\frac{i\Gamma}{2} & M_{12}-\frac{i\Gamma_{12}}{2}\\ M_{12}^*-\frac{i\Gamma_{12}^*}{2} & M-\frac{i\Gamma}{2}\end{pmatrix}\begin{pmatrix}|B_s^0\rangle\\|\bar{B}_s^0\rangle\end{pmatrix}. \tag{1}$$

In the Standard Model, B_s^0-$\bar{B}_s^0$ oscillations are caused by flavor-changing weak interaction box diagrams that induce non-zero off-diagonal elements in the above. The mass eigenstates, defined as the eigenvectors of the above matrix, are different from the flavor eigenstates, with a heavy (H) and light (L) mass eigenstate, respectively:

$$|B_{sH}\rangle = p|B_s^0\rangle - q|\bar{B}_s^0\rangle;\ \ |B_{sL}\rangle = p|B_s^0\rangle + q|\bar{B}_s^0\rangle, \tag{2}$$

with $|p|^2+|q|^2=1$. If CP is conserved in mixing in the B_s^0 system, then $q=p$, and

$$|B_{sH}\rangle = |B_s^{CP-\mathrm{odd}}\rangle;\ \ |B_{sL}\rangle = |B_s^{CP-\mathrm{even}}\rangle. \tag{3}$$

Matrix elements can be extracted experimentally by measuring a mass and width difference between mass eigenstates:

$$\begin{aligned}\Delta m_s &= M_L - M_H &\approx 2|M_{12}|;\\ \Delta\Gamma_s &= \Gamma_L-\Gamma_H &\approx 2|\Gamma_{12}|\cos\phi_s,\end{aligned} \tag{4}$$

where ϕ_s is defined below. Note the sign convention for $\Delta\Gamma_s$ compared to Δm_s. In this convention, the Standard Model (SM) prediction for $\Delta\Gamma_s$ is positive. The current theoretical expectation in the SM is $\Delta\Gamma_s^{\mathrm{SM}} = 2|\Gamma_{12}| = 0.096\pm 0.039\ \mathrm{ps}^{-1}$ [1].

The parameter Γ_{12} is dominated by the quark transition $b\to c\bar{c}s$ in decays into final states common to both B_s^0 ($\bar{b}s$) and $\bar{B}_s^0$ ($b\bar{s}$). Examples of such decays are $B_s^0\to J/\psi\phi$ and $B_s^0\to D_s^{(*)+}D_s^{(*)-}$, as shown in Fig. 1.

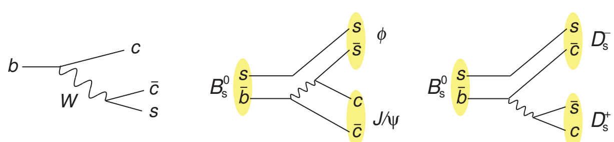

FIG. 1: Example B_s^0 decays giving rise to a non-zero Γ_{12}.

The analogous decay diagram for a width difference in the B_d^0 system substitutes the s quark for a d quark. This decay is Cabibbo suppressed, hence $\Delta\Gamma_d$ is negligible. In the case of $\Delta\Gamma_s$, decays into CP-even final states increase the value of $\Delta\Gamma_s$, while decays into CP-odd final states decrease it.

An average width is defined as $\Gamma_s = (\Gamma_L+\Gamma_H)/2$. The measured lifetime of the B_s^0 will depend on the mix of CP eigenstates involved in its decay. A more fundamental lifetime based on the average width is defined as $\bar{\tau}_s = 1/\Gamma_s$.

A. Weak Phase in B_s^0 Mixing

In general there will be a CP-violating weak phase difference:

$$\phi_s = \arg\left[-M_{12}/\Gamma_{12}\right], \tag{5}$$

between the B_s^0-$\bar{B}_s^0$ amplitude and the amplitudes of the subsequent B_s^0 and $\bar{B}_s^0$ decay to a common final state. In this convention, ϕ_s is defined to fall in the range $[-\pi/2,\pi/2]$. This can affect the observed $\Delta\Gamma_s$ as given above. The SM prediction for this phase is tiny, $\phi_s^{\mathrm{SM}} = 0.004$ [1]; however, new physics in B_s^0 mixing could change this observed phase to

$$\phi_s = \phi_s^{\mathrm{SM}} + \phi_s^{\mathrm{NP}}. \tag{6}$$

The relative phase between the B_s^0 mixing amplitude and that of specific $b\to c\bar{c}s$ quark transitions such as for B_s^0 or $\bar{B}_s^0\to J/\psi\phi$ in the SM is [1, 2]:

$$2\beta_s^{SM} = 2\arg[-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*] \approx 0.04. \tag{7}$$

