CDF note 10444

Combination of CDF top quark mass measurements
(Winter 2011)

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We summarize the CDF measurements of the top-quark mass, M_{top}, performed analyzing $110\,\mathrm{pb}^{-1}$ of data collected at 1.8 TeV center of mass collision energy (Run I), and as much as $5.8\,\mathrm{fb}^{-1}$ of data collected at 1.96 TeV (Run II). We combine the most precise CDF published and preliminary results to obtain $M_{top} = 172.70 \pm 0.63\,(\mathrm{stat}) \pm 0.89\,(\mathrm{syst})\,\mathrm{GeV}/c^2$ which corresponds to a total uncertainty of $1.09\,\mathrm{GeV}/c^2$. The combination provides a relative precision $\Delta M_{top}/M_{top}$ of 0.63%. Rounding off to two significant digits, the combination yields $M_{top} = 172.7 \pm 1.1(\mathrm{stat}+\mathrm{syst})\,\mathrm{GeV}/c^2$. This result includes the new top mass in all-hadronic channel and the first time determined mass in $\not\!\!E_T$+Jets sample.

Preliminary Results for Winter 2011 Conferences

I. INTRODUCTION

We combine the CDF published top-quark mass results from the Tevatron run at 1.8 TeV center of mass collision energy (Run I) performed analyzing 110 pb^{-1} of data [1-3] with two published [4,5] and three preliminary results [6,7,8] that use up to 5.8 fb^{-1} of data collected at the Tevatron run at 1.96 TeV center of mass energy (Run II).

Results from the $t\bar{t} \rightarrow qq'bqq'\bar{b}$ (HAD), $t\bar{t} \rightarrow \ell\nu qq'b\bar{b}$ (LJT), and $t\bar{t} \rightarrow \ell^+\nu b\ell^-\overline{\nu}\bar{b}$ (DIL) final states are included. For first time, additional measurement of the top quark mass from $\not\!\!E_T$+Jets is used in the combination. It is statistically independent from LJT,DIL and HAD. These measurements are combined accounting for statistical and systematic correlations using the method of reference [9].

Relative to the previous CDF combination reported in [10], this combination includes updates of the Run II analyses in the HAD channel (5.8 fb^{-1}) and new result in the $\not\!\!E_T$+Jets channel (5.7 fb^{-1}). Stil, the Summer 2010 measurement in the LJT channel using Matrix Element technique keeps the single most precise result, with a 0.72% relative precision on $\Delta M_{top}/M_{top}$, more precise than the '09 Tevatron top mass combination [11].

The uncertainty categories used in the combination are detailed in Section II while the input measurements themselves are summarized in Section III. The correlations used in the combination are discussed in Section IV and the resulting top-quark mass is given in Section V. Section VI summarizes the results obtained.

II. UNCERTAINTY CATEGORIES

The uncertainty categories did not change from the previous CDF combination [10] and they correspond to the same uncertainty categories as used for the Tevatron world average [11]. They have evolved to include a detailed breakdown of the various sources of uncertainty and aim to lump together sources of systematic uncertainty that share the same or similar origin. For example, the "Signal" category discussed below includes the uncertainties from initial state radiation (ISR), final state radiation (FSR), and parton distribution functions (PDF) - all of which affect the modeling of the $t\bar{t}$ signal. The dominant systematic, the jet energy scale (JES) uncertainty, is sub-divided into several components ("levels") in order to more accurately accommodate our best estimate of the relevant correlations [12]. Additional categories have been added in order to accommodate specific types of correlations. Each uncertainty category is discussed below.

