A Measurement of the Ratio \( \frac{\sigma(B_c^+)*BR(B_c^+\rightarrow J/\psi \mu^+\nu)}{\sigma(B^+)*BR(B^+\rightarrow J/\psi K^+)} \)

Using the Complete CDF Dataset

The CDF Collaboration

URL http://www-cdf.fnal.gov

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This note describes the \( B_c^+ \) production cross section times branching ratio measurement of the decay mode \( B_c^+ \rightarrow J/\psi \mu^+\nu \) relative to the \( B^+ \rightarrow J/\psi K^+ \) decay. Measurements are based on the complete CDF run II data set that for this measurement consists of an integrated luminosity of 8.7 fb\(^{-1}\). We select a sample of 1370 events in which a \( J/\psi \) candidate is matched with a high quality third muon and the \( J/\psi \) decays to two muons for \( p_T(B_c^+) > 6 \text{ GeV/c} \). The background contributions from misidentified \( J/\psi \), misidentified muons, and from the different \( b \)-hadrons are estimated using data and \textsc{pythia} samples. The total background consists of 630.5\( \pm \)14.2 events. We estimate the ratio of the production cross section times branching ratio of \( B_c^+ \rightarrow J/\psi \mu^+\nu \) relative to \( B^+ \rightarrow J/\psi K^+ \) for \( p_T(B_c^+) > 6 \text{ GeV/c} \) and \( |y| < 0.6 \) as \( 0.211 \pm 0.012 \) (stat.) \( ^{+0.021}_{-0.020} \) (syst.).

\textit{Preliminary Results for Spring 2014 Conferences}
I. INTRODUCTION

The $B_c^+$ meson is the most massive of the bottom-flavored mesons, apart from the $b\bar{b}$ charmonia, with a ground state that consists of a $b$ and a $c$ quark [1]. The $B_c^+$ meson was discovered by CDF in Run I using the $B_c^+ \rightarrow J/\psi \mu^+ X$ decay modes [2]. The ratio of the $B_c^+$ production cross section times semileptonic branching ratio in the single muon and electron channels to the production cross section times branching ratio for $B^+ \rightarrow J/\psi K^+$ using an integrated luminosity of 360 pb$^{-1}$ of Run II data was presented in Refs. [3] and [4], respectively. A similar measurement in the muon channel using an integrated luminosity of 1 fb$^{-1}$ of Run II data was presented in [5]. Measurements of the $B_c^+$ lifetime in the $B_c^+ \rightarrow J/\psi \mu^+ X$ and $B_c^+ \rightarrow J/\psi e^+ X$ decay channels using an integrated luminosity of 1 fb$^{-1}$ of Run II data was presented in [6]. In this note we update the \( \frac{\sigma(B_c^+) \cdot BR(B_c^+ \rightarrow J/\psi \mu^+ \nu)}{\sigma(B^+) \cdot BR(B^+ \rightarrow J/\psi K^+)} \) ratio to include the full CDF run II dataset, which for this measurement has an integrated luminosity of $(8.7 \pm 0.5) \text{fb}^{-1}$.

II. EVENT SELECTION

The datasets used in this analysis are collected with the $J/\psi$ di-muon trigger and consist of an integrated luminosity of $8.7 \text{fb}^{-1}$. Our selection requirements closely follow those of the earlier $B_c^+$ semileptonic cross-section [5] and lifetime measurements [6], in which we search for $J/\psi$ particles reconstructed through the $\mu^+ \mu^-$ decay channel that are matched with a third track associated with the $J/\psi$ vertex. The third track might be:

- the muon in the $B_c^+ \rightarrow J/\psi \mu^+ X$ decays, or
- the kaon in the $B^+ \rightarrow J/\psi K^+$ sample, or
- a $\pi^+$, $K^+$ or $p$ for the misidentified muon background calculation

Since the luminosity increases over the time of taking data, we expect that our $B_c^+$ and $B^+$ yields will vary versus the calendar time due to the $J/\psi$ dimuon trigger prescale modification. The question arises as to how stable the ratio of the $B_c^+$ to $B^+$ yields are as a function of time. Figure 1 shows the ratio $N(B_c^+)/N(B^+)$ shown in Fig. 1 yields the slope value $0.00003 \pm 0.00032$ consistent with zero.

The invariant mass distributions of the $J/\psi \mu^+$ and $J/\psi K^+$ event candidates are shown in Fig. 2. There are 1,370 $B_c^+ \rightarrow J/\psi \mu^+ \nu$ event candidates within a $4 - 6 \text{GeV}/c^2$ signal mass window and 14,338 $\pm 125$ events from $B^+ \rightarrow J/\psi K^+$ decays. The fit function for the $J/\psi K^+$ invariant mass
distribution consist of: a double Gaussian for \(B^+ \rightarrow J/\psi K^+\) decays, a mass template for the Cabibbo suppressed \(B^+ \rightarrow J/\psi \pi^+\) contribution within the mass range 5.28-5.4 GeV/\(c^2\) based on Monte Carlo simulation, and a second order polynomial for the continuum background. The Cabibbo suppressed \(B^+ \rightarrow J/\psi \pi^+\) contribution is fixed to 3.83% of \(N(B^+ \rightarrow J/\psi K^+)\) following Ref. [7]. In order to determine the number of \(B_c^+\) signal events, we must calculate the background contributions to the sample and subtract them from the number of candidates.

