CDF note 10456

Measurement of the top quark mass with *in situ* jet energy scale calibration in the all-hadronic channel using the Template Method with 5.8 fb^{-1}

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We present here the measurement of the top quark mass with simultaneous (*in situ*) calibration of the Jet Energy Scale (JES), by the Template Method in the all-hadronic channel, i.e. where both W's decay into $q\bar{q}'$ pairs. The measurement discussed here is performed using about $5.8\,\mathrm{fb}^{-1}$ of $p\bar{p}$ collisions collected with a multijet trigger at $\sqrt{s} = 1.96$ TeV with the Collider Detector at Fermilab (CDF). The method relies on the comparison, for events selected by a Neural Network, of the reconstructed top quark and W boson masses distributions in the data to expectation from signal Monte Carlo and data-driven background events, to extract the top mass and the JES through an unbinned likelihood technique. The measurement gives a top quark mass $M_{top} = [172.5\ \pm 1.4\,(stat)\ \pm 1.4\,(syst)]\,\mathrm{GeV}/c^2$.

Preliminary Results for Winter 2011 Conferences

I. INTRODUCTION

At Fermilab, top quarks are mainly pair produced in $p\bar{p}$ collisions via $q\bar{q}$ annihilation (85%) and gluon-gluon fusion (15%). According to the Standard Model, the top quarks decay into W bosons and b quarks with a branching ratio (BR) about equal to 1. In this analysis we search for events in which both W bosons decay into quark pairs, leading to an all-hadronic final state. This channel has the advantage of the largest BR, about 44%, and of the fully reconstructed kinematics. The major downside is the huge background from QCD multijet production which dominates the signal by three orders of magnitude even after the application of the specific top multijet trigger. A sophisticated event selection based on kinematical and topological variables, followed by the request of identified b-jets, is thus needed in order to further improve the signal to background ratio (S/B).

We present here a measurement of the top quark mass performed using about $5.8\,\mathrm{fb}^{-1}$ of data. Distributions (templates) of variables sensitive to the main observables we want to measure, i.e. the top mass (M_{top}) and the jet energy scale (JES), are built and used to discriminate the possible values of these variables. At the same time, the differences between signal and background distributions allow to estimate the respective average contributions to the observed events, so that the measurement can be obtained by maximizing a likelihood fit of the data to the signal and background templates. As we use templates to measure, above all, two quantities simultaneously, i.e. M_{top} and JES, the technique is referred to as TMT2D (Top Mass Templates 2-Dimensional measurement). The reliability of the method, its expected performance and the main sources of systematic uncertainties have been evaluated by large sets of simulated experiments (*pseudo-experiments*) before the actual measurement on the data.

II. EVENT SELECTION

All data and simulated Monte Carlo events, previously selected by a multijet trigger, have to pass some prerequisites which require a well centered primary vertex and no lepton with high transverse momentum (p_T) identified in the event. The events satisfying this first selection are then required to have a number of detected "tight" jets (i.e. jets with $E_T \geq 15$ GeV, $|\eta| \leq 2.0$) between 6 and 8 with a minimum distance between each pair of jets in the (η, ϕ) plane (ΔR_{min}) larger than 0.5, and no significant missing transverse energy. A number of kinematic variables are then reconstructed using tight jets and serve as inputs to a neural network with 13 input variables, one hidden layer and one output layer. As described in [1], the 13 inputs include both variables depending on energy and direction of jets, and also on their shape. The latter are very effective in distinguishing jets produced by light flavor quark (present in signal events) from the wider jets initiated by gluons, in principle typical of background events only.

Events are selected if the output value from the neural network, N_{out}, exceeds a given threshold. Finally we require the presence of jets tagged as b-jets among the six leading jets, and subdivide our sample in events with exactly one tagged jet (1-tag sample) and two or three tagged jets ($\geq$ 2-tags sample). A jet is tagged if some of its tracks form a secondary vertex significantly displaced from the interaction point. Different values of the N_{out} threshold are chosen for the two categories of tagged events, in such a way to maximize the statistical significance of the mass measurement, as described in section VI.

On signal Monte Carlo samples, generated with values of M_{top} in the range between 160 and 185 GeV/c^2, the event selection is repeated changing the value of the JES from $-2\,\sigma_{\mathrm{JES}}$ to $+2\,\sigma_{\mathrm{JES}}$, in steps of $0.5\,\sigma_{\mathrm{JES}}$, with respect to its central value as measured in [2], where σ_{JES} is the uncertainty on that value itself. In the following we then evaluate the JES in terms of its displacement, ΔJES, from the nominal value (corresponding therefore to $\Delta\mathrm{JES} = 0\,\sigma_{\mathrm{JES}}$) and using σ_{JES} as the unit.

III. BACKGROUND MODELING

The background consists mainly of QCD production of light and heavy flavor quarks. Its modeling and estimate are data-driven and based on a tag rate parametrization derived in a sample of events with exactly 5 jets and therefore dominated by the background. The probability to tag a jet is parametrized according to the jet-E_T, jet track multiplicity, and number of well-defined vertices in the event, and can then be applied to taggable jets (i.e. jets accepted by the b-tagging algorithm) identified in events selected by the kinematic requirements, to evaluate the inclusive number of tagged jets originating from background events. Direct exploitation of the tag rate matrix to predict the number of background events with a given number of tags would give incorrect numbers because the matrix, by construction, refers to an inclusive tagging probability and does not consider that in QCD background real heavy flavour quarks come in pairs and have therefore an enhanced double-tagging probability, so that the probability to tag a pair of jets in the same event is not simply equal to the product of the tag rates of single jets.

To account for this we introduce correction factors to obtain a better estimate for the number of 1-tag and $\geq$ 2-tags events. These factors are derived in a control sample dominated by the background (events with 6-8 jets and $N_{out} \leq 0.25$, where the signal contribution is negligible) and represent average corrections to the probability for a possible "tag configuration", that is for the assumption that given taggable jets in an event in the pretag sample are the only tagged jets in the same event after b-tagging.

The data-driven background prediction must be performed starting from events in the pretag sample, but, as this contains also events from $t\bar{t}$ signal, the raw prediction must be corrected to take these into account.

IV. BUILDING TEMPLATES

The $t\bar{t}$ events under study in this work are characterized by the nominal presence of 6 quarks in the final states, two of which are b-quarks. Therefore, the signal signature would ideally consist of 6 reconstructed jets in the detector, with some being tagged as b-jets. We want to fully reconstruct the kinematics of events passing the kinematical selection, partially described in section II, and exploit the presence of the W and top quark to constrain the event topology. In order to do so we consider only the 6 leading (in E_T) jets in the event to limit the number of ways in which we can combine the jets to reconstruct the events. There are 90 possible permutations of jet-to-parton association with two jet doublets giving a W and two jet triplets giving the top quarks. Since we consider only events with tagged jets, we further reduce the number of permutations by requiring the association of the b-tagged jets to a b quark; we are therefore left with 30 possible parton-jet assignment in 1-tag sample, and 6 or 18 in the $\geq$ 2-tags sample[12].

A. m_t^{rec} templates

We reconstruct the kinematic of the event by a fit based on the following χ^2-like quantity:

$$\chi^2 = \frac{\left(m_{jj}^{(1)} - M_W\right)^2}{\Gamma_W^2} + \frac{\left(m_{jj}^{(2)} - M_W\right)^2}{\Gamma_W^2} + \frac{\left(m_{jjb}^{(1)} - m_t^{rec}\right)^2}{\Gamma_t^2} + \frac{\left(m_{jjb}^{(2)} - m_t^{rec}\right)^2}{\Gamma_t^2} + \sum_{i=1}^{6} \frac{\left(p_{T,i}^{fit} - p_{T,i}^{meas}\right)^2}{\sigma_i^2}$$

where $m_{jj}^{(1,2)}$ are the invariant masses of the dijet systems assigned to light flavor quarks, $m_{jjb}^{(1,2)}$ are the invariant masses of the trijet systems including one b-quark, $M_W = 80.4\,\mathrm{GeV}/c^2$ and $\Gamma_W = 2.1\ \mathrm{GeV}/c^2$ are the measured mass and natural width of the W boson[4], and $\Gamma_t = 1.5\,\mathrm{GeV}/c^2$, is the assumed natural width of the top quark. The measured jet transverse momenta, $p_{T,i}^{meas}$ are free to vary within their known resolution, σ_i. The measured jet transverse momenta can vary, but are constrained to the measured value, $p_{T,i}^{meas}$, within their known resolution, σ_i.

For each permutation of the jet-to-parton assignments in the event, the χ^2 is minimized with respect to 7 free parameters, i.e. the reconstructed top quark mass, m_t^{rec}, and the 6 jets transverse momenta $p_{T,i}^{fit}$ and the combination which gives the lowest χ^2 value is selected. The m_t^{rec} value corresponding to this permutation enters an invariant mass distribution, i.e. the template which will serve as a reference for the M_{top} measurement. This procedure is repeated on selected signal Monte Carlo events with all the different input values of M_{top} and ΔJES and, to parametrize the dependence of the m_t^{rec} templates on these variables, we perform a fit of the distributions to functional forms which vary smoothly with respect to these variables. So, we obtain probability density functions (p.d.f.'s) which we will use to form an unbinned likelihood for the final measurement. The signal p.d.f., $P_s^{m_t^{rec}}(m_t|M_{top}, \Delta\mathrm{JES})$, represents the probability to obtain a value m_t for m_t^{rec}, given a true top quark mass M_{top} and a true value ΔJES of the displacement of the jet energy scale, in a $t\bar{t}$ event.

B. m_W^{rec} templates

Reconstructing the mass of W bosons by dijet systems represents a possibility to obtain a variable in principle insensitive to M_{top} which allows, therefore, a determination of JES not dependent on M_{top} itself.

To build the m_W^{rec} templates we use the same procedure and χ^2 expression considered for m_t^{rec} templates, but now also the W mass is left as a free parameter in the fit (i.e. M_W becomes m_W^{rec}). Again, for each event, the value of m_W^{rec} corresponding to the permutation of the jet-to-parton assignments with the lowest χ^2 enters the template, and this procedure is repeated on selected signal Monte Carlo events with all the different input values of M_{top} and ΔJES. Like for m_t^{rec}, also the m_W^{rec} templates need to be parametrized by functions depending on M_{top} and ΔJES. The m_W^{rec} p.d.f., $P_s^{m_W^{rec}}(m_W|M_{top}, \Delta\mathrm{JES})$, represents the probability to obtain a value m_W for m_W^{rec}, given true inputs M_{top} and ΔJES, in a $t\bar{t}$ event.

C. Background m_t^{rec} and m_W^{rec} templates

As in the case of background normalization, we must build background m_t^{rec} and m_W^{rec} templates considering events and taggable jets (instead of tagged ones) in the pretag sample. In particular all the possible combinations where 1, 2, or 3 *taggable jets* among the 6 leading jets are *assumed* as tagged must be considered, and, for each combination, the same procedures described in sections IV A and IV B must then be repeated to extract corresponding values of m_t^{rec} and m_W^{rec}. These values then enter the templates weighted by the *corrected* probability (see section III) that the jets assumed as tagged in the combination are effectively the tagged ones in the event after b-tagging. Corrections for the presence of signal events in the pretag sample must be taken into account, and the corresponding contribution to the shape subtracted. No dependence on M_{top} and JES is considered for the background templates, but effects of differences due to corrections performed by signal events corresponding to different values of these variables are taken into account by the calibration procedure (section VIII B). The background p.d.f.'s, $P_b^{m_t^{rec}}(m_t)$ and $P_b^{m_W^{rec}}(m_W)$, represent the probabilities to obtain values m_t for m_t^{rec} and m_W for m_W^{rec} respectively, in a background event.

V. BACKGROUND VALIDATION

In order to check how properly our modeling describes the background, we consider events in control regions defined by the N_{out} value, in ranges where the signal presence after tagging is still very low. In these regions the templates, i.e. the distributions which are essential to our measurement, are reconstructed by the procedure described in the previous sections both for the signal and the background. As the final selections of the data samples include cuts on the N_{out} value and on the χ^2 of the fits used to build the m_W^{rec} and m_t^{rec} templates (denoted in the following by $\chi^2(m_W^{rec})$ and $\chi^2(m_t^{rec})$ respectively), as it will be described in section VI, also these distributions are really important. Obviously, as it concerns the background, they must be evaluated by the same procedure of weighting each assumed possible configuration with 1, 2 or 3 tagged jets described in section IV C for the templates.

The agreement between expected and observed distributions is rather good in all the control regions, and this confirms the reliability of the background model. In Fig. 1 the output of the neural network over the whole range of values $N_{out} > 0.5$ is shown, while Figs 2 and 3 show distributions of $\chi^2(m_W^{rec})$, $\chi^2(m_t^{rec})$, m_W^{rec} and m_t^{rec} in one of the control regions both for 1-tag and $\geq$ 2-tags events, where the sum of signal and background is compared to the same distributions reconstructed in the data. In these plots the signal distributions corresponding to $M_{top} = 172.5\,\mathrm{GeV}/c^2$ and $\Delta\mathrm{JES} = 0\,\sigma_{\mathrm{JES}}$ have been normalized assuming $\sigma_{t\bar{t}} = 7.45\,\mathrm{pb}$ [3], while the amount of background events corresponds to the difference beteween the number of observed events and the expected signal.