This angle is analogous to the β angle in the usual CKM unitarity triangle replacing $d \to s$ aside from the negative sign (resulting in a positive angle in the SM). The same additional contribution $\phi_s^{\rm NP}$ due to new physics would show up in this observed phase [1], i.e.:

$$2\beta_s = 2\beta_s^{\rm SM} - \phi_s^{\rm NP}. \tag{8}$$

The current experimental precision does not allow these small CP-violating phases $\phi_s^{\rm SM}$ and $\beta_s^{\rm SM}$ to be resolved, and for large new physics effect, we can approximate $\phi_s \approx -2\beta_s \approx \phi_s^{\rm NP}$, i.e., a significantly large observed phase would indicate new physics.

II. COMBINING EXPERIMENTAL MEASUREMENTS

The most direct and precise experimental results on $\Delta\Gamma_s$ and ϕ_s come from the Tevatron, where reconstructed decays $B_s^0 \to J/\psi\phi$ are separated into CP-even and CP-odd components from fits to angular distributions of J/ψ and ϕ decay products as a function of proper decay time. Including information on the B_s^0 flavor (i.e., B_s^0 or $\bar{B}_s^0$) at production time via flavor tagging improves precision and also resolves the sign ambiguity on the weak phase for a given $\Delta\Gamma_s$. DØ [3] has published such an analysis based on 2.8 fb^{-1} of data, and CDF has a preliminary update [4], also based on 2.8 fb^{-1}, of their previously published result [5]. Both experiments measure the same observed phase angle and report two-dimensional profile likelihoods and confidence-level (CL) contours in the $\phi_s^{J/\psi\phi}$ vs. $\Delta\Gamma_s$ (or $\beta_s^{J/\psi\phi}$ vs. $\Delta\Gamma_s$ where $\phi_s^{J/\psi\phi} = -2\beta_s^{J/\psi\phi}$) plane. Details of the analyses and likelihood fits can be found in the indicated references. The DØ published result [3] imposed weak constraints on the strong phases δ_i, i.e., the phases between polarization amplitudes in the decays, while the CDF analysis [4] allowed these δ_i to float freely in the fit as additional nuisance parameters. In order to produce a coherent constraint-free combination, the DØ likelihood fit was redone by allowing the strong phases to float freely; tables collating the two-dimensional likelihood profile for this case can be found in Ref. [6]. Also, the CL contours of both experiments now explicitly account for the non-Gaussian behavior of the uncertainties (see the following section). Previous HFAG combinations [7] did not include systematic uncertainties in the two-dimensional countours, and also did not correct for coverage using results of toy MC simulations. Our combination of CDF and D0 results is based on the idea of adding the log-likelihoods of independent measurements to produce a combined log-likelihood function. This idea has been extended to the use of profile-likelihood rather than simple likelihoods [7]. These distribution actually depend on systematic parameters, and we account for their presence by picking up the worst-case distribution in each case. All of this is in accordance with the procedure followed by the individual experiments to obtain their results with full account for systematic uncertainties.

A. Correcting for Non-Gaussian Uncertainties

The non-Gaussian behavior of the uncertainties on the fit parameters of the DØ and CDF analyses are different. To combine the two-dimensional likelihood scans in $\beta_s^{J/\psi\phi}$ vs. $\Delta\Gamma_s$ space, the likelihood value at each point needs to be adjusted to put the two scans on the same footing, i.e., to make sure that a particular value of the profile-likelihood ratio in each case has a matching tail probability. In the case of CDF, as indicated in Ref. [4], the two-dimensional profile likelihood with all parameters floating [8] is adjusted to produce a new variable having the chi-squared distribution, since the original variable significantly deviates from the ideal asymptotic behavior. The real profile-likelihood distribution was calculated using 10,000 default Monte Carlo (MC) pseudo-experiments, each having the same sample size as the data, and generated assuming the Standard Model values of $\beta_s^{J/\psi\phi} = 0.02$ and $\Delta\Gamma_s = 0.096$ ps^{-1}. For CDF, the resulting CL contours for the two-dimensional profile likelihood for a given likelihood ratio are shown in Fig. 2(a).