III. \(B_c^+\) BACKGROUND

We consider the following background sources to the semileptonic \(B_c^+\) decays:

- Misidentified \(J/\psi\)
- Misidentified third muon
- \(b\bar{b}\) background
- Contributions from other decay modes

A. Misidentified \(J/\psi\) background

The number of misidentified \(J/\psi\) plus a third muon is estimated using the di-muons from the sidebands of the \(J/\psi\) invariant mass distribution. Our signal di-muon mass region is within \(\pm 0.05\) GeV/\(c^2\) around the PDG mean value of the \(J/\psi\) mass, \(m_{J/\psi}\). The selected sideband regions are: \(|m_{J/\psi} - 0.150\) GeV/\(c^2| < 0.05\) GeV/\(c^2\) and \(|m_{J/\psi} + 0.150\) GeV/\(c^2| < 0.05\) GeV/\(c^2\). The resulting invariant mass distributions of the \(J/\psi_{side}\mu^+\) system can be seen in Fig. 3. We find 11.5\(\pm\)2.4 events below 4 GeV/\(c^2\), 96.5\(\pm\)6.9 events within the 4 - 6 GeV/\(c^2\) signal mass window and 25\(\pm\)3.5 events > 6 GeV/\(c^2\).

B. Misidentified muon background

The misidentified muon contribution to the \(B_c^+\) background is calculated using the following steps:
Reconstruct the $J/\psi$ track system to use as an input sample for the misidentified muon calculation.

Determine the kaon and pion decay-in-flight and punch-through probabilities from a $D^*$ sample having the decay chain $D^* \rightarrow D^0 \pi^+$, $D^0 \rightarrow K^- \pi^+$ using the two track trigger datasets.

Determine the fraction of the events outside of the $D^0$ mass peak using Monte Carlo simulation.

Determine the pion, kaon, and proton fractions in the $J/\psi$ track data using $dE/dx$ and time-of-flight tools.

The invariant mass distribution of the $J/\psi$ track system is shown in Fig. 4. In the fit procedure the muon mass was assigned to the third track. For each third track in this sample we assign a weight $W$ to calculate the misidentified muon background:

$$W = \epsilon_\pi \cdot (1 + F_{x,\pi}^{\text{out}}) \cdot F_x + \epsilon_K \cdot (1 + \alpha F_{K}^{\text{out}}) \cdot F_K + \epsilon_p \cdot F_p,$$

where $\epsilon_{(\pi,K,p)}$ is the probability for a given particle type to be misidentified as a muon, $F_{x,\pi}^{\text{out}}$ is the fraction of misidentified events outside of the $D^0$ mass peak for a given particle type, $\alpha=1$ for $K^-$, $\alpha=\epsilon_K^-/\epsilon_K^+$ for $K^+$, and $F(\pi,K,p)$ is the fraction of a given particle type within the $J/\psi$ track sample. The weight $W$ is applied on a track-by-track basis to calculate the weighted $J/\psi$ track mass distribution.

The misidentification probabilities derived from data assume that the $\pi/K$ track, even after a possible kink in the $\pi(K) \rightarrow \mu + X$ decay-in-flight, will allow the reconstructed $D^0$ mass value to remain under the peak. The fraction of the events outside of the $D^0$ mass peak are accounted for with a special simulation study.

The proton probability to make a misidentified muon via the punch-through process is estimated using the proton tracks in $\Lambda \rightarrow p\pi$ decays. We are able to establish only an upper limit for this process since it is so rare.

The muon misidentification probabilities and the fraction of the muon associated events outside of the $D^0$ mass peak as a function of hadron $p_T$ are shown in Fig. 5.

The $\pi$ fraction $F_\pi$ in the $J/\psi$ track sample is determined using a $dE/dx$ method. The $K$ and $p$ energy losses in the COT for our $p_T > 3$ GeV/c range are insufficient to separate them in the $dE/dx$ distribution. Consequently, the $K$ and $p$ fractions $F_{K+p}$ are combined. The proton fraction $F_p$ is estimated only within the $2 - 3$ GeV/c momentum region using a simultaneous fit of the $dE/dx$ and time-of-flight data. For higher momenta ($p > 3$ GeV/c) we follow the predictions for $F_p$ from PYTHIA.

In order to estimate the $\pi$ and $K+p$ fractions, we initially intended to fit the $dE/dx$ distribution with two Gaussians, where the mean for the pion’s is zero and the mean for $K+p$ is about -1.5.
FIG. 4: The invariant mass distribution of the $J/\psi$ track system. This sample is an input to the misidentified muon calculation.