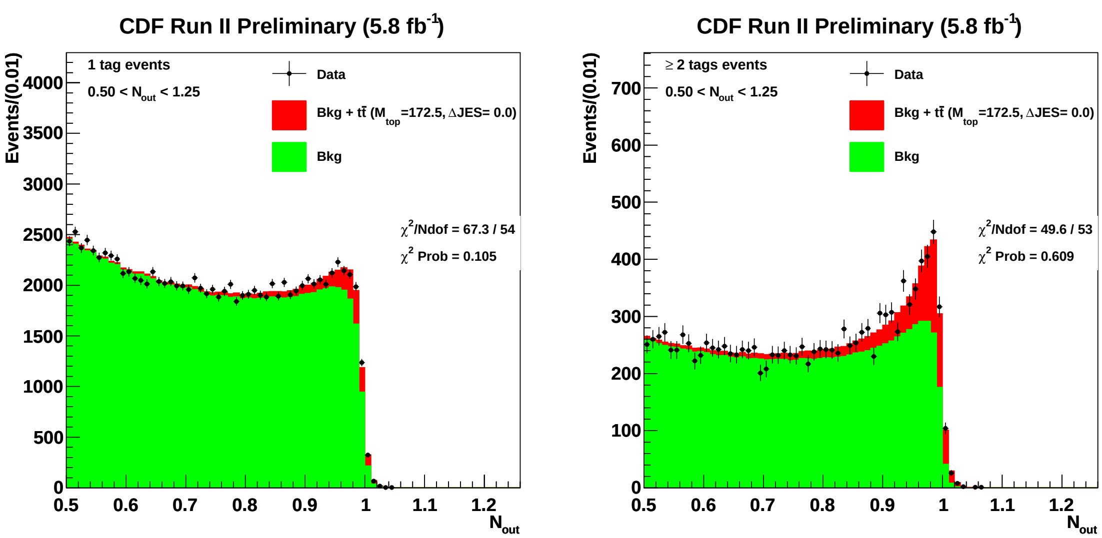

FIG. 1: Distributions of the output from the Neural Net, N_{out}, for 1-tag events, left plot, and $\geq$ 2-tags events, right plot, are shown in the whole region defined by $N_{out} > 0.5$. Along with the data are plotted the corrected expected background and the signal contribution. We see that the agreement is generally good. The value of the purely statistical χ^2 probability is also reported on each plot.

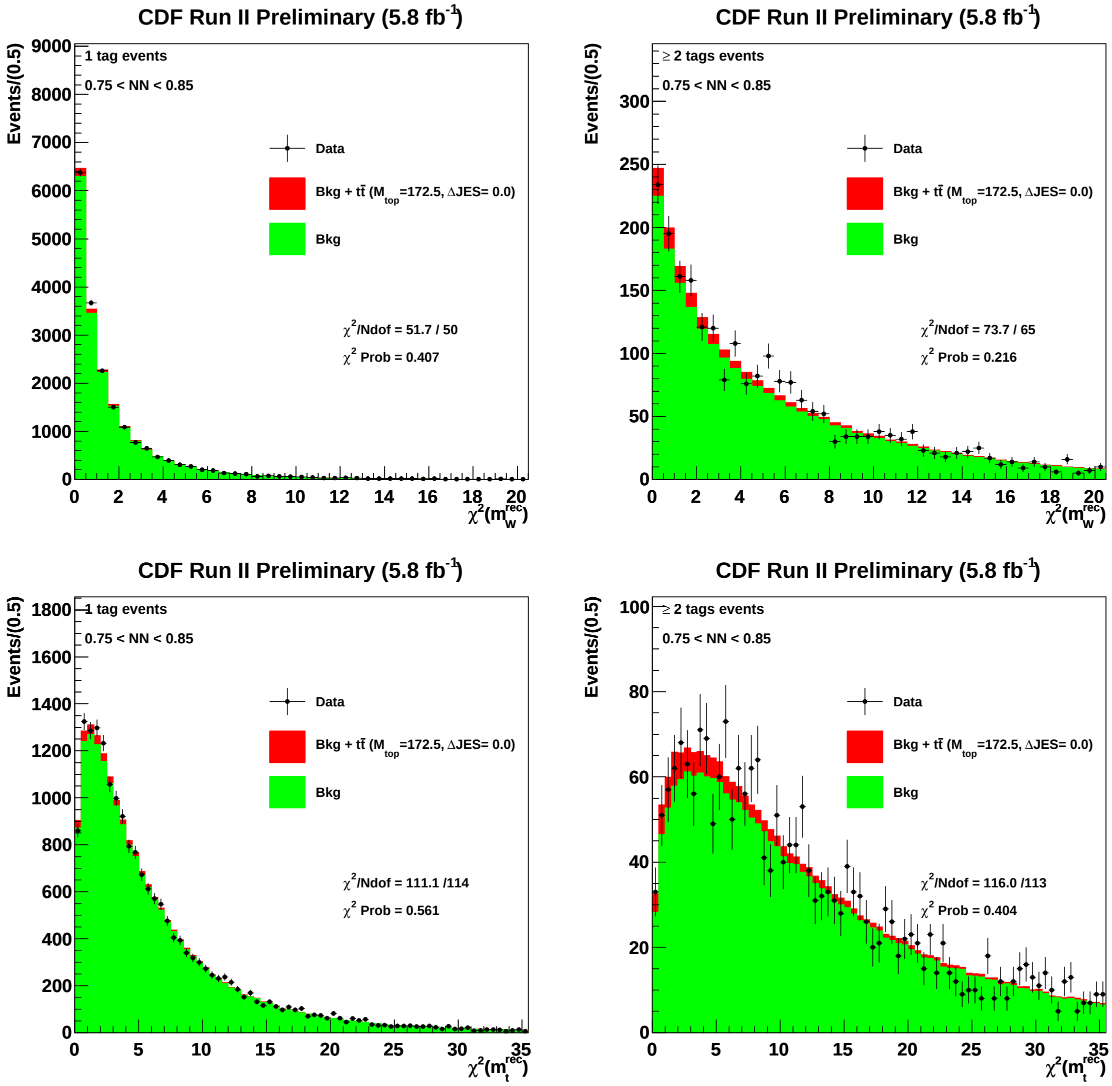

FIG. 2: Distributions of the χ^2 of the fit used to build the m_W^{rec} (upper plots) and m_t^{rec} (lower plots) templates, are shown in a control region defined by $0.75 \leq N_{out} < 0.85$ both for 1-tag events, left plots and $\geq$ 2-tags events, right plots. Along with the data are plotted the corrected expected background and the signal contribution. We see that the agreement is generally good. The value of the purely statistical χ^2 probability is also reported on each plot.

VI. EVENTS SAMPLES

In order to obtain the best performance from our method, we performed sets of pseudo-experiments (PEs) to find the requirements on the values of N_{out}, $\chi^2\left(m_W^{rec}\right)$, and $\chi^2\left(m_t^{rec}\right)$ which minimize the statistical uncertainty on the top mass measurement.

Two different samples of events, denoted by S_{JES} and $S_{M_{top}}$, are defined and used to build the m_W^{rec} and m_t^{rec} templates respectively. The set S_{JES} is selected by requirements on N_{out} and $\chi^2\left(m_W^{rec}\right)$, while $S_{M_{top}}$ by a *further* requirement on $\chi^2\left(m_t^{rec}\right)$, so that $S_{M_{top}} \subseteq S_{\mathrm{JES}}$. As S_{JES} is somehow used to calibrate the JES, while $S_{M_{top}}$ is more strictly related to the top quark mass measurement, we also refer to S_{JES} and $S_{M_{top}}$ as "JES-sample" and "M_{top}-sample" respectively.

The procedure used in PEs to obtain the cuts optimization is totally similar, being a simplified version, to the one described in section VIII A exploiting a binned version of the same likelihood function. It is applied separately to 1-tag and $\geq$ 2-tags samples and considers many different combinations $\left\{N_{out},\, \chi^2\left(m_W^{rec}\right), \chi^2\left(m_t^{rec}\right)\right\}$ of cuts. The smallest values of uncertainties are obtained using $\left\{N_{out} \geq 0.97,\, \chi^2\left(m_W^{rec}\right) \leq 2,\, \chi^2\left(m_t^{rec}\right) \leq 3\right\}$ in the 1-tag sample

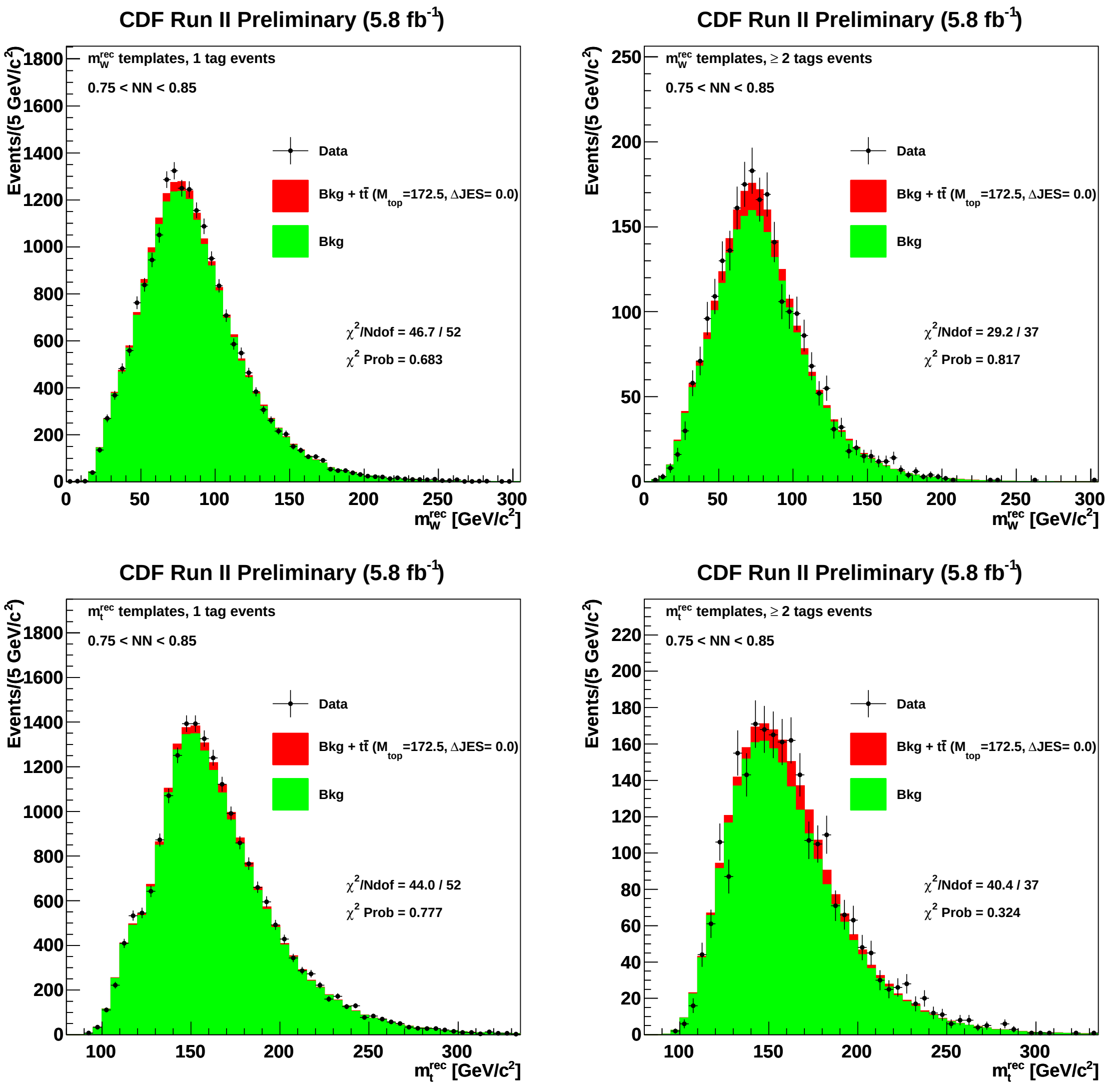

FIG. 3: Templates of the reconstructed W mass, m_W^{rec} (upper plots), and top quark mass, m_t^{rec} (lower plots), are shown in a control region defined by $0.75 \leq N_{out} < 0.85$ both for 1-tag events, left plots, and $\geq$ 2-tags events, right plots. Along with the data are plotted the corrected expected background and the signal contribution. We see that the agreement is generally good. The value of the purely statistical χ^2 probability is also reported on each plot.

and $\left\{N_{out} \geq 0.94,\, \chi^2\left(m_W^{rec}\right) \leq 3,\, \chi^2\left(m_t^{rec}\right) \leq 4\right\}$ in the $\geq$ 2-tags sample.

We therefore finally set, besides the prerequisites described in section II :

- 1-tag events :
  - S_{JES} sample : $N_{out} \geq 0.97$, $\chi^2\left(m_W^{rec}\right) \leq 2$ and 1 tagged jet;
  - $S_{M_{top}}$ sample : $N_{out} \geq 0.97$, $\chi^2\left(m_W^{rec}\right) \leq 2$, $\chi^2\left(m_t^{rec}\right) \leq 3$ and 1 tagged jet;
- $\geq$ 2-tags events :
  - S_{JES} sample : $N_{out} \geq 0.94$, $\chi^2\left(m_W^{rec}\right) \leq 3$ and 2 or 3 tagged jets;
  - $S_{M_{top}}$ sample : $N_{out} \geq 0.94$, $\chi^2\left(m_W^{rec}\right) \leq 3$, $\chi^2\left(m_t^{rec}\right) \leq 4$ and 2 or 3 tagged jets;

as the requirements and the samples to be used in our analysis.