Similarly, in the case of DØ, 2,000 MC pseudo-experiments, generated with the same statistics as for the DØ analysis at the same Standard Model values are used to find the coverage, i.e., the CL value that corresponds to a given profile-likelihood ratio value in the two-dimensional likelihood scans. This curve is shown in Fig. 2(b). While the distribution could in principle depend on the assumed values of β_s and $\Delta\Gamma_s$, extensive MC tests have not shown any such dependency.

B. Including Systematic Uncertainties

A large number of different systematic effects influence the results of these measurements. In order to ensure that the quoted contours actually have the confidence level that is nominally assigned to them (that is, that they have the correct coverage), both experiments include systematics by taking a projection on the physics parameters of the many-dimensional confidence region that includes all possible systematic parameters, that is, they use the worst-case p-value. The use of the profile likelihood as an ordering function in this multidimensional space has almost-optimal efficiency properties and allows to use quite conservative uncertainties on the systematic parameters without weakening significantly the resolution of the measurements [9].

In the case of CDF, as described in Ref. [4], systematic uncertainties are treated by the description of signal models, background models, Δm_s, etc., as nuisance parameters in the fit. CDF generates 5,300 pseudo-experiments conducted in sixteen "alternative universes", in which the nominal values of all nuisance parameters have been picked at random within $\pm 5\sigma$ flat range. The adjustment curves for the profile likelihood for the different cases are shown in Fig. 2(a). The most conservative value of the $(1-CL)$ value (i.e., largest value) for each likelihood ratio value is then selected. Detailed descriptions of systematic uncertainties and their numerical values on one-dimensional estimates of the extracted parameters are found in Ref. [4].

Dominant systematic uncertainties for the DØ analysis are also included as nuisance parameters. The largest effect is the inclusion of the uncertainty on $\Delta m_s = 17.77 \pm 0.12$ ps^{-1}, which is allowed to vary within a Gaussian constraint of 0.12 ps^{-1}. Parameters in the signal and background models and their systematic uncertainties are also treated as nuisance parameters in the fit. Explicit variations to take into account other dominant systematic uncertainties are also used to generate curves of $(1-CL)$ versus likelihood ratio in "alternative universes". As above, the most conservative value is taken, i.e., the largest value of $(1-CL)$ for a given likelihood ratio. 2,000 pseudo-experiments are generated for each of four different acceptance parameterizations, and for an alternative parameterization of the function used to estimate dilution of the flavor tag. Fig. 2(b) shows the resulting adjustment curve including systematic uncertainties.