FIG. 5: The $\pi^\pm$ and $K^\pm$ muon misidentification probabilities (left) and the fraction of the muon associated events outside of the $D^0$ mass peak (right) as a function of hadron $p_T$.

However, we are faced with the challenge that the sum of two single Gaussians does not well describe the pion’s positive tail, or the kaon’s either. As another option we considered the use of a Gamma distribution. Finally, the the $dE/dx$ data were fitted with following formula:

$$N_{ev} = N_{fit} \times [F_{\pi} \times \text{GPDF}(\gamma_{\pi}, \mu_{\pi}, \beta_{\pi}) + (1 - F_{\pi}) \times \text{GPDF}(\gamma_{K+p}, \mu_{K+p}, \beta_{K+p})],$$

where $N_{fit}$ is the number of events, $F_{\pi}$ is the pion fraction, $F_{K+p} = 1 - F_{\pi}$, and GPDF is the probability density function of the Gamma distribution. There are only two free parameters in the fit: $N_{fit}$ and $F_{\pi}$. The rest of the quantities are fixed: $\gamma_{\pi} = 6.8$, $\beta_{\pi} = 1.11$, and $\mu_{\pi} = -\gamma_{\pi} \beta_{\pi}$ for pions; $\gamma_{K+p} = 23$, $\beta_{K+p} = \frac{1.15}{\sqrt{K+p}}$, and $\mu_{K+p} = k\text{pMean} - \gamma_{K+p} \beta_{K+p}$ for $K + p$, where $k\text{pMean}$ is the $K + p$ momentum dependent position with respect to the pion and changes slowly for each specific momentum bin. These settings will lead to the mean for the pion to be zero, the mean for the $K + p$ to be $k\text{pMean}$, and widths of 1.11 and 1.15 for pions and $K + p$, respectively. Figure 6 shows the distributions of the $dE/dx$ data for the positively charged tracks in various momentum ranges. The Gamma distribution for kaon’s alone is well calibrated by using $dE/dx$ data from an independent sample of $B^+ \rightarrow J/\psi K^+$ events.

Figure 7 shows the $F_{\pi}$, $F_{K}$, and $F_{p}$ fractions for the positively (left) and negatively (right) charged third tracks for the $J/\psi$ track system for track momenta $> 3$ GeV/c.
FIG. 6: The distributions of the \( dE/dx \) data for the positively charged tracks in various momentum ranges. The fitted function consists of two Gamma distributions, one for pions and a second one for the combined \( K + p \). The parameters are: \( \gamma_p = 6.8, \beta_p = 1.11, \mu_p = -\gamma_p \beta_p \) for pions; \( \gamma_{K+p} = 23, \beta_{K+p} = \frac{1.15}{\sqrt{K+p}} \) and \( \mu_{K+p} = Kp \text{Mean} - \gamma_{K+p} \beta_{K+p} \) for \( K + p \), where \( Kp \text{Mean} \) is the \( K + p \) momentum dependent position with respect to the pion and changes slowly for each specific momentum bin. The fit returns the \( F_x \) for pions, where the fraction \( F_{K+p} \) is defined as 1 - \( F_x \).

For the misidentified muon uncertainties the following options were considered to estimate the various components of the systematic uncertainty:

- For the \( \pi/K \) muon misidentification probabilities, the daughter distributions associated with a high quality muon are fitted with the double Gaussian templates derived from the non-muon sample and compared with the results from the single Gaussian fits.
• For the fraction of the muon associated events outside of the $D^0$ mass peak, the Monte Carlo mass distributions fitted with single Gaussians forced to have the same widths as the experimental data and were compared with single Guassian fits where the Monte Carlo simulation was allowed to set the widths of the fits. The resulting differences are used to estimate the systematic uncertainty for this part of the mididentified muon calculation.

• For the particle fractions ($\pi$ and $K+p$) in the $J/\psi$track system, fits of the $dE/dx$ data with a single Gaussian function are compared to the fits with Gamma distributions to determine the systematic uncertainty.

• For the proton fractions, the data normalized PYTHIA simulation is bounded conservatively from above and below to determine the systematic uncertainty. From below we follow the slope of the Pythia simulation in the region 3-4.2 GeV/c assuming that $F_p = 0$ beyond 5.5 GeV/c. From above we assume a straight line connecting the lowest and highest momentum points in the Pythia simulation (see the dashed lines in Fig. 8).

![CDF Run II Monte Carlo](image_url)

FIG. 8: The $F_p$ fractions for the charged third tracks of the $J/\psi$track system. The dashed lines are used for the systematic uncertainty studies.

Both the misidentified $J/\psi$ and misidentified muon backgrounds have one common subsample: misidentified $J/\psi$ with a misidentified third $\mu$. To avoid double counting, it is necessary to subtract it only once. To calculate the doubly fake background, we apply the same weighting procedure to the sideband $J/\psi$ plus track sample using the misidentification probabilities and the $\pi$, $K$, and $p$ fractions.