For $t\bar{t}$ events corresponding to $M_{top} = 172.5\,\mathrm{GeV}/c^2$ and $\Delta\mathrm{JES} = 0\,\sigma_{\mathrm{JES}}$, the values of the efficiencies of the JES-sample selections are 2.1% and 1.1% for 1-tag and $\geq$ 2-tags respectively, while for the corresponding M_{top}-samples we obtain 1.4% and 0.7%. For the same M_{top} and ΔJES, the fraction of events of the JES-sample selected by the requirements on $\chi^2\,(m_t^{rec})$ only, and therefore belonging to the M_{top}-sample, are 68.5% and 67.6%, as can be inferred by the ratios of the absolute efficencies. These latter acceptances will be denoted by $\mathcal{A}_s$ in the following and their values generally depend on M_{top} and ΔJES.

A. Expected background

Given the final requirements we can evaluate, as described in section III, the average amounts of background events expected in the selected samples. As already metioned, the raw prediction obtained applying the corrected tag rate on jets and events of the pretag data sample, must be corrected for the presence of $t\bar{t}$ events. The correction to obtain central values is performed assuming $M_{top} = 172.5\,\mathrm{GeV}/c^2$, $\Delta\mathrm{JES} = 0\ \sigma_{\mathrm{JES}}$ and the theoretical signal cross section as calculated in [3]. Uncertainties due to these assumptions are taken into account, together with the discrepancy between the observed number of events in the data and the sum of the predicted background and the expected contribution from the signal.

The numbers of expected background events are summarized in Tab. I, together with the observed data and expected signal selected for this analysis.

Sample		N_{obs}	Exp Bkg (B)	Exp $t\bar{t}$ (S) ($M_{top} = 172.5$, $\Delta\mathrm{JES} = 0$)	S/B
1-tag	S_{JES}	4368	3652 ± 181	881 ± 73	0.24
	S_{Mtop}	2256	1712 ± 77	604 ± 50	0.35
$\geq$ 2-tags	S_{JES}	1196	718 ± 14	468 ± 38	0.65
	S_{Mtop}	600	305 ± 22	316 ± 26	1.04

TABLE I: Numbers of observed data (N_{obs}) and expected amount of background and signal events in the samples salected for this analysis. The signal-to-background ratios (S/B) are also shown.

From these values we can derive also for the background the values of the acceptances $\mathcal{A}_b$, given by the ratios of the number of events expected in the M_{top} and JES samples. Taking into account the correlations among the uncertainties we obtain $\mathcal{A}_b = (46.9 \pm 0.7)\,\%$ for 1-tag events and $\mathcal{A}_b = (42.5 \pm 3.6)\,\%$ for $\geq$ 2-tags events.

B. Parametrizations

Having defined the best requirements for this analysis, we can proceed to build the signal and background templates from events in the selected samples and, for the signal, to parametrize their dependence on M_{top} and ΔJES into smooth probability density functions. The method have been already described in section IV. Figures 4 and 5 show the fitted p.d.f.' s superimposed to the m_t^{rec} and m_W^{rec} signal templates respectively for different M_{top} and ΔJES values.

The background m_t^{rec} and m_W^{rec} templates and the corresponding fitted parametrized p.d.f.'s for the signal region are shown in Figure 6 both for 1-tag and $\geq$ 2-tags events.

For signal events, also the acceptances $\mathcal{A}_s$ defined in section VI depend on M_{top} and ΔJES, with values in the range between 60% and 70% in the ranges $160 \leq M_{top} \leq 185$ and $-2 \leq \Delta\mathrm{JES} \leq +2$. Therefore, as they appear in the likelihood function described in section VII, also their values must be parametrized and this is done by polynomial functions.

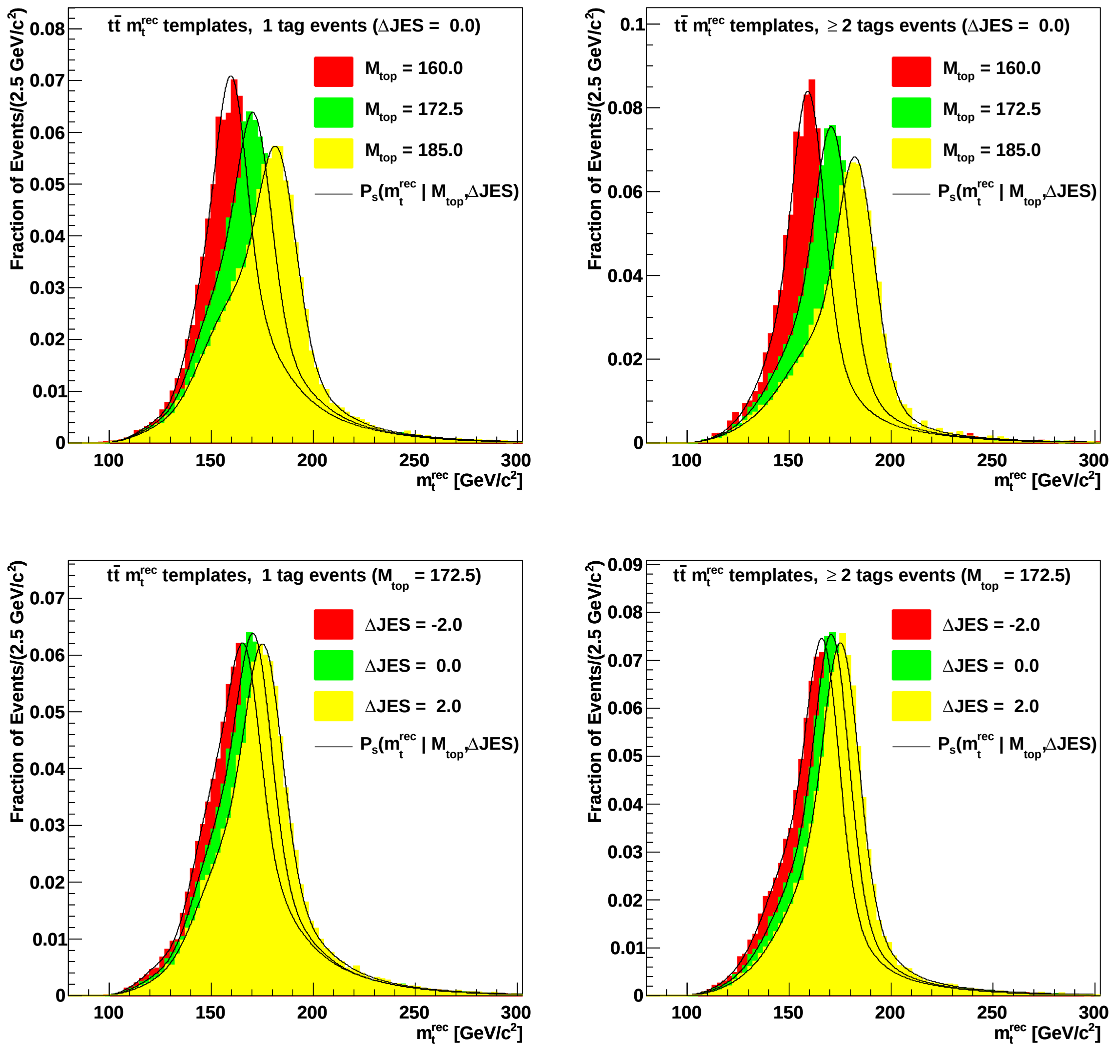

FIG. 4: Probability density functions for the signal m_t^{rec} templates for 1-tag (left plots), and $\geq$ 2-tags events (right plots) for a constant ΔJES value ($0\,\sigma_{\rm JES}$), but varying the input top quark mass (upper plots) and for a constant M_{top} value ($172.5\,{\rm GeV}/c^2$), but varying the input jet energy scale (lower plots).

VII. LIKELIHOOD

The simultaneous measurement of the top quark mass and the jet energy scale by the template method (TMT2D) consists in finding the values of M_{top}, JES, and the number of signal (n_s) and background (n_b) events for each tagging category which best reproduce the observed distributions of m_t^{rec} and m_W^{rec}, as reconstructed in the selected data samples, given the p.d.f.'s expected for signal and background.

This is done by performing a fit where a likelihood function is maximized, or, equivalently, its negative logarithm is minimized. This function is divided into 3 main parts: the first two terms are the ones strictly needed for the M_{top} and the JES *in situ* measurements, where the probability for the observed distributions are calculated as a function of the free parameters (M_{top}, ΔJES, n_s^{1tag}, n_b^{1tag}, $n_s^{\geq 2tags}$ and $n_b^{\geq 2tags}$) for the two tagging categories, taking also into account the *a priori* expectation for the background normalizations and their errors, while the third one constrains the JES parameter to the *a priori* independent measurement [2] (i.e. $\Delta{\rm JES} = 0\,\sigma_{\rm JES}$ in our notation) to reduce the uncertainty on this variable.

Namely the likelihood, $\mathcal{L}$, is written as:

$$\mathcal{L} = \mathcal{L}_{1\,tag} \times \mathcal{L}_{\geq 2\,tags} \times \mathcal{L}_{\Delta{\rm JES}_{constr}}$$

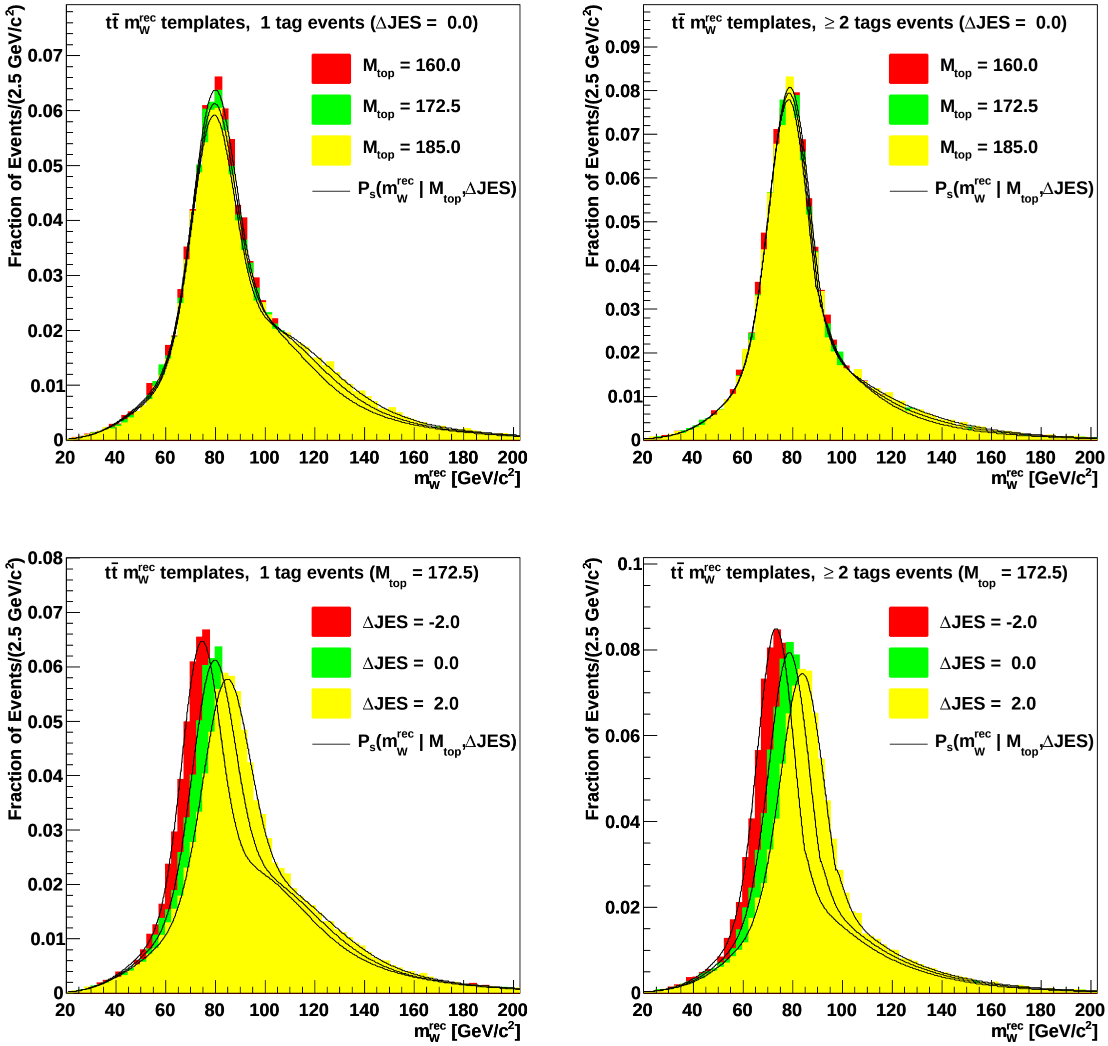

FIG. 5: Probability density functions for the signal m_W^{rec} templates for 1-tag (left plots), and ≥ 2-tags events (right plots) for a constant ΔJES value ($0\,\sigma_{\rm JES}$), but varying the input top quark mass (upper plots) and for a constant M_{top} value ($172.5\,{\rm GeV}/c^2$), but varying the input jet energy scale (lower plots).