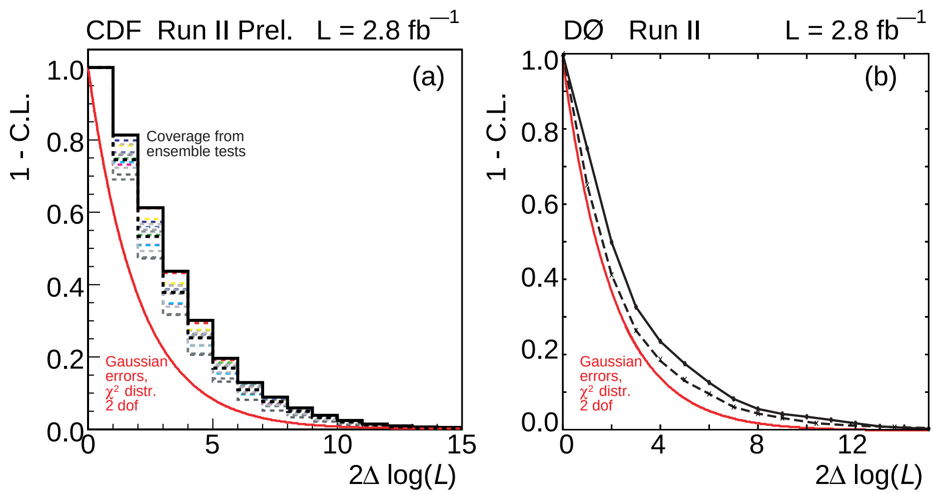

FIG. 2: (a) For the CDF analysis [4], default/nominal correspondence (black dashed histogram) between (1−confidence level) to likelihood ratio for the two-dimensional profile likelihood of $\beta_s^{J/\psi\phi}$ and $\Delta\Gamma_s$ compared to that expected for Gaussian uncertainties (red line). Analogous relations for all of the "alternate universes" described in the text (colored histograms), and the conservative choice used to adjust the likelihoods including systematics (black solid histogram). (b) For the DØ analysis [3], default/nominal correspondence (dashed line) and the conservative choice of "alternate universes" for a given $\Delta \log L$ including systematics (solid line).

C. Adjusted Two-Dimensional Profile Likelihoods

Using the curves above, the likelihood values in the $\beta^{J/\psi\phi}$ vs. $\Delta\Gamma_s$ scans from CDF and DØ are adjusted to correspond to those expected for Gaussian errors corresponding to a given CL. An example is shown in Fig. 3 where ensemble coverage tests for CDF indicate that a value of $2\Delta \log L = 8.42$ corresponds to 95% CL. In the two-dimensional likelihood scans, a value of 8.42 is then replaced with 5.99, the value of $2\Delta \log L$ expected for Gaussian errors (i.e., χ^2 with two degrees of freedom). Although CDF and DØ start with different coverage, the use of the different curves ensures that, after adjustment, a $\Delta \log L$ value in each case has the same CL value.

We have used a parameterization of Fig. 2(a) to adjust the CDF likelihood scans to give the CL contours shown in Fig. 4. The comparison of this result with the original calculation, shown in Fig. 11 of Ref. [4] shows good agreement

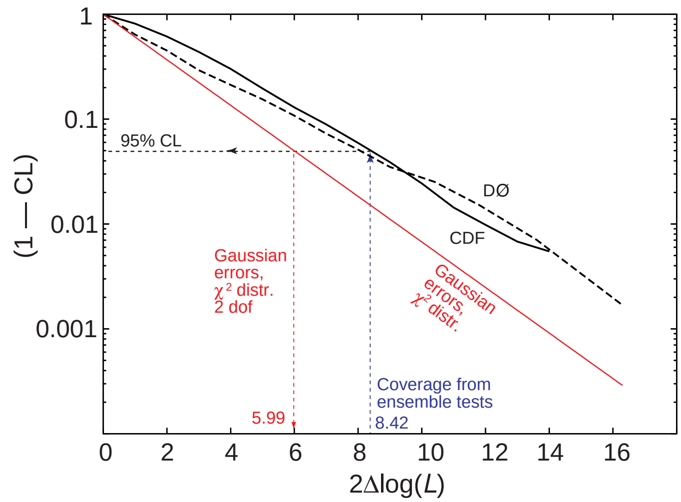

FIG. 3: An example of adjusting the likelihood ratio value in each scan to correspond to expected Gaussian uncertainties according to relevant coverage.

and gives confidence in the correctness of the present procedure.

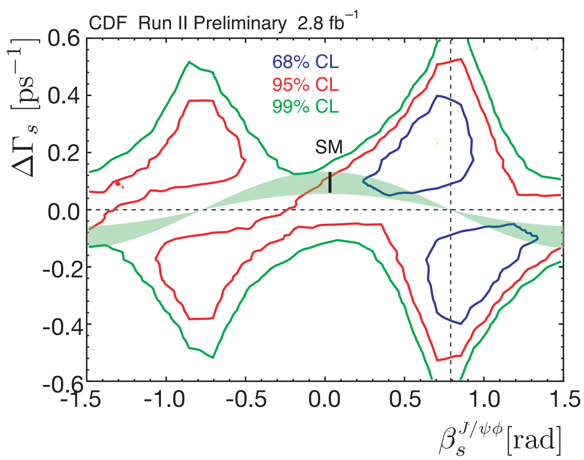

FIG. 4: Adjusted two-dimensional profile likelihood as confidence contours of $\beta_s^{J/\psi\phi}$ and $\Delta\Gamma_s$ for CDF's analysis using 2.8 fb^{-1} of data (to be compared to Fig. 11 of Ref. [4]). The Standard Model expectation and uncertainty is indicated by the black line. The region allowed in new physics models given by $\Delta\Gamma_s = 2|\Gamma_{12}|\cos\phi_s$ (i.e., CP violation in the interference between mixing and decay amplitudes) is also shown (light green band).