Statistical: The statistical uncertainty associated with the M_{top} determination.

iJES: The statistical uncertainty on the JES arising from the in-situ $W \rightarrow qq'$ calibration alone. Residual JES uncertainties, which arise from effects not considered in the in-situ calibration, are included in the Method category below.

aJES: This is specific to DØ Run II and is only included here in order to be consistent with reference [11].

bJES: The systematic uncertainty specific to the modeling of b-jets. This includes uncertainties arising from variations in the semi-leptonic branching fraction, b-fragmentation modeling, and differences in the color flow between b-quark jets and light-quark. This is usually labeled "B Jet" for CDF Run II analyses.

cJES: The systematic uncertainty on the JES arising from the modeling of the out-of-cone corrections. This is the quadrature sum of the L7 and L8 JES uncertainties for CDF Run II analyses.

dJES: The systematic uncertainty on the JES arising from the relative corrections. This is the L1 JES uncertainties for CDF Run II analyses.

rJES: The systematic uncertainty on the JES arising from the modeling of the calorimeter response, the underlying event, and the multiple interaction corrections. This is the quadrature sum of the L4, L5, and L6 JES uncertainties for CDF Run II analyses.

Lepton P_T: The systematic uncertainty arising from uncertainties in the scale of the lepton transverse momentum measurements. This is an important uncertainty in the analysis with minimal dependence on the jet energy scale. It was not considered a source of systematic uncertainty in the Run I measurements.

Signal: The systematic uncertainty arising from uncertainties in the modeling of the $t\bar{t}$ signal including variations in the ISR, FSR, and PDF descriptions used to generate the $t\bar{t}$ Monte Carlo samples that calibrate each method.

Generator: The systematic uncertainty associated with variations of the physics model used to calibrate the fit methods and correlated across all measurements. It includes variations observed when substituting Pythia [13–15] (Run I and Run II) or ISAJET [16] (Run I) for HERWIG [17, 18] when modeling the $t\bar{t}$ signal.

Uranium noise and multiple interactions (UN/MI): This is specific to DØ Run I and is only included here in order to be consistent with reference [11].

Background: The systematic uncertainty arising from uncertainties in modeling the dominant background sources, including q^2 variations. This is the quadrature sum of the "Background Shape" and "Background normalization" uncertainties for most CDF Run II analyses.

Method: The systematic uncertainty arising from any source specific to a particular fit method, including the variations in B-tagging efficiency and the finite Monte Carlo statistics available to calibrate each method. This is the quadrature sum of the "Method", "B-tag", and "MC Statistics" categories for most CDF Run II analyses.

Color Reconnections (CR): The systematic uncertainty arising from a variation of the phenomenological description of color reconnection between final state particles [19]. This is obtained taking the difference between PYTHIA 6.4 tune "Apro" and PYTHIA 6.4 tune "ACRpro" that only includes a change in the color reconnection model. Monte Carlo generators which explicitly include CR models for hadron collisions have recently become available. This was not possible in Run I and these measurements do not include this source of systematic uncertainty.

Multiple Hadron Interactions (MHI): The systematic uncertainty arising from a mismodeling of the distribution of number of collision per bunch crossing due to the change in the collider instantaneous luminosity during data-taking. It has been separated from other sources to account for the fact that it is uncorrelated with D0's measurements.

These categories represent the current preliminary understanding of the various uncertainty categories and their correlations. We expect these to evolve as we continue to probe each method's sensitivity to the various systematic sources with ever improving precision. Small variations in the assignment of uncertainties to the uncertainty categories and in the correlations assumed negligibly affect the combination.

III. INPUT MEASUREMENTS

For this combination we use eight measurements: three published Run I [1-3] results, two published Run II results [4,5] and three preliminary Run II results [6,7,8]. They are summarized in Table I. The correlations between the various inputs are described in the next section. Based on studies described in reference [20] the statistical correlation between the L_{xy} + P_T^{lep} and LJT inputs is set to zero in the combination. Variations of ±0.5, which cover the full range observed in these studies, negligibly affect the combination reported here. Additionally, the new measurement from the $\not\!\!E_T$+Jets sample is treated as statistically uncorrelated with the other samples.

There are other CDF measurements in the LJT, DIL, HAD, and minimal-JES-dependence channels. We have chosen to include in this combination the analysis methods which yielded the best *expected* sensitivity for each channel. The channels themselves are statistically independent of each other and are treated as such.