The $\mathcal{L}_{1,\,\geq 2\,tags}$ terms further consist of other factors :

$$\mathcal{L}_{1,\geq 2\,tags} = \mathcal{L}_{\Delta \rm JES} \times \mathcal{L}_{M_{top}} \times \mathcal{L}_{evts} \times \mathcal{L}_{N^{bkg}_{constr}}$$

where the four terms assume the following form (the superscripts referring to the tag sample are omitted) :

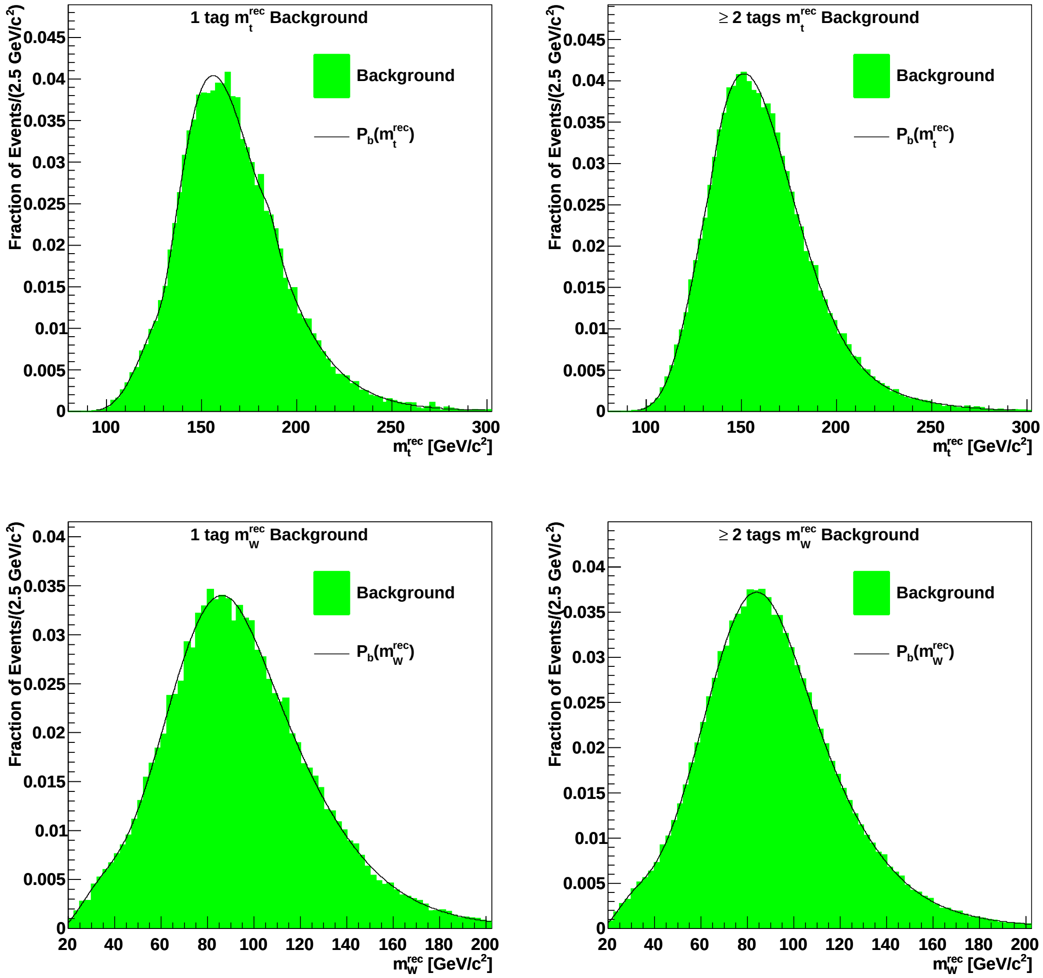

FIG. 6: Background m_t^{rec} (upper plots) and m_W^{rec} (lower plots) templates with the corresponding fitted p.d.f. for 1-tag events, left plots, and $\geq$ 2-tags events, right plots.

$$\mathcal{L}_{\Delta\mathrm{JES}} = \prod_{i=1}^{N_{obs}^{S_{\mathrm{JES}}}} \frac{n_s \cdot P_s^{m_W^{rec}}(m_{W,\,i}\,|\,M_{top},\,\Delta\mathrm{JES}) + n_b \cdot P_b^{m_W^{rec}}(m_{W,\,i})}{n_s + n_b}$$

$$\mathcal{L}_{M_{top}} = \prod_{i=1}^{N_{obs}^{S_{M_{top}}}} \frac{\mathcal{A}_s(M_{top},\,\Delta\mathrm{JES}) \cdot n_s \cdot P_s^{m_t^{rec}}(m_{t,\,i}\,|\,M_{top},\,\Delta\mathrm{JES}) + \mathcal{A}_b \cdot n_b \cdot P_b^{m_t^{rec}}(m_{t,\,i})}{\mathcal{A}_s(M_{top},\,\Delta\mathrm{JES}) \cdot n_s + \mathcal{A}_b \cdot n_b}$$

$$\mathcal{L}_{evts} = \sum_{r_s + r_b = N_{obs}^{S_{JES}}} P(r_s,\,n_s) \cdot P(r_b,\,n_b) \cdot \left[\sum_{\substack{t_s \leq r_s,\ t_b \leq r_b \\ t_s + t_b = N_{obs}^{S_{M_{top}}}}} B\,(t_s,\,r_s,\,\mathcal{A}_s) \cdot B\,(t_b,\,r_b,\,\mathcal{A}_b) \right]$$

$$\mathcal{L}_{N_{constr}^{bkg}} = e^{-\frac{(n_b - n_{(b,\,exp)})^2}{2\sigma^2_{n_{(b,\,exp)}}}}$$

In the first term the probability to observe the set $m_{W,\,i}$, $(i = 1, ..., N_{obs}^{S_{\mathrm{JES}}})$ of m_W^{rec} values reconstructed in the data JES-sample is calculated by the signal and background p.d.f.'s, $P_s^{m_W^{rec}}$ and $P_b^{m_W^{rec}}$ respectively, as a function of the free parameters of the fit. In the second the same is done for the distributions of the observed reconstructed top masses in the M_{top}-sample $m_{t,\,i}$, $(i = 1, ..., N_{obs}^{S_{M_{top}}})$, and the m_t^{rec} p.d.f.'s. The third term, $\mathcal{L}_{evts}$, gives the probability to observe simultaneously the number of events selected in the data for the JES-sample and the M_{top}-sample, given the assumed values for the average number of signal (n_s) and background (n_b) events to be expected in S_{JES} and the acceptances $\mathcal{A}_s(M_{top},\, \Delta\mathrm{JES})$ and $\mathcal{A}_b$. It depends on the Poisson (P) and Binomial (B) probabilities

$$P(r,\, n) \;=\; \frac{e^{-n} \cdot n^r}{r\,!}$$

$$B\,(t,\, r,\, \mathcal{A}) \;=\; \binom{r}{t} \cdot \mathcal{A}^t \cdot (1 - \mathcal{A})^{r-t}$$

In the last term, $\mathcal{L}_{N_{constr}^{bkg}}$, the parameter n_b is constrained by a Gaussian to the *a priori* background estimate given in section VI A, i.e. $n_{(b,\, exp)} = 3652 \pm 181$ for 1-tag events and $n_{(b,\, exp)} = 718 \pm 14$ for $\geq$ 2-tags events.

Finally, $\mathcal{L}_{\Delta\mathrm{JES}_{constr}}$ is a Gaussian term constraining the parameter JES to the value measured and reported in [2], which is equivalent, in our notation, to constrain the parameter ΔJES to 0:

$$\begin{aligned}
\mathcal{L}_{\Delta\mathrm{JES}_{constr}} &= e^{-\frac{(\mathrm{JES}-\mathrm{JES}_{constr})^2}{2\sigma^2_{\mathrm{JES}}}} \\
&= e^{-\frac{[(\mathrm{JES}_{constr}+\Delta\mathrm{JES}\cdot\sigma_{\mathrm{JES}})-\mathrm{JES}_{constr}]^2}{2\sigma^2_{\mathrm{JES}}}} \\
&= e^{-\frac{[\Delta\mathrm{JES}]^2}{2}} \\
&= e^{-\frac{[\Delta\mathrm{JES}-\Delta\mathrm{JES}_{constr}]^2}{2}}
\end{aligned}$$

where, generally, $\Delta\mathrm{JES}_{constr} = 0$.

In order to facilitate the computation, we minimize the negative logarithm of the likelihood using MINUIT. The uncertainties on the parameters are given by MINOS taking positive and negative statistical error as the difference between the observable (O) central value and the values O^+ and O^- for which stands the relation $-lnL(O^{\pm}) + lnL(O) = -1/2$. Following [5] we then take as unique, symmetric errors the average between O^+ and O^- for each parameter. By construction, the MINOS uncertainties take into account the correlations among all the parameters, so that the error on each fitted variable includes both the statistical contribution and the systematic one due to the uncertainties on the other parameters.

VIII. SANITY CHECKS AND EXPECTED PERFORMANCE

We want to investigate the possible presence of biases in the top mass and jet energy scale measurements introduced by our method, as well as to have an estimate of the TMT2D method statistical power before performing the measurement on the actual data sample. To do so, we run realistic pseudo-experiments where "pseudo-data" are extracted from simulated signal and data-driven background templates corresponding to known values of M_{top} and ΔJES (M_{top}^{in}, $\Delta\mathrm{JES}^{in}$) and used as inputs to the likelihood fit to perform the measurement. Also the other parameters of the fit, i.e. the average numbers of input signal and background events, are modified. The results obtained from the fit can then be compared to the true values of the input parameters to study the behavior of the machinery.

A. Pseudo-experiments setup

Sets of about 2000 PEs have been performed at many "points" in the six-dimensional space of the fit parameters. Actually, given the practical impossibility to consider all the possible simultaneous variations of the parameters, we vary pairs of variables, grouping together the two with the largest mutual correlation, while the remaining ones are kept constant to their central values. In particular sets of PEs have been performed with variations of the pairs $\{M_{top}^{in}\,\Delta\mathrm{JES}^{in}\}$, $\left\{n_s^{1\,tag,\,in},\, n_b^{1\,tag,\,in}\right\}$, and $\left\{n_s^{\geq 2\,tags,\,in},\, n_b^{\geq 2\,tags,\,in}\right\}$.

The procedure is the same for each PE of any set:

1. For each tagging category we generate the actual number $N_{(s,\,obs)}^{S_{\mathrm{JES}}}$ of signal events in the JES-sample by a Poisson distribution with mean n_s^{in}, i.e. the n_s input value; the same is repeated for the actual number of background events $N_{(b,\,obs)}^{S_{\mathrm{JES}}}$ by using a Poisson with mean n_b^{in}.

2. The number of signal events in the M_{top}-sample, $N^{S_{M_{top}}}_{(s,\,obs)}$, is generated by a Binomial distribution, given $N^{S_{\rm JES}}_{(s,\,obs)}$ and the acceptance $\mathcal{A}_s$ corresponding to the input values M^{in}_{top} and $\Delta\mathrm{JES}^{in}$. Again, the same procedure is repeated for the background, to obtain $N^{S_{M_{top}}}_{(b,\,obs)}$, obviously using $N^{S_{\rm JES}}_{(b,\,obs)}$ and $\mathcal{A}_b$.

3. The generated number of signal events must correspond to the same numbers of reconstructed masses, with average distributions given by the signal templates. In particular we have one m^{rec}_W value for each event in $S_{\rm JES}$ and one m^{rec}_t value for each event in $S_{M_{top}}$. More precisely, as being $S_{M_{top}} \subseteq S_{\rm JES}$, values of both m^{rec}_W and m^{rec}_t exist for each event in the M_{top}-sample, while for events belonging to $S_{\rm JES}$ *but NOT* to $S_{M_{top}}$ one has a value of m^{rec}_W only.

   Then, to take into account correlations between m^{rec}_t and m^{rec}_W in the same event, $N^{S_{M_{top}}}_{(s,\,obs)}$ m^{rec}_W and m^{rec}_t values are both drawn from signal two-dimensional histograms where m^{rec}_W *vs* m^{rec}_t are plotted for each event in $S_{M_{top}}$. Finally, the missing $N^{S_{\rm JES}}_{(s,\,obs)} - N^{S_{M_{top}}}_{(s,\,obs)}$ values of m^{rec}_W are drawn from distributions of m^{rec}_W obtained from events belonging to $S_{\rm JES}$ *but NOT* to $S_{M_{top}}$ (this set is simply denoted by $S_{\rm JES} - S_{M_{top}}$ in the following). Obviously all the histograms used here correspond to the input values $\{M^{in}_{top}, \Delta\mathrm{JES}^{in}\}$.

4. The same procedure just outlined for the signal events is repeated to generate the $N^{S_{\rm JES}}_{(b,\,obs)}$ and $N^{S_{M_{top}}}_{(b,\,obs)}$ m^{rec}_W and m^{rec}_t values respectively by the background templates. We remind here that the raw shapes must be corrected by the presence of signal in the pretag sample, like mentioned in section IV C. In performing PEs the corrections is done coherently to the input values of the parameters, so that effects due to possible variations of the corrected background templates can be calibrated.

5. The actual values of $n_{(b,\,exp)}$ to be used in $\mathcal{L}_{N^{bkg}_{constr}}$ are generated from a Gaussian distribution of mean $n_{(b,\,exp)}$ and width $\sigma_{n_{(b,\,exp)}}$ both for 1-tag sample and $\geq$ 2-tags sample. $n_{(b,\,exp)}$ is the true input value n^{in}_b and $\sigma_{n_{(b,\,exp)}}$ is the uncertainty evaluated on central values $\sigma_{n_{(b,\,exp)}} = 181$ for 1-tag and $\sigma_{n_{(b,\,exp)}} = 14$ for $\geq$ 2-tags.