The misidentified muon and “double fake” backgrounds versus the invariant mass value of the $J/\psi$track system and the associated systematic uncertainties are shown in Fig. 9. The numerical results are given in Table I.

<table>
<thead>
<tr>
<th>“Misidentified” backgrounds</th>
<th>$M &lt; 4 \text{ GeV}/c^2$</th>
<th>$4 &lt; M &lt; 6 \text{ GeV}/c^2$</th>
<th>$M &gt; 6 \text{ GeV}/c^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Misidentified muon</td>
<td>$86.7_{-2.4}^{+2.7}$</td>
<td>$344.4_{-10.6}^{+9.6}$</td>
<td>$32.1_{-1.3}^{+1.9}$</td>
</tr>
<tr>
<td>Double fake</td>
<td>$5.1_{-0.2}^{+0.1}$</td>
<td>$19.0_{-0.9}^{+1.0}$</td>
<td>$5.2_{-0.3}^{+0.1}$</td>
</tr>
</tbody>
</table>

TABLE I: The misidentified muon and “double fake” backgrounds within the signal and sideband mass regions and the associated systematic uncertainties.
C. The $b\bar{b}$ background

We must also account for cases when the $J/\psi$ is produced by a $b$ hadron and the third muon is produced by a $\bar{b}$ hadron (or vice versa) in the same event. The basic procedure for the calculation is described in the semileptonic $B^+_c$ lifetime measurement [6]. However, we updated many steps of this calculation. Therefore, we describe the full procedure here. The $b\bar{b}$ background calculation is based on a PYTHIA Monte Carlo simulation. The question is what fractions of QCD production processes “flavor creation” (FC), “flavor excitation” (FE), and “gluon splitting” (GS) should be used in determining the $b\bar{b}$ background for $B^+_c \rightarrow J/\psi \mu^+ \nu$ decays. A lengthy study led to the conclusion that a more experimentally driven choice is to constrain the QCD fractions by requiring them to give a good description of the distributions for the unvertexed $J/\psi$ pairs in the experimental data. Fig. 10 shows the $\Delta\phi$ distributions for the unvertexed $J/\psi \mu^+$ pairs in the experimental data. Fig. 10 shows the $\Delta\phi$ distributions for the unvertexed $J/\psi \mu^+$ pairs in the experimental data. The plots represent the complete CDF dataset.

Using the PYTHIA generator, we simulate the $J/\psi$ production from one $b$ hadron and force the other $\bar{b}$ hadron in the same event to decay semileptonically via a muon. The $b$ hadron decaying to the $J/\psi$ includes the $J/\psi K^+$ decay chain.

The fit of the $\Delta\phi$ distributions shown in the Fig. 10 (right) with all three QCD production mechanisms lead to the result where the experimental data rejected the “flavor excitation” contribution.
The fit returned for the associated FE scale factor the value $S_{FE} = -0.11 \pm 0.10$. Therefore, we fit the $\Delta \phi$ distribution determined from data using only the FC and GS contributions. The predicted number of $b\bar{b}$ events, $N_{bb}^{pred}$, for a given $\Delta \phi$ bin is given by the formula:

$$N_{bb}^{pred} = C_{norm}(S_{FC}N_{bb}^{FC} + S_{GS}N_{bb}^{GS}) \times \frac{N_{B+}^{data}}{S_{FC}N_{B+}^{FC} + S_{GS}N_{B+}^{GS}};$$

(3)

Here, $C_{norm} = 0.76 \pm 0.07$ is a parameter that accounts for the uncertainties in the simulation of $J/\psi$ production in $B$ decays relative to the $B^+ \rightarrow J/\psi K^+$ branching fraction. In the fit the value of $C_{norm}$ is constrained by it’s uncertainty. The parameters $S_{FC}$ and $S_{GS}$ are the scale factors for the FC and GS QCD processes in PYTHIA. The fit procedure allows the scale factors to float. $N_{bb}^{FC}$ and $N_{bb}^{GS}$ are the number of PYTHIA events in a given $\Delta \phi$ bin from FC and GS, respectively. $N_{B+}^{data}$ is the total number of $B^+ \rightarrow J/\psi K^+$ decays in the data shown in Fig. 2 (right). $N_{B+}^{FC}$ and $N_{B+}^{GS}$ are the numbers of $B^+ \rightarrow J/\psi K^+$ decays produced by the two QCD processes in PYTHIA. This last term provides for the normalization of the PYTHIA sample to data.