Figure 5 shows the same adjusted CL contours for DØ, both with and without systematic uncertainties included, as described above. Note that these results allow the strong phases δ_i to float, and are hence different from those reported in the DØ publication [3] where weak constraints were imposed on δ_i.

D. Combined Result

The CDF and DØ two-dimensional adjusted log likelihoods are then added for a combined result shown in Fig. 6. For this combination, the p-value at the Standard Model central point is 3.4% or 2.12σ (this takes as reference the central predicted value of $\Delta\Gamma_s^{\rm SM}$, without accounting for the associated theoretical uncertainty. If the lower bound given by the theoretical uncertainty of $\Delta\Gamma_s = 0.096 \pm 0.039$ is instead taken, the p-value is found to be 4.2% or 2.03σ).

The likelihood profile for $\beta_s^{J/\psi\phi}$ alone where $\Delta\Gamma_s$ is allowed to float is shown in Fig. 7. From this, the 68% CL interval for $\beta_s^{J/\psi\phi}$ is $[0.27, 0.59] \cup [0.97, 1.30]$ and the 95% CL interval is $[0.10, 1.42]$. In this projection, the p-value for the Standard Model point is 2.0% or 2.33σ. It is worth noting that, although a correct estimate of the combined result,

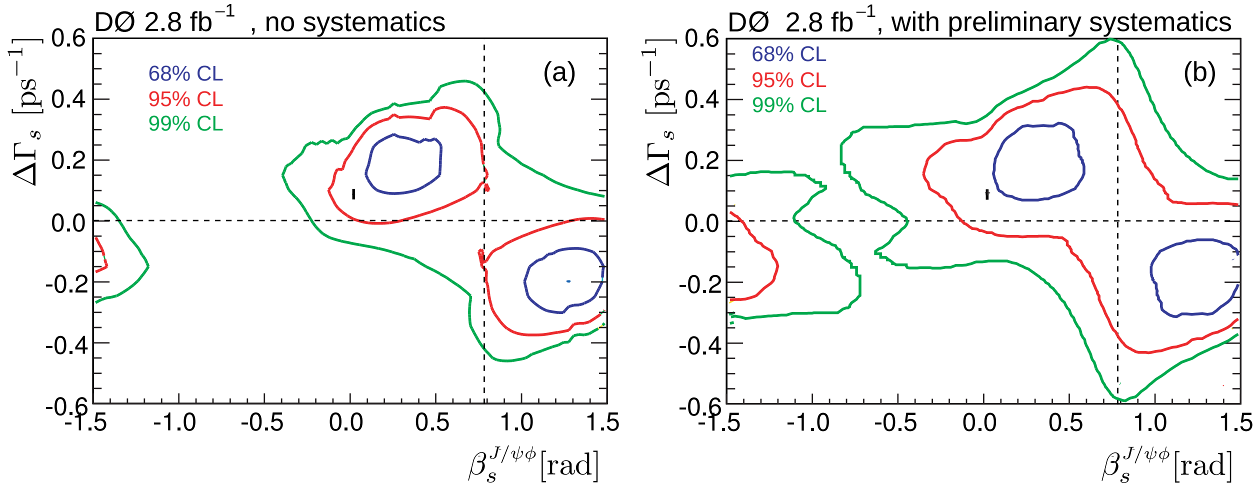

FIG. 5: Adjusted two-dimensional profile likelihood as confidence contours of $\beta_s^{J/\psi\phi}$ and $\Delta\Gamma_s$ for DØ's published analysis using 2.8 fb^{-1} of data [3], but allowing strong phases, δ_i to float when systematic uncertainties are (a) not included, and (b) included. (Note: systematic uncertainties on the two-dimensional profile likelihood implemented after publication, hence are preliminary.) The Standard Model expectation and uncertainty is indicated by the black line.

this is not the most complete and optimal way to combine CDF and D0 data, being a combination of two-dimensional slices of a much higher dimensional likelihood function. An example is the fitted value of $\bar{\tau}_s$, that will not necessarily be the same for CDF and DØ for any given $(\beta_s^{J/\psi\phi}, \Delta\Gamma_s)$ point. Work is currently ongoing towards implementing a combined fit to the CDF and DØ data sets in all dimensions, effectively providing a combined analysis of both data samples, thus yielding the maximum achievable sensitivity.