6. The actual value of $\Delta\mathrm{JES}_{constr}$ to be used in the term $\mathcal{L}_{\Delta\mathrm{JES}_{constr}}$ is extracted from a Gaussian of mean $\Delta\mathrm{JES}^{in}$ and width 1;

7. $-\log\mathcal{L}$ is simultaneously minimized with respect to the 6 free parameters, M_{top}, ΔJES, $n^{1\,tag}_s$, $n^{1\,tag}_b$, $n^{\geq 2\,tags}_s$, and $n^{\geq 2\,tags}_b$.

Histograms are filled by outputs from each PE and then used to study the average behavior of the measurement machinery with respect to the true input quantities. Uncertainties on variables extracted from these histograms and related to the limited statistic of the samples used to build the templates[13], are evaluated by a *bootstrap* procedure [6, 7], that is fluctuating the contents of each bin in the templates by its statistical uncertainty and performing PEs extracting data from the set of "fluctuated" templates. This is repeated 200 times, and the RMS of variables extracted from histograms are taken as the statistical uncertainties.

B. Calibration

There are many factors which may introduce a bias using the TMT2D method and, given our machinery, the most likely is an unappropriate parametrizations of the templates by smooth p.d.f.'s. We take advantage of the PEs procedure to find calibration functions to be applied to the outputs of a measurement to obtain, on the average, more reliable estimates of the true input values of the fitted parameters. As it concerns in particular M_{top} and ΔJES, the calibrated values will be denoted by M^{corr}_{top} and $\Delta\mathrm{JES}^{corr}$ respectively. Obviously, also the uncertainties from the likelihood fit have to be propagated through the calibration. To test the goodness of the calibration we performed a complete set of PEs where it is applied PE by PE. In Fig. 7 we show examples of the residuals of M_{top} and ΔJES after the calibration. These plots show how the applied corrections get rid of most of the average biases.

To check that the calibrated uncertainties are unbiased we consider the width of M^{corr}_{top} and $\Delta\mathrm{JES}^{corr}$ pull distributions. Fig. 8 shows examples of the values of the M_{top} and ΔJES pull widths as a function of the input top mass, M^{in}_{top} and of the input ΔJES, $\Delta\mathrm{JES}^{in}$, after the calibration. To derive a correction we average the pull widths over all the M^{in}_{top} and $\Delta\mathrm{JES}^{in}$ values, setting to 1 possible values smaller than 1. This procedure leads to multiplicative correction factors equal to 1.06 for δM^{corr}_{top} and to 1.07 for $\delta\Delta\mathrm{JES}^{corr}$.

Figures 9 shows examples of the expected uncertainties after both the calibration and the pull width correction have been applied. The values of these average expected uncertainty on the measured top quark mass and jet energy scale displacement for true Mtop and ΔJES around 172.5 GeV/c^2 and 0 $\sigma_{\rm JES}$, are:

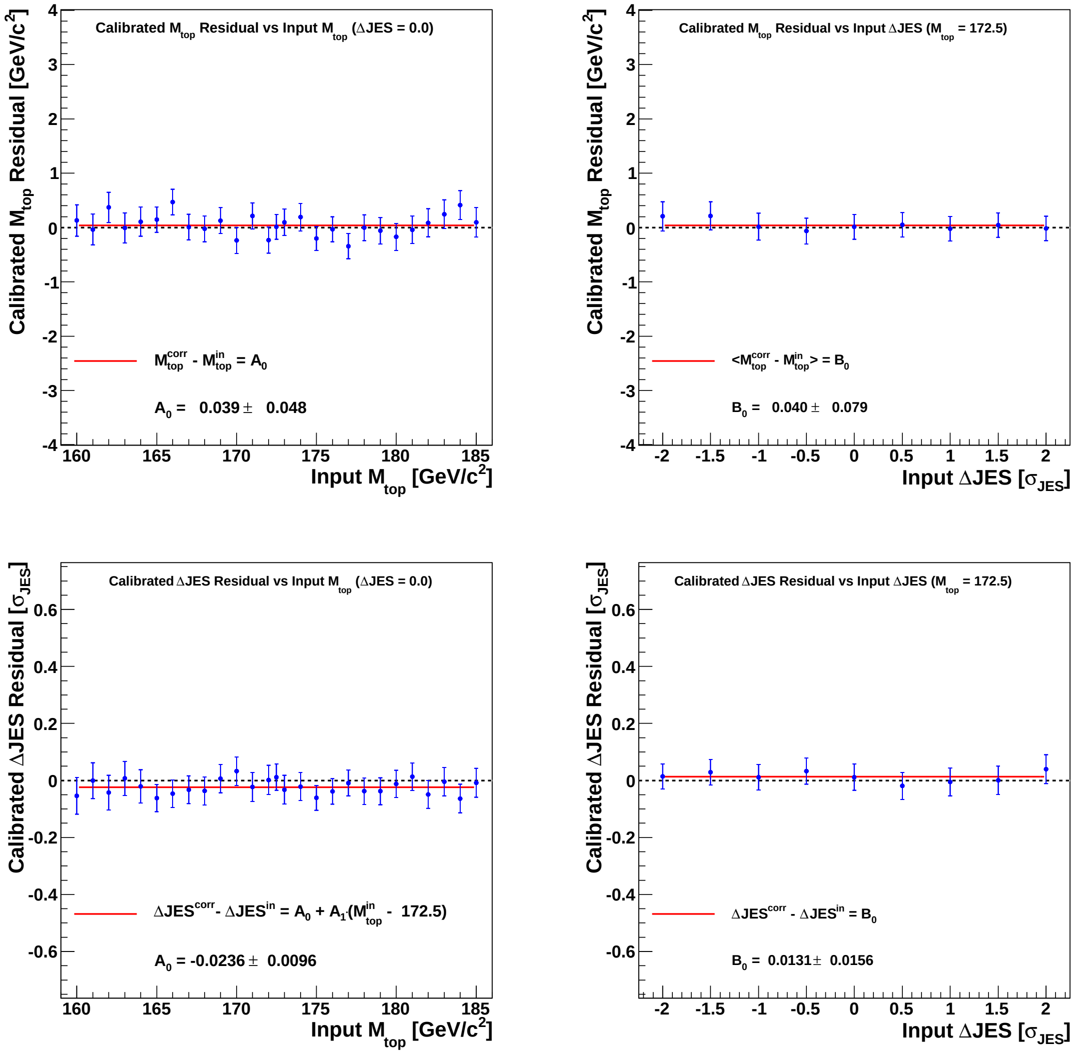

FIG. 7: Examples of residuals of the calibrated top quark mass ($M_{top}^{corr} - M_{top}^{in}$, upper plots) and jet energy scale displacement ($\Delta\mathrm{JES}^{corr} - \Delta\mathrm{JES}^{in}$, lower plots) as a function of the input M_{top} (left plots), and of input ΔJES (right plots). The results of fits by a straight line are superimposed.

$$
\begin{aligned}
\delta M_{top}^{corr}\,(stat + \mathrm{JES}) &\simeq 1.5\,\mathrm{GeV}/c^2 \\
\delta\Delta\mathrm{JES}^{corr}\,(stat + M_{top}) &\simeq 0.34\,\sigma_{\mathrm{JES}}
\end{aligned}
$$

These uncertainties actually include the systematic contributions due to all the parameters of the fit, but the contributions from the n_s and n_b parameters are negligible with respect to the one coming from JES for M_{top}^{corr} and from M_{top} for $\Delta\mathrm{JES}^{corr}$.

IX. SYSTEMATIC UNCERTAINTIES ON THE TOP QUARK MASS AND THE JET ENERGY SCALE

Various sources of systematic uncertainties might affect the top quark mass and the jet energy scale measurements. The main possible effect have been studied and are summarized in this section. These arise mostly from the measurement technique itself, from uncertainties in the simulation of the $t\bar{t}$ events, from mismodeling in the simulation of the

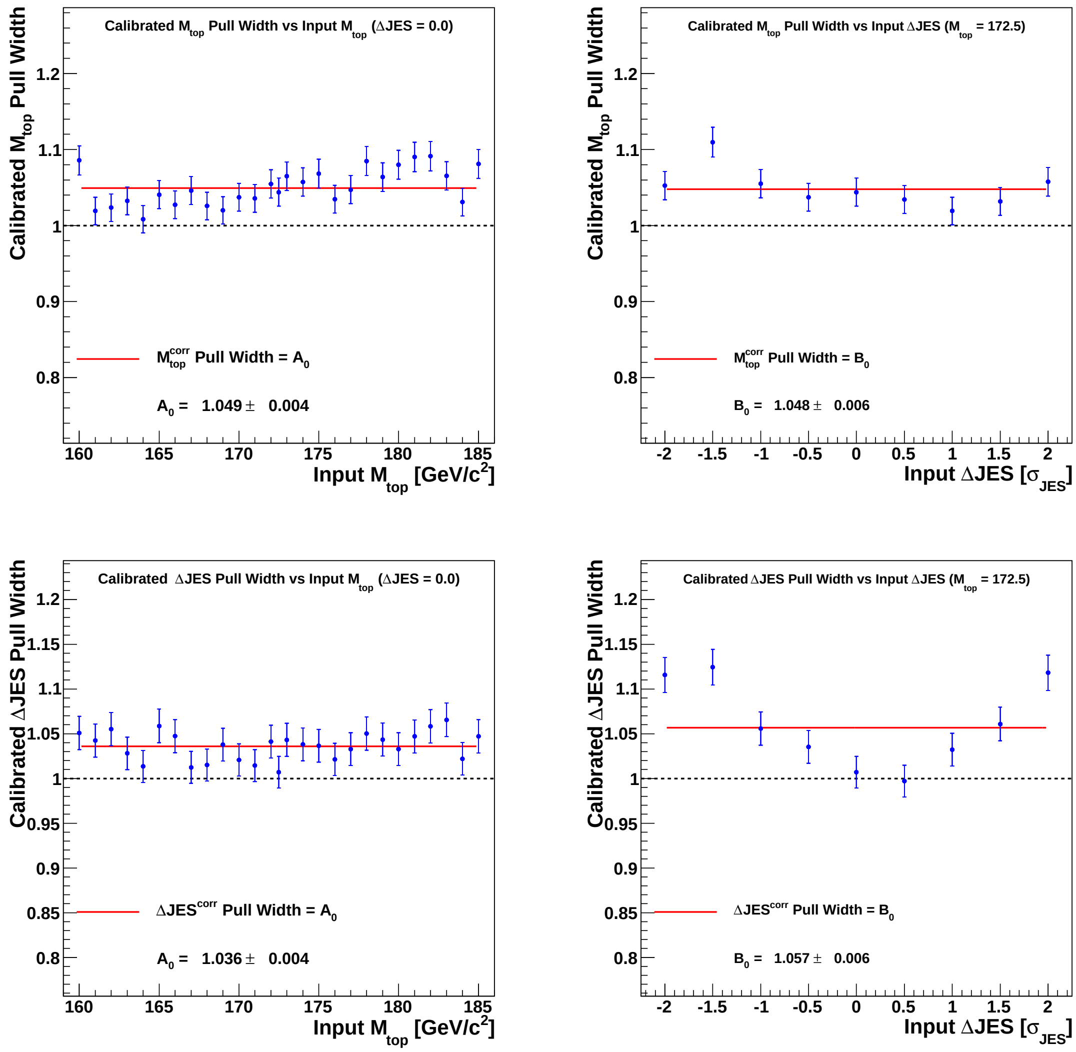

FIG. 8: Widths of the pull distributions for M_{top}^{corr} (upper plots) and $\Delta\mathrm{JES}^{corr}$ (lower plots) as a function of the input M_{top} at constant $\Delta\mathrm{JES} = 0\ \sigma_{\mathrm{JES}}$ (left plots) and of the input $\Delta\mathrm{JES}$ for $M_{top}^{in} = 172.5\,\mathrm{GeV}/c^2$ (right plots). The straight lines denote the fit by a constant function.

detector response and from uncertainty on the shapes of signal and background templates used to derive the p.d.f.'s and to calibrate the measurement.

They are usually evaluated by performing PEs extracting pseudo-data from templates built using signal samples where the possible systematic effects have been considered and included. Corresponding corrections to the shape of raw background templates, are performed to obtain also the corrected background templates in agreement with the effect one wants to study. On the contrary, nothing is changed in the measurement machinery, i.e. in the elements of the likelihood fit, because it is this machinery that we want to apply to the data and that, therefore, we have to test in front of possible mismodeling of the data themeselves.