For the L_{xy}+P_T^{lep} measurement, we include the systematic uncertainty associated with the potential mis-modeling of the background decay length distribution in the "Method" category, as it is a source of uncertainty unique to this method.

For the Run I measurements, we back propagate the systematic uncertainty specific to B-jets (bJES) as determined in Run II and then correct the Run I absolute corrections (rJES) to keep the total JES uncertainty constant. We perform a series of cross-check to probe the robustness of the result. We propagate using the estimation of the color reconnection systematic performed in Run II back to Run I measurements as well, finding no noticeable difference in the combination. Another cross-check is to back-propagate the Lepton P_T systematic back to Run I results; again, no noticeable difference was found in the combination. Variations of the above assumptions were explored and found to negligibly affect the final value.

IV. CORRELATIONS

The following correlations are used when making the combination:

Input	Run II Published LJT	L_{xy}+P_T^{lep}	Run II Preliminary DIL	HAD	$\not\!\!E_T$+Jets	Run I Published LJT	DIL	HAD
M_{top}	173.00	170.70	170.56	172.47	172.32	176.1	167.4	186.0
Statistical	0.65	6.30	2.19	1.43	1.80	5.1	10.3	10.0
iJES	0.58	0.0	0.0	0.95	1.54	-	-	-
aJES	-	-	-	-	-	-	-	-
bJES	0.26	0.0	0.35	0.15	0.29	0.6	0.8	0.6
cJES	0.27	0.06	2.01	0.24	0.20	2.7	2.6	3.0
dJES	0.01	0.02	0.64	0.04	0.05	0.7	0.6	0.3
rJES	0.41	0.11	1.98	0.38	0.45	3.4	2.7	4.0
Lepton P_T	0.14	1.20	0.31	-	0.00	-	-	-
Signal	0.21	0.86	0.36	0.22	0.29	2.6	2.8	1.8
Generator	0.37	0.30	0.57	0.48	0.65	0.1	0.6	0.8
UN/MI	-	-	-	-	-	-	-	-
Background	0.34	1.70	0.27	0.75	0.12	1.3	0.3	1.7
Method	0.10	1.13	0.05	0.38	0.14	0.0	0.7	0.6
Color Reconnections	0.37	0.40	0.61	0.32	0.20	-	-	-
Multiple Hadron Interactions	0.10	0.70	0.27	0.08	0.16	-	-	-
Statistical	0.65	6.20	2.19	1.43	1.80	5.1	10.3	10.0
Systematics	1.06	3.10	3.09	1.49	1.82	5.3	4.9	5.7
Total	1.24	6.90	3.79	2.06	2.56	7.3	11.4	11.5

TABLE I: The measurements used to determine the CDF combined top-quark mass. All numbers are in units of GeV/c^2. The uncertainty categories and their correlations are defined in the text. Some of the systematic sources were not accounted for in Run I measurements, due to them being negligible with respect to the actual measurements.

- The uncertainties in the Statistical, Method, and iJES categories are taken to be uncorrelated among the measurements.
- The uncertainties in the aJES, dJES, Lepton P_T and Multiple Hadron Interaction categories are taken to be 100% correlated among all Run I and all Run II measurements, but uncorrelated between Run I and Run II.
- The uncertainties in the Background category are taken to be 100% correlated among all measurements in the same channel.
- The uncertainties in the bJES, cJES, rJES, Signal, Generator and Color Reconnection categories are taken to be 100% correlated among all measurements.

Using the inputs from Table I and the correlations specified here, the resulting matrix of total correlation coefficients is given in Table II.

To help quantify how sensitive the combination is to these assumptions we vary some of these correlations and redo the combination. In Summer 2010 combination, we changed all correlations that were set to be 100%, to a 50% value and found a shift in the combined M_{top} of less than 50 MeV/c^2, and a change in precision of less than 10 MeV/c^2. We also varied the statistical correlation for the two very different LJT analyses from 0% to 50%, and found a shift and change in precision less than 10 MeV/c^2.