Figure 11 shows the result of the fit to the $\Delta \phi$ distribution presented in Fig. 10 (right) with the two QCD production mechanisms FC and GS. The results of the fits are shown in Table II.

\begin{table}[h]
\centering
\begin{tabular}{l l l}
\hline
$C_{norm}$ & $0.76 \pm 0.07$  \\
$S_{FC}$ & $2-S_{GS}$  \\
$S_{GS}$ & $1.02 \pm 0.03$  \\
$\chi^2/ndf$ & 39.6/34  \\
\hline
\end{tabular}
\caption{The fit results for the data shown in Fig. 11. The expected value for $C_{norm}$ is $0.76 \pm 0.07$. The fit parameters are used to calculate the central value of the $b\bar{b}$ background.}
\end{table}

The final results for the $b\bar{b}$ background in the signal region are summarized in Table III. The uncertainty in the $b\bar{b}$ background is due to several sources. There are statistical uncertainties in the four PYTHIA simulated samples: $N_{bb}^{FC}$, $N_{bb}^{GS}$, $N_{B+}^{FC}$, and $N_{B+}^{GS}$. There is the statistical uncertainty in the determination of the $B^+ \rightarrow J/\psi K^+$ sample in the experimental data. Finally there are uncertainties in the parameters $C_{norm}$, $S_{FC}$ and $S_{GS}$ that are determined by the fit to the $\Delta \phi$ distribution in the unvertexed $J/\psi \mu^+$ sample plus additional correlations between all these quantities that are introduced by the fitting procedure. The resulting invariant mass distribution of the $b\bar{b}$ background is shown in Fig. 12.
Flavor creation (FC) 12.9 ± 4.1
Flavor excitation (FE) 0
Gluon splitting (GS) 165.7 ± 11.7
Total 178.6 ± 12.4

TABLE III: The expected number of $b\bar{b}$ background events in the signal region. Uncertainties are statistical due to the sizes of the three muon systems, the number of $B^+$, and the statistical uncertainty of the scale factors.

<table>
<thead>
<tr>
<th>Backgrounds</th>
<th>$M &lt; 4$ GeV/c²</th>
<th>$4 &lt; M &lt; 6$ GeV/c²</th>
<th>$M &gt; 6$ GeV/c²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Misidentified $J/\psi$</td>
<td>11.5 ± 2.4</td>
<td>96.5 ± 6.9</td>
<td>25 ± 3.5</td>
</tr>
<tr>
<td>Misidentified muon</td>
<td>96.7</td>
<td>344.4</td>
<td>32.1</td>
</tr>
<tr>
<td>Double fake</td>
<td>-5.1</td>
<td>-19.0</td>
<td>-5.2</td>
</tr>
<tr>
<td>$b\bar{b}$ background</td>
<td>12.4 ± 2.4</td>
<td>178.6 ± 12.4</td>
<td>110.4 ± 10.7</td>
</tr>
<tr>
<td>Sum of Misid.+$b\bar{b}$</td>
<td>105.5 ± 3.4</td>
<td>600.5 ± 14.2</td>
<td>162.3 ± 11.3</td>
</tr>
</tbody>
</table>

TABLE IV: Summary of the misidentified $J/\psi$, misidentified muon, double fake, and $b\bar{b}$ backgrounds. The doubly misidentified contribution is subtracted to avoid double counting. All uncertainties are statistical.
The excess of the $B_c^+ \rightarrow J/\psi\mu^+X$ candidates over the backgrounds described above, $N_{obs}$, is presented in Table V. The top line in Table V represents the reconstructed number of $B_c^+ \rightarrow J/\psi\mu^+X$ candidates shown in Fig. 2 (left). The quantity $N_{obs}$ is used to calculate the final $B_c^+ \rightarrow J/\psi\mu^+\nu$ yield.

### IV. CONTRIBUTIONS FROM OTHER DECAY MODES

After subtracting the backgrounds, the basic tri-muon sample that is reconstructed from data may still have contributions from other $B_c^+$ decay modes. For example, a $B_c^+$ might decay into $\psi(2S) + \mu^+ + \nu$ followed by $\psi(2S)$ decay into $J/\psi + \ldots$. Another example is a $B_c^+$ decay into $J/\psi + \tau^+ + \nu$ followed by the $\tau$ decay into a muon. The probability of events from these decays to survive our selection requirements is small, but non-zero.

In order to determine contributions from other decay modes, we generate $2.2 \times 10^7 B_c^+ \rightarrow J/\psi\mu^+X$ decays associated with 11 other decay modes that may end-up in the tri-muon system. The branching fractions of these decay modes are taken from theoretical predictions [8].

Our method uses the number of observed $B_c^+$ candidates, $N_{obs}$, in the data after all the other backgrounds have been subtracted except for the other decay modes as shown in Table V. The number of $B_c^+ \rightarrow J/\psi\mu^+\nu$ events in the data is given by $N_{B_c^+} = N_{obs} - N_{other}$ where $N_{other}$ is the number due to other decay modes. Study shows that the surviving fractions of the other decay modes combined with respect to the $B_c^+ \rightarrow J/\psi\mu^+\nu$ channel are: 9.9% in the $3 - 4 \text{ GeV}/c^2$ mass range, 3.9% in the signal mass range, and no contribution in the $> 6 \text{ GeV}/c^2$ mass range. Finally, the other decays are: $769.5 \times (1 - 0.961) = 30.0$ in the signal mass range and $26.5 \times (1 - 0.901) = 2.6$ events in the $3 - 4 \text{ GeV}/c^2$ mass range.