The results from these PEs are then compared to the ones obtained by using default templates, and the shifts in the average M_{top}^{corr} and $\Delta\mathrm{JES}^{corr}$ values are taken as the estimate of the systematic uncertainty. In some cases the statistical uncertainty on the shifts may be larger than the shifts themselves and therefore we use conservatively the former as systematic uncertainty.

a. **Residual bias** The calibration gets rid of the *average* biases, related especially to the templates parametrization by smooth probability density functions. Anyway, as can be observed in Fig. 7, residual biases usually exist at single $\{M_{top}^{in}, \Delta\mathrm{JES}^{in}\}$ points, and have to be taken into account. Similarly to what done to define a correction for

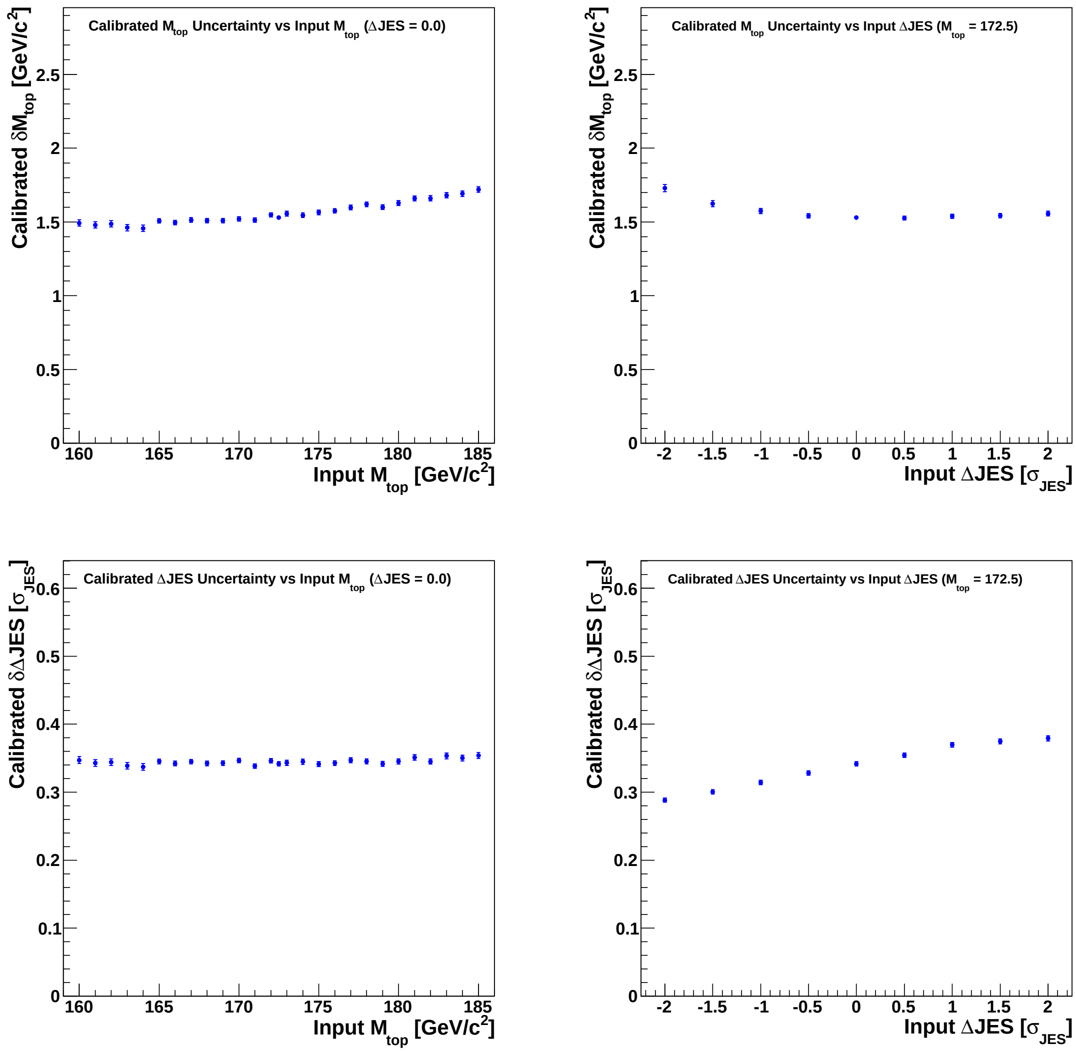

FIG. 9: The expected uncertainties on the top mass ($\delta M_{top}\,(stat{+}\mathrm{JES})$) and on the jet energy scale displacement ($\delta\Delta\mathrm{JES}\,(stat{+}M_{top})$) are shown as a function of M_{top}^{in} (at constant $\Delta\mathrm{JES} = 0$, left) and of $\Delta\mathrm{JES}^{in}$ for $M_{top}^{in} = 172.5\,\mathrm{GeV}/c^2$ (right), after both the calibration and the pull width corrections have been applied.

the calibrated uncertainties in section VIII B, to evaluate the residual bias we consider the *mean* of pull distributions at all different $\left\{M_{top}^{in},\, \Delta\mathrm{JES}^{in}\right\}$ points. Examples of pull means are shown in figure 10.

To consider properly the local biases, we perform separate averages of positive and negative pull means. This leads to

$$\begin{aligned}\delta M_{top}^{syst}\,(\mathrm{Res.\,Bias}) &\simeq \left({}^{+0.10}_{-0.14}\right)\cdot \delta M_{top}^{corr}\,(stat + \mathrm{JES})\\ \delta\Delta\mathrm{JES}^{syst}\,(\mathrm{Res.\,Bias}) &\simeq \left({}^{+0.12}_{-0.08}\right)\cdot \delta\Delta\mathrm{JES}^{corr}\,(stat + M_{top})\end{aligned}$$

This means that, at central points like $\left\{M_{top}^{in} = 172.5\,\mathrm{GeV}/c^2,\ \Delta\mathrm{JES}^{in} = 0\ \sigma_{\mathrm{JES}}\right\}$, systematic “residual bias” uncertainties of about $\left({}^{+0.15}_{-0.21}\right)\mathrm{GeV}/c^2$ for M_{top} and $\left({}^{+0.040}_{-0.025}\right)\ \sigma_{\mathrm{JES}}$ for ΔJES may be expected.

b. **Calibration** The uncertainties on the parameters of the calibration give a small uncertainty on the corrected values M_{top}^{corr} and $\Delta\mathrm{JES}^{corr}$. This can be calculated in each single measurement. At $M_{top}^{in} = 172.5\,\mathrm{GeV}/c^2$ and $\Delta\mathrm{JES}^{in} = 0\,\sigma_{\mathrm{JES}}$ we obtain on average $\delta M_{top}^{syst}\,(\mathrm{Calib}) \simeq 0.18\,\mathrm{GeV}/c^2$ and $\delta\Delta\mathrm{JES}^{syst}\,(\mathrm{Calib}) \simeq 0.020\ \sigma_{\mathrm{JES}}$ by this source of uncertainty.

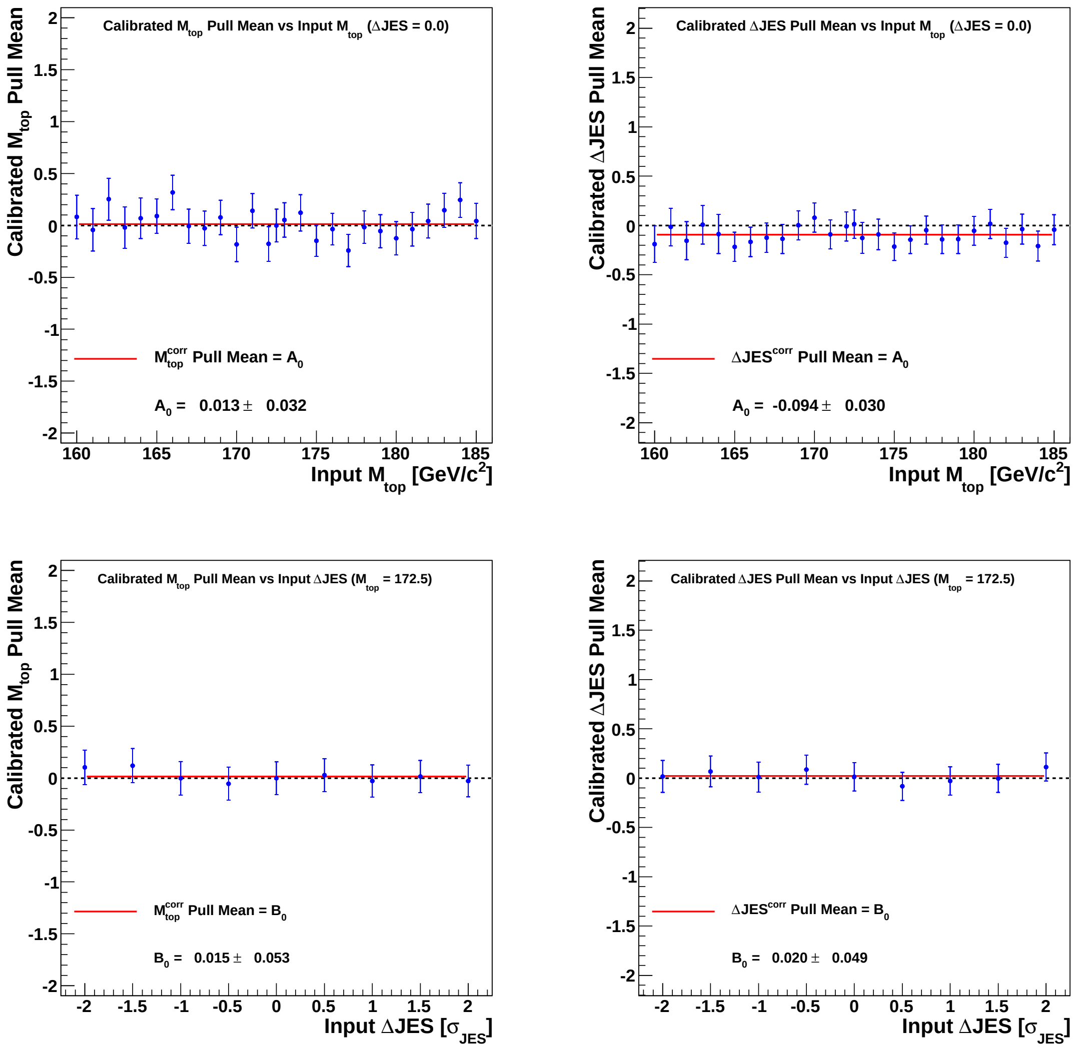

FIG. 10: M_{top} (on the left) and ΔJES (on the right) pull means as a function of M_{top}^{in} at constant $\Delta\text{JES} = 0\ \sigma_{\text{JES}}$ (upper plots) and as a function of ΔJES^{in} for $M_{top}^{in} = 172.5\,\text{GeV}/c^2$ (lower plots). The straight lines denote the fit by a constant function.

c. **Generator** Many sources of systematic effects arise from uncertainty in the Monte Carlo modeling of the hard interaction. PYTHIA and HERWIG generators differ in their hadronization schemes and in their description of the underlying event and multiple interactions. The default signal Monte Carlo samples have been generated by PYTHIA. Templates are built for events generated by HERWIG (at $M_{top}^{in} = 172.5\,\text{GeV}/c^2$, $\Delta\text{JES}^{in} = 0\ \sigma_{\text{JES}}$) and PEs are performed drawing pseudo-data from these distributions. By considering the shift with respect to the default, the estimated systematic uncertainties due to this source are $\delta M_{top}^{syst}\,(\text{Generator}) \simeq 0.48\,\text{GeV}/c^2$ and $\delta\Delta\text{JES}^{syst}\,(\text{Generator}) \simeq 0.211\ \sigma_{\text{JES}}$.

d. **Initial and final state radiation (IFSR)** Additional jets coming from possible emission of hard gluons might fall among the six leading jets and populate the tails in the top quark invariant mass distribution. The amount of radiation from partons in the initial (ISR) or final (FSR) state is set by parameters of the PYTHIA generator used to simulate signal events. To study these effects, templates are built using samples where the values of the parameters have been changed with respect to the default, to increase or to decrease the amount of radiation. Again, PEs are performed where pseudo-data are drawn from these modified templates and the results compared to the default. The resulting uncertainties are $\delta M_{top}^{syst}\,(\text{ISR/FSR}) \simeq 0.10\,\text{GeV}/c^2$ and $\delta\Delta\text{JES}^{syst}\,(\text{ISR/FSR}) \simeq 0.040\ \sigma_{\text{JES}}$.

e. ***b*-jets energy scale** Since the default jet energy corrections are derived on data samples deprived of heavy flavors, an additional uncertainty comes from considering the different properties of b quarks. We account for the uncertainties due to the b-quark semileptonic branching ratios, the fragmentation modeling, and the response of the calorimeters to b and c hadrons. Templates are built varying the default assumption for the three mentioned sources, and PEs are performed drawing pseudo-data from these modified distributions. The comparison to the default results gives systematic uncertainties $\delta M_{top}^{syst}\,(b\text{−jets}) \simeq 0.15\,\mathrm{GeV}/c^2$ and $\delta\Delta\mathrm{JES}^{syst}\,((b\text{−jets}) \simeq 0.050\;\sigma_{\mathrm{JES}}$.

f. ***b*-tagging efficiency** The different efficiency of the b-tagging algorithm on data and Monte Carlo simulated events is usually considered a constant Scale Factor (b-tag SF). However this value might have a dependence on the transverse energy of jets, leading to possible variations in the shapes of m_t^{rec} and m_W^{rec} templates. Since the background estimate is data-driven, the analysis is sensitive to an overall uncertainty in the b-tagging scale factor only through signal shapes. Signal templates are built taking into account the possible dependence of the SF on the jet E_T and then used in PEs. The corresponding systematic effects have been evaluated to be $\delta M_{top}^{syst}\,(b\text{−tag SF}) \simeq 0.09\,\mathrm{GeV}/c^2$ and $\delta\Delta\mathrm{JES}^{syst}\,(b\text{−tag SF}) \simeq 0.007\;\sigma_{\mathrm{JES}}$.

g. **Residual JES** Our templates are built displacing the value of the jet energy scale by fractions of its uncertainty σ_{JES}, as estimated in [2]. However σ_{JES} results from many independent effects with different behavior with respect to properties of jets like E_T and η, and represents therefore a leading order estimate. So, second order effects can arise from uncertainties on single levels of correction of the jet energies. To evaluate these possible effects, we build signal templates by varying separately by $\pm 1\,\sigma$ the single corrections and PEs were then performed by using these templates and not applying the constraint $\mathcal{L}_{\Delta\mathrm{JES}_{constr}}$ in the likelihood fit. The resulting uncertainties have been added in quadrature to obtain a "Residual JES" uncertainty on the top mass: $\delta M_{top}^{syst}\,(\text{Res. JES}) \simeq 0.45\,\mathrm{GeV}/c^2$

h. **Parton distribution functions** The choice of parton distribution functions (PDF) inside the proton can affect the kinematics of $t\bar{t}$ events and thus the top quark mass measurement. We estimate the uncertainty resulting from the possible PDF models by using our standard signal Monte Carlo samples and reweighting the events by their probability to occur according many different PDF's. Templates are built by weighted events, PEs are performed by extracting pseudo-data from these modified distributions and the shifts in the average M_{top}^{corr} and $\Delta\mathrm{JES}^{corr}$ values are taken as systematic uncertainties. We considered four sources of uncertainties:

1. The difference arising from the use of the default CTEQ5L [9] PDF and the one calculated from the MRST group, MRST72 [10].
2. The uncertainty depending on the value of α_s. This is evaluated by the difference between the use of MRST72 and MRST75 PDF's.
3. The uncertainty depending on the differences between the leading order (LO) and next-to-leading order (NLO) calculations of PDF, evaluated by the difference between using default CTEQ5L (LO) and CTEQ6M (NLO) PDF.
4. The uncertainties on PDF deriving from experimental data uncertainties. These are encoded by 20 pairs of values, where each pair corresponds to variations of $\pm 1\,\sigma$ of the experimental uncertainties on CTEQ6M PDF.