	Run II Published LJT	L_{xy}+P_T^{lep}	Run II Preliminary DIL	HAD	$\not\!\!E_T$+Jets	Run I Published LJT	DIL	HAD
LJT	1							
L_{xy}+P_T^{lep}	0.15	1						
DIL	0.43	0.06	1					
HAD	0.24	0.04	0.24	1				
$\not\!\!E_T$+Jets	0.22	0.04	0.21	0.14	1			
LJT	0.36	0.04	0.47	0.17	0.16	1		
DIL	0.20	0.03	0.28	0.11	0.10	0.29	1	
HAD	0.23	0.15	0.35	0.18	0.13	0.32	0.19	1

TABLE II: The resulting matrix of total correlation coefficients used to determine the CDF combined top quark mass.

V. RESULTS

Using the measurements of Table I and the correlations of Section IV the CDF combined top mass is

$$M_{top} = 172.70 \pm 0.63\,(\text{stat}) \pm 0.89\,(\text{syst})\ \text{GeV}/c^2 \quad (1)$$
$$= 172.70 \pm 1.09\ \text{GeV}/c^2 \quad (2)$$

with $\chi^2/dof = 3.4/7$, which corresponds to a chi-squared probability of about 85%, indicating good agreement among all the input measurements. The combined value is 0.4 GeV/c^2 lower than the previous CDF combination [10], due to an downward shift in the mass determination in the Run, II HAD channel [8]. It is also 0.3 GeV/c^2 lower than [4] due to lower masses in HAD and $\not\!\!E_T$+Jets.

This CDF-only combination has a relative precision of $\Delta M_{top}/M_{top} = 0.63\%$. The combined value presented here improves the precision by 13% with respect to the single most precise measurement [5]. This CDF combination improves the precision on M_{top} by 6% with respect to the previous CDF best determination [10].

The method of reference [9] decomposes the total uncertainty into the contributions from the various uncertainty categories; the results can be found in Table III.

Results	CDF combined values
M_{top}	172.70
iJES	0.51
aJES	-
bJES	0.23
cJES	0.11
dJES	0.02
rJES	0.24
Lepton P_T	0.10
Signal	0.13
Generator	0.42
UM	-
Background	0.27
Fit	0.11
CR	0.35
MHI	0.11
Syst.	0.89
Stat.	0.63
Total	1.09

TABLE III: Summary of the Tevatron combined world average M_{top} . The error categories are described in the text. The total systematic uncertainty and the total uncertainty are obtained by adding the relevant contributions in quadrature. All numbers are in units of GeV/c^2.

The corresponding pull and weight for each of the inputs are listed in Table IV.

	Run II Published		Run II Preliminary			Run I Published		
	LJT	L_{xy}+P_T^{lep}	DIL	HAD	$\not\!\!E_T$+Jets	LJT	DIL	HAD
Pull	0.51	-0.30	-0.59	-0.13	-0.16	0.47	-0.47	1.16
Weight (%)	+76.52	0.46	-2.78	19.25	10.65	-2.65	-0.46	-0.99

TABLE IV: The pull and weight for each of the inputs used to determine the CDF combined top quark mass.

The input measurements, the output combination and the χ^2/dof are summarized in Figure 2.

The weights of some of the measurements are negative. In general, this situation can occur if the correlation between two measurements is larger than the ratio of their total uncertainties. This is indeed the case here. In these instances the less precise measurement will usually acquire a negative weight. While a weight of zero means that a particular input is effectively ignored in the combination, a negative weight means that it affects the resulting central value and helps reduce the total uncertainty. See Reference [22] for further discussion of negative weights. To visualize the weight each measurement carries in the combination, weights have been divided by the sum of the modulo of all the input measurements weights. The result is shown in Figure 1.