Note that the surviving fractions of the other decay modes based on the prediction [9] are: 7.5% in the $3 - 4 \text{ GeV}/c^2$ mass range, 2.6% in the signal mass range, and no contribution in the $> 6 \text{ GeV}/c^2$ mass range. Then, the Ivanov prediction for other decay modes is $769.5 \times (1 - 0.974) = 20.0$ events.

### V. THE $B_C^+$ EXCESS

The $N_{obs}$ from Table V is used as an input to calculate the other decay modes contribution. The other decay modes and the $B_C^+$ excess, $N_{B_C^+}$, is shown in Table VI.

The invariant mass distribution of the $J/\psi\mu^+$ candidate events is shown in Fig. 13 with Monte Carlo simulated signal and the backgrounds superimposed. “Misid. Muon” stands for the misidentified muon background with the doubly misidentified background subtracted, while “Other modes” indicates the contribution from the the other decay modes. “$B_c$ Monte Carlo” stands for the simulated $B_c^+ \rightarrow J/\psi\mu^+\nu$ decays. The simulated sample size is set to the number of signal events in the data after background subtraction.
**TABLE VI:** $B_c^+$ excess from the $B_c^+ \rightarrow J/\psi \mu^+ \nu$ decay, $N_{B_c^+}$. All uncertainties are statistical. The last line presents the Monte Carlo simulated events in the different mass regions, they are scaled down so that the number in the signal region is consistent with the experimental data.

<table>
<thead>
<tr>
<th>Mass (GeV/c^2)</th>
<th>$N_{obs}$</th>
<th>Other decay modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 - 4$</td>
<td>$26.5 \pm 12.0$</td>
<td>$2.6 \pm 0.1$</td>
</tr>
<tr>
<td>$4 - 6$</td>
<td>$769.5 \pm 39.6$</td>
<td>$30.0 \pm 0.2$</td>
</tr>
<tr>
<td>$&gt; 6$</td>
<td>$45.7 \pm 18.3$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

$N(B_c^+ \rightarrow J/\psi \mu^+ \nu)$, Monte Carlo: $22.8 \pm 0.6$.

**FIG. 13:** The invariant mass distribution of the $B_c^+ \rightarrow J/\psi \mu^+ \nu$ candidate events using the full CDF data sample with a Monte Carlo simulated signal sample and the calculated backgrounds superimposed. Misid. Muon contains both the misidentified muon and the "double fake" correction. The $B_c^+ \rightarrow J/\psi \mu^+ \nu$ Monte Carlo simulation shows the shape of the simulated $B_c^+ \rightarrow J/\psi \mu^+ \nu$ decays, while the number of events is consistent with the $B_c^+$ excess. The error bars are the statistical uncertainties of the data and background predictions combined.

**VI. RELATIVE EFFICIENCY**

The ratio of the production cross section times branching ratio of the $B_c^+ \rightarrow J/\psi \mu^+ \nu$ relative to the $B^+ \rightarrow J/\psi K^+$ can be written as

$$\frac{\sigma(B_c^+) \cdot BR(B_c^+ \rightarrow J/\psi \mu^+ \nu)}{\sigma(B^+) \cdot BR(B^+ \rightarrow J/\psi K^+)} = \frac{N_{B_c^+}}{N_{B^+}} \times \epsilon_{rel},$$

where $N_{B_c^+}$ is finalized in Table VI within the signal mass region ($4 - 6$ GeV/c^2), the $N_{B^+}$ is given in Fig. 2 (right) and $\epsilon_{rel} = \frac{\epsilon_{B_c^+} \times \epsilon_{CMUP}}{\epsilon_{B^+} \times \epsilon_{CMUP}}$. Here $\epsilon_{B_c^+}$ is the CDF acceptance for $B^+ \rightarrow J/\psi K^+$ decay, $\epsilon_{B_c^+}$ is the CDF acceptance for $B_c^+ \rightarrow J/\psi \mu^+ \nu$ decay, $\epsilon_{CMUP}$ is the third muon detection efficiency by the CMUP detectors. As the CMUP efficiency we use $0.962 \pm 0.007$ (stat) $\pm 0.021$ (sys) [10]. This efficiency is normalized to the Monte Carlo simulation.

As an input spectrum for the $B^+ \rightarrow J/\psi K^+$ acceptance, the FONLL spectrum from Ref. [11] is used. We find that the FONLL spectrum shows some discrepancies in the low $p_T$ region with respect to the our data. Thus, a corrected FONLL spectrum was used to determine the $B^+ \rightarrow J/\psi K^+$ acceptance.