The resulting total uncertainties due to parton distributions are $\delta M_{top}^{syst}\,(\mathrm{PDF}) \simeq \left({}^{+0.23}_{-0.16}\right)\,\mathrm{GeV}/c^2$ and $\delta\Delta\mathrm{JES}^{syst}\,(\mathrm{PDF}) \simeq \left({}^{+0.026}_{-0.051}\right)\;\sigma_{\mathrm{JES}}$.

i. **Multiple Hadron Interactions** The probability to have multiple $p\bar{p}$ interactions during the same bunch-crossing is a function of the istantaneous luminosity. We account for the fact that our nominal MC for the signal description does not model the actual luminosity profile of the data, and that there is a residual dependence in the jet energy response in the MC as a function of the reconstructed number of primary vertices, even after specific corrections. The systematic due to the above effects is estimated to be $\delta M_{top}^{syst}\,(\mathrm{MHI}) \simeq 0.08\,\mathrm{GeV}/c^2$ and $\delta\Delta\mathrm{JES}^{syst}\,(\mathrm{MHI}) \simeq 0.036\;\sigma_{\mathrm{JES}}$.

j. **Color Reconnections** Uncertainties from modeling of color reconnections effects [11] are estimated by comparing the results of two sets of PEs performed drawing pseudo-data from templates built by Monte Carlo samples where different tunes of parameters have been set, corresponding to different models of color reconnections. This gives $\delta M_{top}^{syst}\,(\text{Color Reconn.}) \simeq 0.32\,\mathrm{GeV}/c^2$ and $\delta\Delta\mathrm{JES}^{syst}\,(\text{Color Reconn.}) \simeq 0.116\;\sigma_{\mathrm{JES}}$ for this source of uncertainties.

k. **Templates statistics** As mentioned in section VIII A, the shapes of signal and background templates are affected by uncertainties due to the limited statistics of the Monte Carlo (for the signal) and data (for the background) samples used to build them. These uncertainties affect the results of a measurement, which is performed by an unbinned likelihood where parametrized p.d.f.'s, fitted to default templates, are evaluated. We address this effect obtaining 200 sets of templates by statistical fluctuations of default ones, and performing pseudo-experiments drawing data from each of these sets separately. The spread in the average values of M_{top}^{corr} and $\Delta\mathrm{JES}^{corr}$ distributions is taken as systematic uncertainty. This was repeated at many (M_{top}^{in}, $\Delta\mathrm{JES}^{in}$) points and an average gives δM_{top}^{syst} (Templ. Stat.) $\simeq 0.27\,\mathrm{GeV}/c^2$ and $\delta\Delta\mathrm{JES}^{syst}$ (Templ. Stat.) $\simeq 0.052\ \sigma_{\mathrm{JES}}$.

l. **Background** Different kinds of systematic effects could be related to the modeling of the background. Uncertainties on the shape of background templates are taken into account by the "Templates statistics" systematic and, as it concerns the correction needed for the presence of signal in the pretag sample (section IV C) by the pseudoexperiments and calibration procedure, as described in section VIII A. The effects of the overall uncertainty on the background normalization are included in the uncertainty from the likelihood fit, as being the average number of background events a parameter of the fit itself. Anyway that parameter corresponds to the number of events in the JES-sample, while the numbers for the M_{top}-sample are derived from these. Infact in the likelihood function (section VII) the signal and background acceptances, $\mathcal{A}_s$ and $\mathcal{A}_b$, appear. The meaning of these variables are explained in sections VI and VI A. For the background the values of $\mathcal{A}_b$ have rather large uncertainty, but in the likelihood fit, as well as during the default PEs procedure, the values of $\mathcal{A}_b^{1\,tag}$ and $\mathcal{A}_b^{\geq 2\,tags}$ are kept constant to their central values, i.e. 46.9% and 42.5% respectively, section VI A. We perform PEs changing the input values of $\mathcal{A}_b$ by $\pm 1\,\sigma$ and consider the shifts of M_{top}^{corr} and $\Delta\mathrm{JES}^{corr}$ with respect to the default to obtain δM_{top}^{syst} (Background) $\simeq 0.55\,\mathrm{GeV}/c^2$ and $\delta\Delta\mathrm{JES}^{syst}$ (Background) $\simeq 0.112\,\sigma_{\mathrm{JES}}$.

m. **Trigger** The multijet trigger, used for the first online selection of $t\bar{t}$ candidate events in the data, is simulated on signal Monte Carlo events. Uncertainties on this simulation, possibly related to mismodeling of the energy deposition in the calorimeters and/or changes of the trigger algorithms and requirements not faithfully reproduced in the default Monte Carlo samples, are taken into account. Templates are built by events where the trigger simulation has been modified and PEs performed drawing pseudo-data from them. Comparison to the default PEs leads to uncertainties δM_{top}^{syst} (Trigger) $\simeq 0.20\,\mathrm{GeV}/c^2$ and $\delta\Delta\mathrm{JES}^{syst}$ (Trigger) $\simeq 0.042\ \sigma_{\mathrm{JES}}$.

A. Total systematic uncertainty

Table II shows a summary of all the systematic uncertainties and their quadrature sum, which gives a total systematic uncertainty of $1.1\,\mathrm{GeV}/c^2$ for the M_{top} measurement and $0.3\ \sigma_{\mathrm{JES}}$ for the ΔJES, where the "residual bias" uncertainty, depending on the statistical errors, is already evaluated at the values given by the measurement on the data, described in section X.

X. THE TOP QUARK MASS MEASUREMENT

After the kinematic selections with $N_{out} \geq 0.97$ ($N_{out} \geq 0.94$), $\chi^2\,(m_W^{rec}) \leq 2.0$ ($\chi^2\,(m_W^{rec}) \leq 3.0$) and $\chi^2\,(m_t^{rec}) \leq 3.0$ ($\chi^2\,(m_t^{rec}) \leq 4.0$) for events with 1 tag ($\geq$ 2 tags), we are left with 4368 and 2256 events in the JES-sample and M_{top}-sample with 1 tag respectively, and 1196 and 600 events in the corresponding samples with $\geq$ 2 tags. The expected background, corrected for the contribution due to $t\bar{t}$ events amounts to 3652 ± 181 (1-tag JES-sample), 1712 ± 77 (1-tag M_{top}-sample), 718 ± 14 ($\geq$ 2-tag JES-sample), and 305 ± 22 ($\geq$ 2-tag M_{top}-sample) events. The likelihood fit described in Sec. VII has been applied to the data samples to derive the best top quark mass and jet energy scale displacement from the default value to be

$$\begin{aligned} M_{top}^{fit} &= 172.45 \pm 1.48\,(stat + \mathrm{JES})\,\mathrm{GeV}/c^2 \\ \Delta\mathrm{JES}^{fit} &= -0.038 \pm 0.285\,(stat + M_{top})\,\sigma_{\mathrm{JES}} \end{aligned}$$

Figure 11 shows the behavior of the likelihood as a function of the M_{top} and ΔJES parameters and the contours corresponding to variations of one, two and three standard deviations of the same parameters with respect to the values maximizing the likelihood itself (before the calibration).

Source	δM_{top}^{syst} (GeV/c^2)	$\delta\Delta\text{JES}^{syst}$ (σ_{JES})
Residual bias	0.2	0.03
Calibration	0.1	0.01
Generator	0.5	0.21
ISR/FSR	0.1	0.04
b-jets energy scale	0.2	0.05
SF E_T dependence	0.1	0.01
Residual JES	0.4	−−
PDF	0.2	0.04
Multiple Hadron Interactions	0.1	0.04
Color Reconnections	0.3	0.12
Templates Statistics	0.3	0.05
Background	0.6	0.11
Trigger	0.2	0.04
Total	1.1	0.29

TABLE II: Breakdown of *observed* systematic uncertainties from different sources and their respective amount. The contribution depending on the statistical errors (i.e. the "residual bias") has been calculated here by the values observed in the measurement on data. The total uncertainty is obtained by the quadrature sum of single contributions.

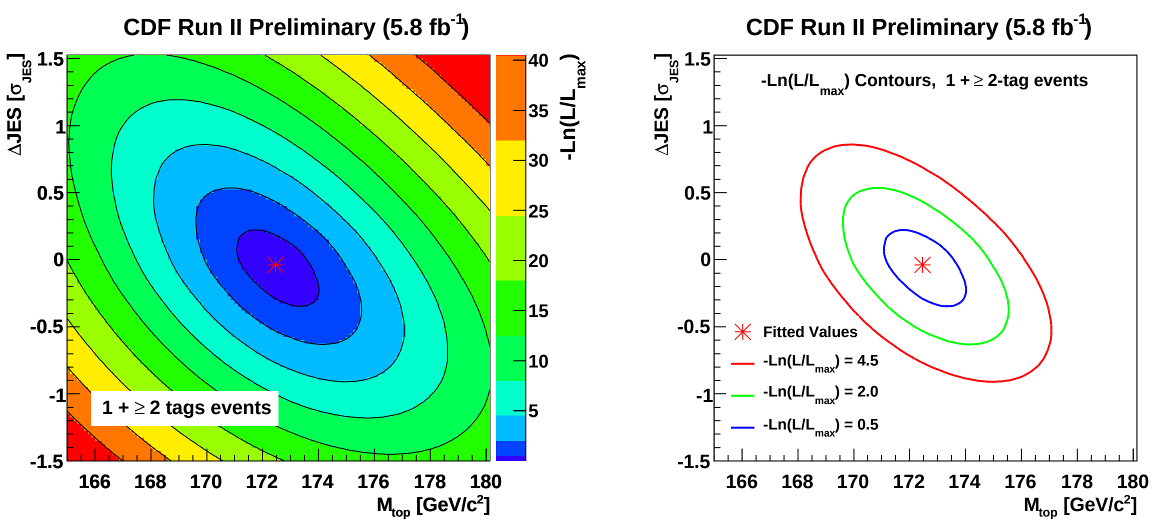

FIG. 11: Measured likelihood as a function of the M_{top} and ΔJES parameters (left) and contours corresponding to variations of the same parameters of one, two and three standard deviations as given by MINOS (right). The fitted central values, corresponding to the maximum likelihood (or minimum $-\ln\mathcal{L}$), are also shown.

These values have to be calibrated and the uncertainties have then also to be corrected by multiplicative factors 1.06 and 1.07 for M_{top} and ΔJES respectively, as mentioned in section VIII B, so that we finally obtain:

$$M_{top}^{corr} = 172.47 \pm 1.72\,(stat + \text{JES})\,\text{GeV}/c^2$$
$$\Delta\text{JES}^{corr} = -0.105 \pm 0.331\,(stat + M_{top})\,\sigma_{\text{JES}}$$

The purely statistical part of the uncertainty can be isolated and the results written as:

$$
\begin{aligned}
M_{top}^{corr} &= 172.47 \pm 1.43\,(stat) \;\; \pm 0.96\,(\mathrm{JES})\,\mathrm{GeV}/c^2 \\
\Delta\mathrm{JES}^{corr} &= -0.105 \pm 0.276\,(stat) \;\; \pm 0.183\,(M_{top})\,\sigma_{\mathrm{JES}}
\end{aligned}
$$

The whole set of parameters, as measured in the data by the likelihood fit, is summarized in Table III together with the corrected values.

Variable	Fitted value	Calibrated value
M_{top}	172.46 ± 1.48	172.47 ± 1.72
ΔJES	-0.04 ± 0.285	-0.10 ± 0.33
$n_s^{1\,tag}$	925 ± 86	904 ± 97
$n_b^{1\,tag}$	3463 ± 92	3482 ± 102
$n_s^{\geq 2\,tags}$	449 ± 31	446 ± 32
$n_b^{\geq 2\,tags}$	724 ± 13	725 ± 14

TABLE III: The values of free parameters and their uncertainties as fitted by MINUIT in the data by the likelihood fit, and their values after the calibration. For δM_{top}^{corr} and $\delta\Delta\mathrm{JES}^{corr}$ also the multiplicative correction factors evaluated by the pull widths have been applied.