Acknowledgments

The DØ Collaboration thanks the staffs at Fermilab and collaborating institutions, and acknowledge support from the DOE and NSF (USA); CEA and CNRS/IN2P3 (France); FASI, Rosatom and RFBR (Russia); CAPES, CNPq, FAPERJ, FAPESP and FUNDUNESP (Brazil); DAE and DST (India); Colciencias (Colombia); CONACyT (Mexico); KRF and KOSEF (Korea); CONICET and UBACyT (Argentina); FOM (The Netherlands); PPARC (United Kingdom); MSMT (Czech Republic); CRC Program, CFI, NSERC and WestGrid Project (Canada); BMBF and DFG (Germany); SFI (Ireland); Research Corporation, Alexander von Humboldt Foundation, and the Marie Curie Program.

The CDF Collaboration thanks the Fermilab staff and the technical staffs of the participating institutions for their vital contributions. This work was supported by the U.S. Department of Energy and National Science Foundation; the Italian Istituto Nazionale di Fisica Nucleare; the Ministry of Education, Culture, Sports, Science and Technology of Japan; the Natural Sciences and Engineering Research Council of Canada; the National Science Council of the Republic of China; the Swiss National Science Foundation; the A.P. Sloan Foundation; the Bundesministerium für Bildung und Forschung, Germany; the Korean Science and Engineering Foundation and the Korean Research Foundation; the Science and Technology Facilities Council and the Royal Society, UK; the Institut National de Physique Nucleaire et Physique des Particules/CNRS; the Russian Foundation for Basic Research; the Ministerio de Ciencia e Innovación, and Programa Consolider-Ingenio 2010, Spain; the Slovak R&D Agency; and the Academy of Finland.

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[1] A. Lenz and U. Nierste, J. High Energy Physics **06**, 072 (2007) [arXiV:hep-ph/0612167].

[2] As determined before direct experimental inputs of $2\beta_s^{J/\psi\phi} = -\phi_s^{J/\psi\phi}$: CKMfitter Group (J. Charles *et al.*), Eur. Phys. J. C **41**, 1 (2005) [hep-ph/0406184], updated results and plots available at: `http://ckmfitter.in2p3.fr`, Summer 2007 result; M. Bona *et al.* [UTfit Collaboration], arXiv:hep-ph/0606167.

[3] V.M. Abazov *et al.* (DØ Collaboration) Phys. Rev. Lett. **101**, 241801 (2008) [arXiv.org:0802.2255].

[4] T. Aaltonen *et al.* (CDF Collaboration) CDF Public Note CDF/ANAL/BOTTOM/PUBLIC/9458, August 7, 2008.

[5] T. Aaltonen *et al.* (CDF Collaboration) Phys. Rev. Lett. **100**, 161802 (2008) [arXiv:0712.2397v1].