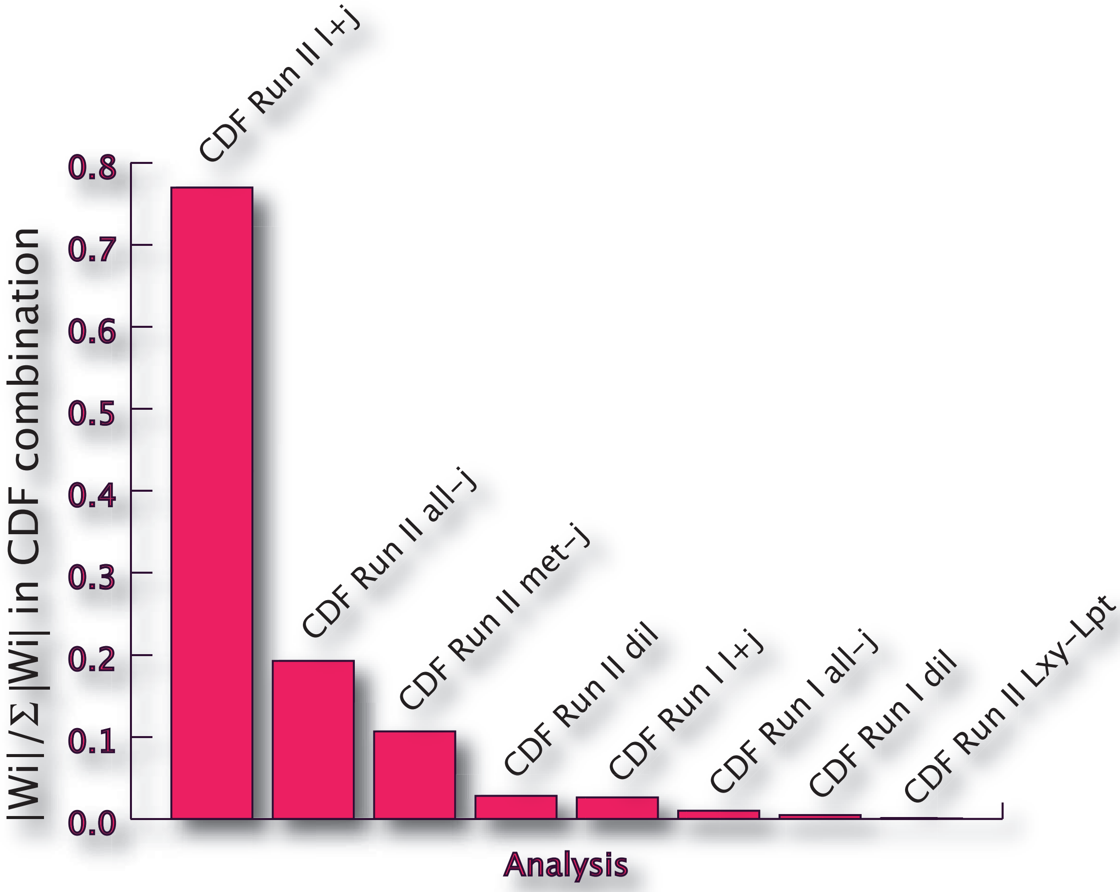

FIG. 1: Relative weights of the input measurements in the combination. The relative weights have been obtained dividing each measurement weight by the sum over all measurements of the modulo of their weights.

Although the chi-squared from the combination of all measurements indicates that there is good agreement among them, and no input has an anomalously large pull, it is still interesting to also fit for the top mass in the LJT, DIL, HAD and $\not\!\!E_T$+jets channels separately. We use the same methodology and include the systematic correlations among the measurements as described in Section IV. The results are shown in Table V.

Using the expression in reference [21] we calculate the following chi-squares $\chi^2(LJT-DIL) = 1.62/1$, $\chi^2(LJT-HAD) = 0.02/1$, $\chi^2(LJT- \not\!\!E_T + jets) = 0.07/1$, $\chi^2(DIL-HAD) = 1.07/1$, $\chi^2(DIL- \not\!\!E_T + Jets) = 0.69/1$ and $\chi^2(HAD- \not\!\!E_T + Jets) = 0.02/1$. These correspond to chi-squared probabilities of 20%, 89%, 79%, 30%, 41%, 89% respectively, and indicate that all channels are reasonably consistent with each other.

		correlations			
	fit value (GeV/c^2)	M(LJT)	M(DIL)	M(HAD)	$\not\!\!E_T$+Jets
M(LJT)	172.69 ± 1.20	1			
M(DIL)	168.77 ± 3.31	0.38	1		
M(HAD)	172.43 ± 2.04	0.22	0.22	1	
$\not\!\!E_T$+Jets	172.00 ± 2.53	0.19	0.17	0.12	1

TABLE V: The results of a fit to determine the top-quark mass in the three final states separately.