Summarizing, including the systematic uncertainties, the measured values for the top quark mass and the jet energy scale are:

$$
\begin{aligned}
M_{top} &= 172.5 \;\; \pm 1.7\,(stat + \mathrm{JES}) \pm 1.1\,(syst)\,\mathrm{GeV}/c^2 \\
\Delta\mathrm{JES} &= -0.10 \;\; \pm 0.3\,(stat + M_{top}) \pm 0.3\,(syst)\,\sigma_{\mathrm{JES}}
\end{aligned}
$$

or, dividing completely the statistical and systematic contibutions

$$
\begin{aligned}
M_{top} &= 172.5 \;\; \pm 1.4\,(stat) \;\; \pm 1.4\,(syst)\,\mathrm{GeV}/c^2 \\
\Delta\mathrm{JES} &= -0.10 \;\; \pm 0.3\,(stat) \;\; \pm 0.3\,(syst)\,\sigma_{\mathrm{JES}}
\end{aligned}
$$

The plots in Fig. 12 show the m_t^{rec} and m_W^{rec} distributions for the data compared to the probability density functions corresponding to the fitted values of M_{top} and ΔJES, while in Fig. 13 the N_{out} distributions are shown for a top quark mass of $172.5\,\mathrm{GeV}/c^2$ and a jet energy scale displacement of $0\ \sigma_{\mathrm{JES}}$, that is the values of simulated M_{top} and ΔJES as close as possible to the measurements in the data. In all these plots the signal and background contributions are normalized to the respective number of events as fitted in the data.

The plots in Fig. 14 compare the observed calibrated uncertainties, to the expected distribution from default pseudo-experiments using as input mass $M_{top} = 172.5\,\mathrm{GeV}/c^2$ and $\Delta\mathrm{JES} = 0\ \sigma_{\mathrm{JES}}$, i.e. the available templates with input top quark mass and ΔJES as close as possible to the values measured in the data. We find that the probability of achieving a better sensitivity is 89.2% for M_{top} and 35.3% for ΔJES.

XI. CONCLUSIONS

We described in this note the Template Method technique with *in situ* calibration used to measure the top quark mass on the latest available data sample, corresponding to an integrated luminosity of $5.8\,\mathrm{fb}^{-1}$. The method has been studied and calibrated through thousands of pseudo-experiments and the systematic uncertainties estimated by the same procedure. We then applied the technique to the data to measure a top quark mass of $[172.5\;\pm 1.4\,(stat)\;\pm 1.4\,(syst)]\,\mathrm{GeV}/c^2$ and a displacement of the jet energy scale from the value measured in [2] of $[-0.1\;\pm 0.3\,(stat)\;\pm 0.3\,(syst)]\,\sigma_{\mathrm{JES}}$, in units of the uncertainty on that value itself.

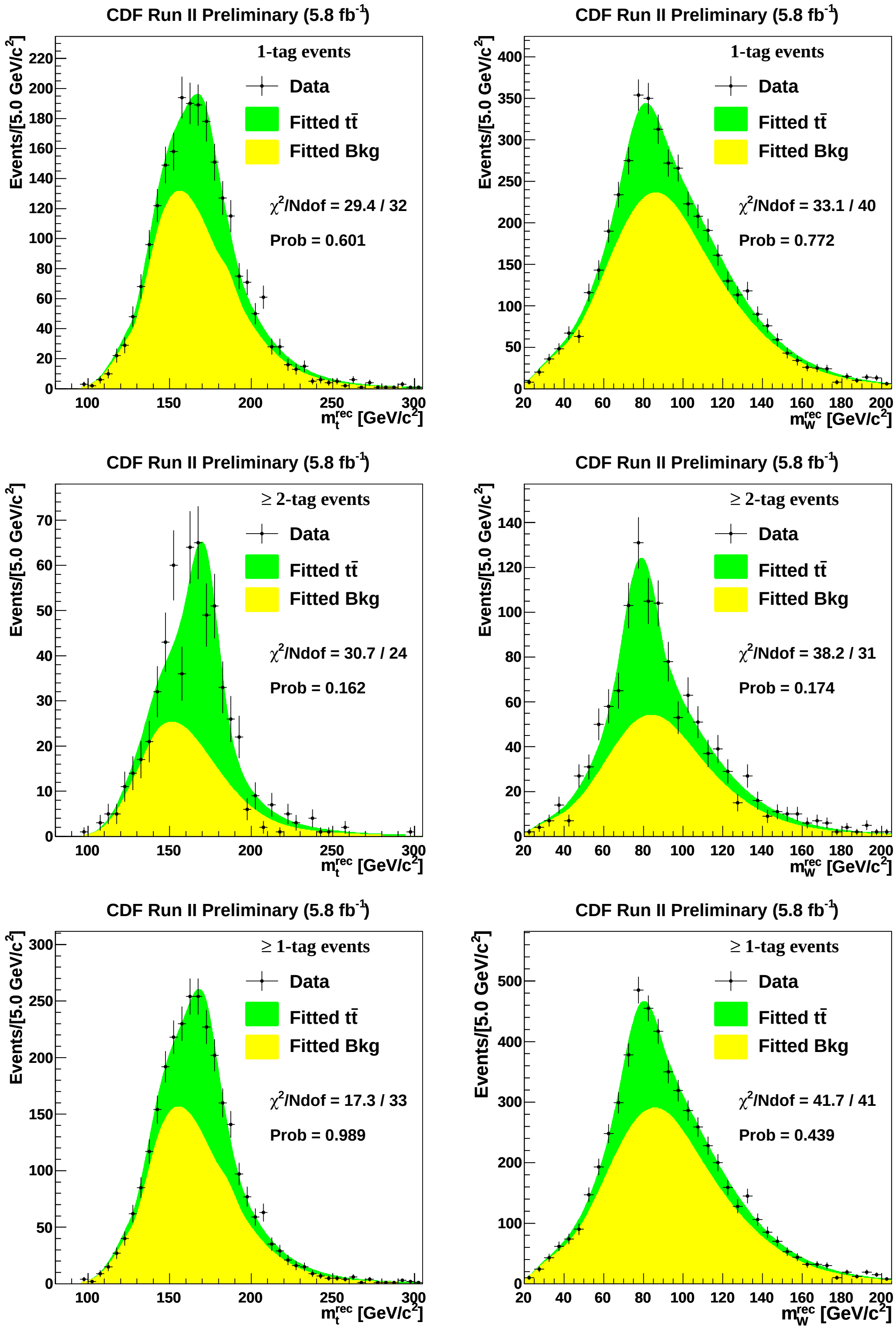

FIG. 12: Distributions of m_t^{rec} (left plots) and m_W^{rec} (right plots) as obtained in the data (black points) are compared to the probability density functions from signal and background corresponding, both in shape and normalization, to the likelihood fit parameters measured in the data. The upper and middle plots show distributions for the 1-tag and $\geq$ 2-tags samples respectively, while the lower plots are their sum.

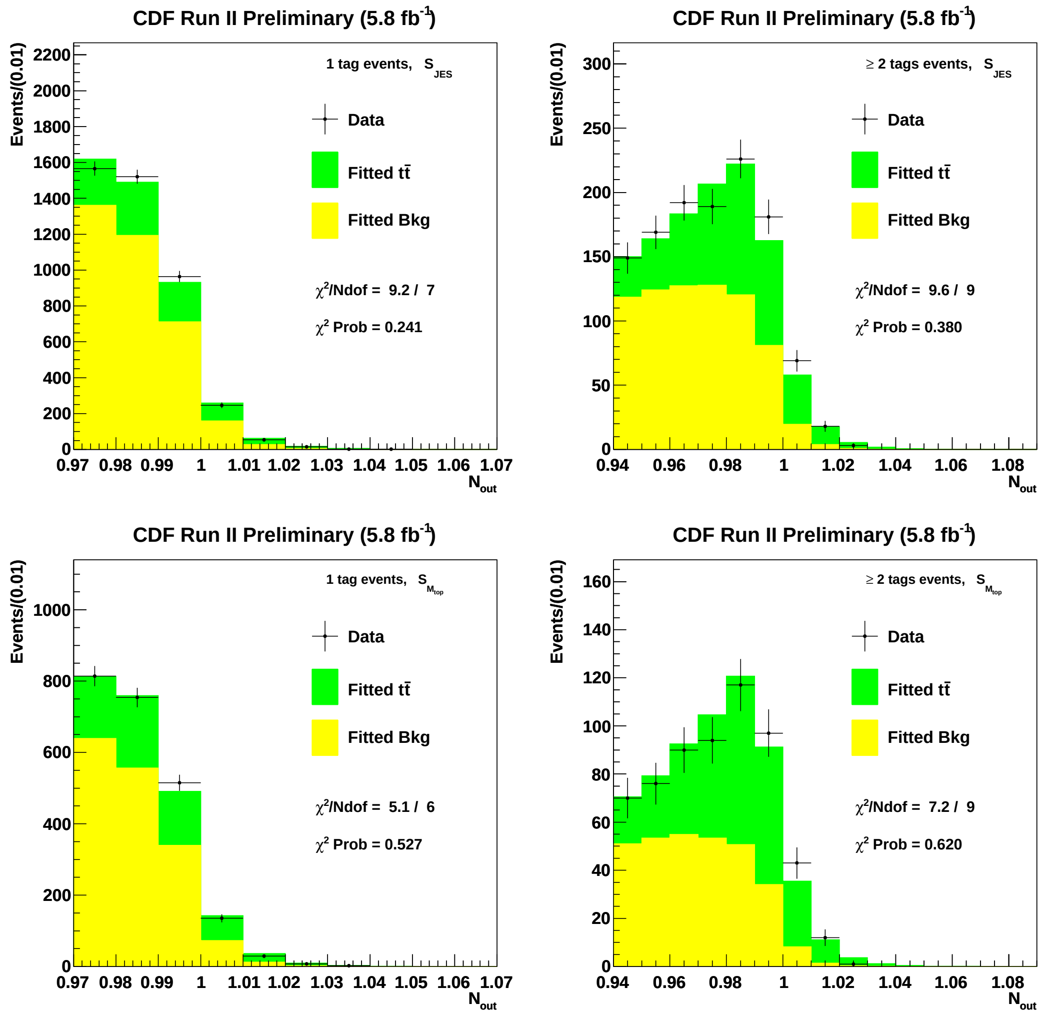

FIG. 13: Distributions of the output from the Neural Net as obtained in the data (black points) are compared to the distributions from signal and background corresponding to $M_{top} = 172.5$ GeV/c^2 and ΔJES $= 0\ \sigma_{\rm JES}$, i.e. the values of simulated M_{top} and ΔJES as close as possible to the measurements in the data. The expected histograms are normalized to the measured values for the average number of signal and background events. The upper plots show the distributions in the JES-sample for the 1-tag (left) and $\geq$ 2-tags events (right) respectively, while the lower plots show the same distributions for events in the M_{top}-sample.

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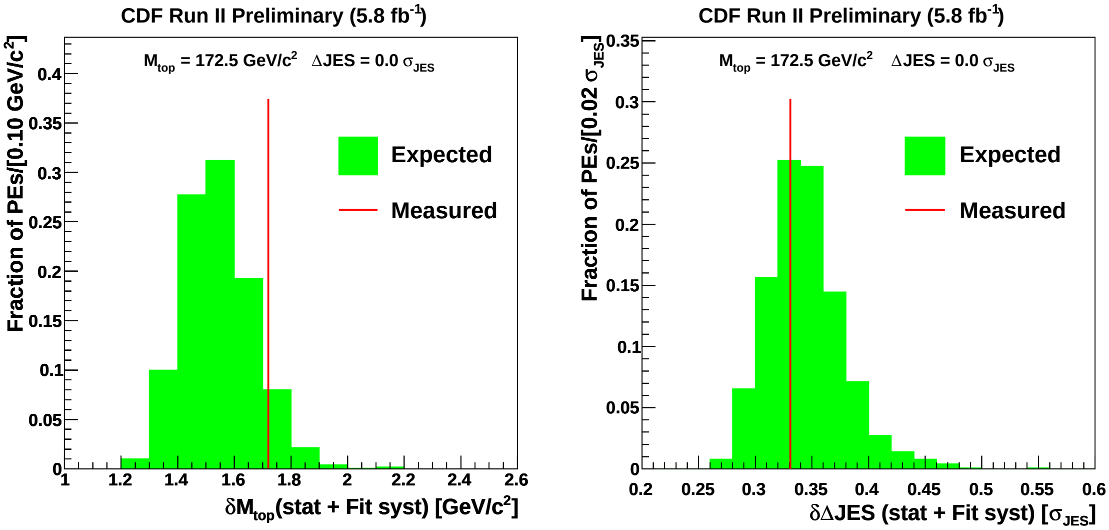

FIG. 14: Uncertainties on the top quark mass (left) and the jet energy scale displacement (right) as measured in default PEs performed at $M_{top}^{in} = 172.5\,\mathrm{GeV}/c^2$ and JES $= 0\ \sigma_{\mathrm{JES}}$, i.e. using the available set of PEs with input top quark mass and ΔJES as close as possible to the values measured in the data. The red lines indicate the uncertainties obtained in the data.

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[12] The $\geq$ 2-tags sample actually consists of events with 2 or 3 tagged jets. When 3 tags are present, the 3 different possible assignments of two out of three jets to b quarks are also tested, with the remaining tagged jet considered as a light quark
[13] Given the large number of pseudo-experiments, fluctuations due to the PEs statistic are negligible