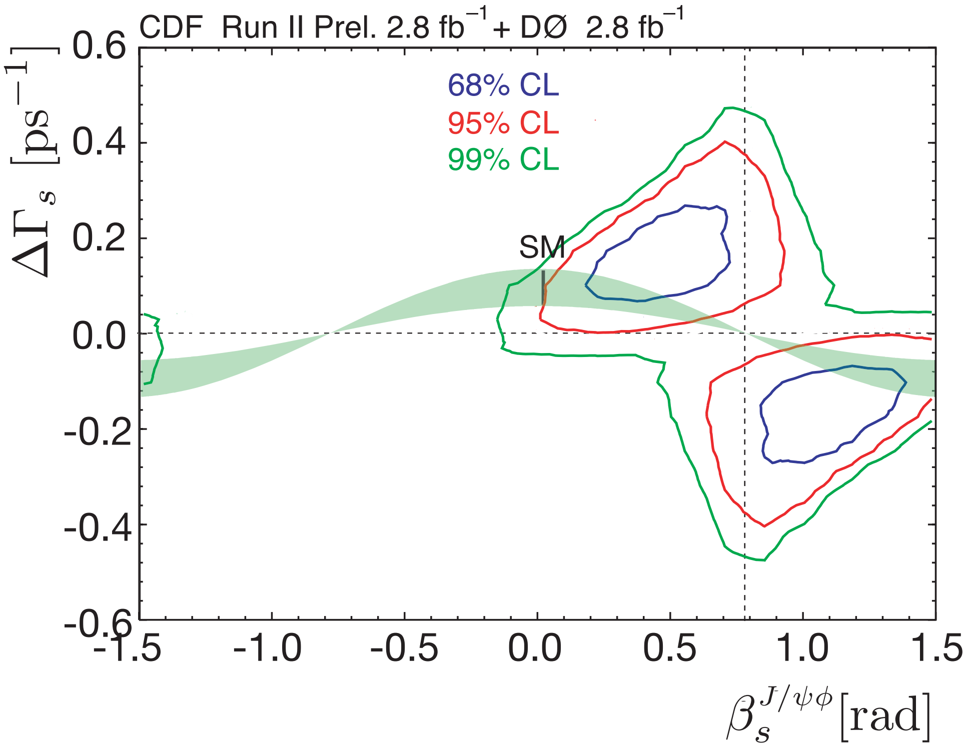

FIG. 6: Combined two-dimensional profile likelihood as confidence contours of $\beta_s^{J/\psi\phi}$ and $\Delta\Gamma_s$ for DØ's published analysis using 2.8 fb^{-1} of data [3] and CDF's preliminary analysis also using 2.8 fb^{-1} of data [4]. The Standard Model expectation and uncertainty is indicated by the black line. The region allowed in new physics models given by $\Delta\Gamma_s = 2|\Gamma_{12}|\cos\phi_s$ (i.e., CP violation in the interference between mixing and decay amplitudes) is also shown (light green band).

[6] `http://www-d0.fnal.gov/Run2Physics/WWW/results/final/B/B08A/likelihoods/index.html`
[7] E. Barberio *et al.* [Heavy Flavor Averaging Group], arXiv:0808.1297 [hep-ex].
[8] `http://www-cdf.fnal.gov/physics/new/bottom/071214.blessed-tagged_BsJPsiPhi/Combination_Info/Combination_main.html`
[9] K.S. Cranmer, PHYSTAT-2003-WEMT004, proceedings of PHYSTAT 2003, Statistical problems in particle physics, astrophysics and cosmology [arXiv:physics/0310108]; G. Punzi, in Oxford 2005, Statistical problems in particle physics, astrophysics and cosmology, 88-92 [arXiv:physics/0511202].

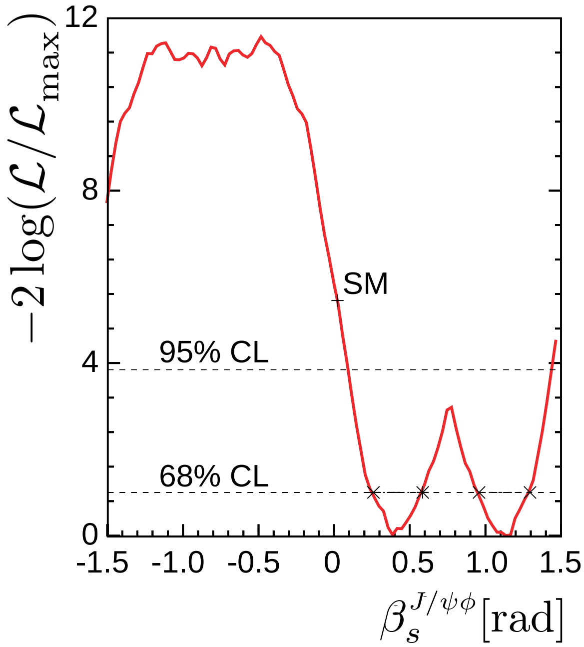

FIG. 7: One-dimensional likelihood profile for $\beta_s^{J/\psi\phi}$ for DØ's published analysis using 2.8 fb^{-1} of data [3] and CDF's preliminary analysis also using 2.8 fb^{-1} of data [4].