Using only the Run II measurements of Table I and the correlations of Section IV the CDF combined Run II top mass is

$$M_{top} \; = \; 172.97 \pm 1.12 \,\mathrm{GeV}/c^2 \tag{3}$$

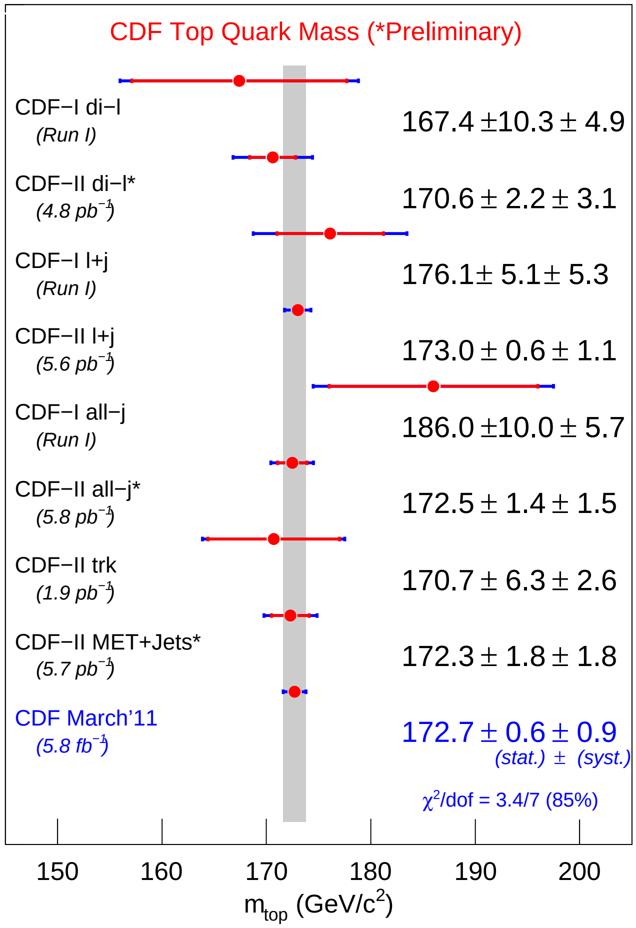

FIG. 2: A summary of the input measurements utilized in this combination, and the resulting CDF combined top-quark mass. The preliminary results are the ones that carry the asterisk sign. All other results have been published.

with $\chi^2/dof = 0.678/4$, which corresponds to a chi-squared probability of about 95%, indicating good agreement among all the input measurements. The combined value is $300\,\mathrm{MeV}/c^2$ higher than the Run I plus Run II CDF combination, and has only $20\,\mathrm{MeV}/c^2$ larger uncertainty.

VI. CONCLUSION

We have combined CDF Run I and Run II top-quark mass measurements from four final states, LJT, DIL, HAD and $\not\!\!E_T$ to get a CDF combined top-quark mass of $172.70 \pm 1.09\,\mathrm{GeV}/c^2$. In this combination, for first time, a new sample of $\not\!\!E_T$+Jets events was used. This CDF-only combination has thus a relative precision of $\Delta M_{top}/M_{top} = 0.63\%$. This corresponds to a relative improvement of 7% with respect to the previous CDF combination [10] and 3% worse with respect to the previous 2010 Tevatron combination [11]. Rounding off to two significant digits, the combination yields $M_{top} = 172.7 \pm 1.1(\mathrm{stat} + \mathrm{syst})\ \mathrm{GeV}/c^2$.

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