As the input spectrum for the $B_c^+$, a theoretically-predicted spectra from Ref. [12] is used. Here we also use our background subtracted data to make some corrections in the low $p_T$ region. Comparison of the corrected theoretically-predicted spectra with our data for the $B^+ \rightarrow J/\psi K^+$ and for the
$B_c^+ \to J/\psi \mu^+ \nu$ decays are shown in Fig. 14

![PT spectrum for J/ψK and J/ψμ](image)

**Fig. 14:** The left plot illustrates the $p_T$ spectra for $J/\psi K^+$ and the right plot - for the $J/\psi \mu$ samples. Both plots have been background subtracted. Both the theoretically-predicted spectra are corrected using our data.

In the acceptance calculation for both $B_c^+ \to J/\psi \mu^+ \nu$ and $B^+ \to J/\psi K^+$ decays we also take into account the fact that XFT efficiencies for kaons and muons, which exist in the data, due to the number of hits required by the XFT in the COT, are not well modeled in the simulation [13].

Finally, the result for $\epsilon_{rel} = \frac{\epsilon_{B_c^+}}{\epsilon_{B_c^+} \times \epsilon_{CMUP}}$ with $p_T(B) > 6$ GeV/c cut within the $|y| < 0.6$ range is $4.093 \pm 0.038$ (stat.)

**VII. SYSTEMATIC UNCERTAINTIES**

We divide the systematic uncertainties into two categories. The first represents the uncertainties on the number of $B_c^+$ signal events $N_{B_c^+}$. This includes the $B_c^+$ background systematic uncertainties. The second represents the uncertainties in the relative efficiency. In this case we consider the $\epsilon_{rel}$ variations due to uncertainties in the knowledge of the $B_c^+$ lifetime and production spectrum, the $B^+$ production spectrum, the relative efficiencies of kaons and muons due to triggering effects at the first level of the CDF II trigger (XFT), and the muon efficiency.

**A. $B_c^+$ background systematics**

Below we discuss the following $B_c^+$ background systematics:

- Misidentified $J/\psi$
  - As this is derived from data, we do not assign a systematic uncertainty.

- Misidentified muon
  - See discussions in Section III B. Results are given in Table I.

- $b\bar{b}$ background
  - See discussions in Section III C.

- Other decay modes
The difference between two predictions on $\text{BR}(B_c^+ \rightarrow J/\psi \mu^+ \nu)$ ([8] and [9]) is 9%. This lead to the systematic uncertainty at the level of 2.2% of $N_{B_c^+}$ or 16.3 events.

Table VII summarizes all $B_c^+$ background systematics assigned.

<table>
<thead>
<tr>
<th>$B_c^+$ background</th>
<th>Systematic uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Misidentified $\mu$</td>
<td>$-16.5$</td>
</tr>
<tr>
<td>Doubly misidentified</td>
<td>$-0.9$</td>
</tr>
<tr>
<td>$b\bar{b}$</td>
<td>$\pm 5.8$</td>
</tr>
<tr>
<td>Other decay modes</td>
<td>$\pm 16.3$</td>
</tr>
<tr>
<td>Total</td>
<td>$-24.9$</td>
</tr>
</tbody>
</table>

TABLE VII: Systematic uncertainties assigned for various backgrounds.

B. Relative efficiency systematics

We consider following sources of uncertainty:

- $B_c^+$ lifetime
- $B_c^+$ production spectrum
- $B^+$ production spectrum
- $K$ and $\mu$ differences in XFT simulation
- CMUP efficiency

The total $\epsilon_{rel}$ systematic uncertainty are summarized in Table VIII. Details of the different systematic uncertainties are given in the following sections.

<table>
<thead>
<tr>
<th>Variation variable</th>
<th>Systematic uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c^+$ lifetime</td>
<td>$+0.184$</td>
</tr>
<tr>
<td>$B_c^+$ spectrum</td>
<td>$+0.336$</td>
</tr>
<tr>
<td>$B^+$ spectrum</td>
<td>$-0.363$</td>
</tr>
<tr>
<td>XFT trigger</td>
<td>$\pm 0.055$</td>
</tr>
<tr>
<td>CMUP efficiency</td>
<td>$+0.092$</td>
</tr>
<tr>
<td>Total</td>
<td>$+0.401$</td>
</tr>
</tbody>
</table>

TABLE VIII: $\epsilon_{rel}$ systematic uncertainties.

1. $B_c^+$ lifetime systematics

The $B_c^+$ lifetime systematic uncertainty is estimated by varying the $B_c^+$ lifetime within $\pm 9 \mu m$ relative to the world average value, $\sigma(B_c^+) = 137 \mu m$. To determine the systematic uncertainty, we generate two $B_c^+$ Monte Carlo samples with $B_c^+$ lifetimes, $\sigma(B_c^+)$, of 128 $\mu m$ and 146 $\mu m$. 
2. $B_c^+$ and $B^+$ production spectrum systematic uncertainties

For both the $B_c^+$ and $B^+$ production spectrum systematic uncertainty calculation, we applied a similar approach: the systematic uncertainty is derived from comparing the $p_T$ spectrum given by the data directly with that of simulated events in the detector produced from the corrected theoretical input production spectra. The data to Monte Carlo ratio plots for both cases are used to estimate an average ratio for $p_T(B) > 6$ GeV/$c$: $\tilde{R} = \frac{\sum w_i R_i}{\sum w_i}$, where $w_i = \frac{1}{\sigma_i^2}$ and $\sigma_{\tilde{R}} = \sqrt{\frac{\sum \sigma_i^2}{n(n-1)}}$. Thus, we find $\tilde{R}(B_c^+) = 1.00 \pm 0.08$ and $\tilde{R}(B^+) = 0.999 \pm 0.0133$. We assign a systematic uncertainties of 8% and 1.3% for the $B_c^+$ and $B^+$, respectively. The $B_c^+$ and $B^+$ spectrum systematic uncertainties are shown in Table VIII.

3. $K$ and $\mu$ differences in XFT simulation

$K$ and $\mu$ differences in the XFT simulation are taken into account in the CDF detector acceptance calculation for both $B_c^+ \rightarrow J/\psi \mu^+ \nu$ and $B^+ \rightarrow J/\psi K^+$ decays in Section VI. We find that the $\epsilon_{rel}$ difference between using and not using the XFT correction is $\pm 0.140$, and 50% of this difference was assigned as the systematic uncertainty.

C. $\frac{\sigma(B_c^+) \times BR(B_c^+ \rightarrow J/\psi \mu^+ \nu)}{\sigma(B^+) \times BR(B^+ \rightarrow J/\psi K^+)}$ ratio systematic uncertainties

The total systematic uncertainty for the $\frac{\sigma(B_c^+) \times BR(B_c^+ \rightarrow J/\psi \mu^+ \nu)}{\sigma(B^+) \times BR(B^+ \rightarrow J/\psi K^+)}$ ratio are summarized in Table IX.

<table>
<thead>
<tr>
<th>Variation variable</th>
<th>Systematic uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c^+$ background</td>
<td>$0.0037$</td>
</tr>
<tr>
<td>$\epsilon_{rel}$</td>
<td>$0.0028$</td>
</tr>
<tr>
<td>$\epsilon_{rel}$</td>
<td>$0.0153$</td>
</tr>
<tr>
<td>Total</td>
<td>$0.0214$</td>
</tr>
</tbody>
</table>

TABLE IX: Systematic uncertainties for $\frac{\sigma(B_c^+) \times BR(B_c^+ \rightarrow J/\psi \mu^+ \nu)}{\sigma(B^+) \times BR(B^+ \rightarrow J/\psi K^+)}$ ratio.

VIII. CROSS SECTION RESULTS

Using $B_c^+$ and $B^+$ yields from Table VI and Fig. 2 (right), respectively, and the $\epsilon_{rel}$ we calculate the ratio of the production cross section time branching ratio of the $B_c^+ \rightarrow J/\psi \mu^+ \nu$ relative to the $B^+ \rightarrow J/\psi K^+$. The final cross section is presented in Table X.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(B_c^+ \rightarrow J/\psi \mu^+ \nu)$</td>
<td>$739.5 \pm 39.6 (stat) \pm 12.3 (sys)$</td>
</tr>
<tr>
<td>$N(B^+ \rightarrow J/\psi K^+)$</td>
<td>$14338 \pm 125 (stat)$</td>
</tr>
<tr>
<td>$\epsilon_{rel}$</td>
<td>$4.093 \pm 0.038 (stat) +0.401 +0.359 (sys)$</td>
</tr>
<tr>
<td>$\frac{\sigma(B_c^+) \times BR(B_c^+ \rightarrow J/\psi \mu^+ \nu)}{\sigma(B^+) \times BR(B^+ \rightarrow J/\psi K^+)}$</td>
<td>$0.211 \pm 0.012 (stat) +0.021 -0.020 (sys)$</td>
</tr>
</tbody>
</table>

TABLE X: Results of $B_c^+$ production cross section times the branching ratio to $J/\psi \mu^+ \nu$ over $B^+$ production cross section times the branching ratio to $J/\psi K^+$. 
We have performed a measurement of the ratio \( \frac{\sigma(B^+) * BR(B^+ \rightarrow J/\psi \mu^+ \nu)}{\sigma(B^+) * BR(B^+ \rightarrow J/\psi K^+)} \) using the complete CDF dataset. We have identified a sample of 1370 events with an estimated background from all sources of 630.5 ± 14.2 events. We obtain for \( p_T > 6 \text{ GeV}/c \) and \(|y| < 0.6\):

\[
\frac{\sigma(B^+) * BR(B^+ \rightarrow J/\psi \mu^+ \nu)}{\sigma(B^+) * BR(B^+ \rightarrow J/\psi K^+)} = 0.211 \pm 0.012 \text{ (stat.)}^{+0.021}_{-0.020} \text{ (syst.)}
\]

[1] Charge-conjugate states are implied throughout the paper unless otherwise specified.
[2] F. Abe et al. (CDF Collaboration), Phys. Rev. Lett. 81, 2432 (1998);