



## Combination of CDF top quark mass measurements using up to $3.2 \text{ fb}^{-1}$ of data

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We summarize the CDF measurements of the top-quark mass,  $M_{top}$ , based on data sets including as much as  $3.2 \text{ fb}^{-1}$  of data. We combine the most recent preliminary Run-II results with the published Run-I results to obtain,  $M_{top} = 172.6 \pm 0.9 \text{ (stat)} \pm 1.2 \text{ (syst)} \text{ GeV}/c^2$ , which corresponds to a total uncertainty of  $1.5 \text{ GeV}/c^2$  and a relative precision  $\Delta M_{top}/M_{top}$  of 0.85%.

*Preliminary Results for Winter 2009 Conferences*

## I. INTRODUCTION

We combine the CDF published top-quark mass results from Run-I [1-3] with four preliminary Run-II results using up to  $3.2 \text{ fb}^{-1}$  of data [4-7]. Results from the  $t\bar{t} \rightarrow qq' bqq'\bar{b}$  (HAD),  $t\bar{t} \rightarrow \ell\nu qq' b\bar{b}$  (LJT), and  $t\bar{t} \rightarrow \ell^+ \nu b \ell^- \bar{\nu} \bar{b}$  (DIL) final states are included. These measurements are combined accounting for statistical and systematic correlations using the method of reference [8]. The Run-II measurement in the LJT channel yields the single most precise result. Relative to the previous CDF combination reported in [9], this combination includes updates of the Run-II analyses in the LJT, HAD channels, and a new measurement with low dependency on the jet energy scale.

The error categories used in the combination are detailed in Section II while the input measurements themselves are summarized in Section III. The correlations used in the combination are discussed in Section IV and the resulting top-quark mass is given in Section V.

## II. ERROR CATEGORIES

We employ slightly different error categories as used for the Tevatron world average [10]. They have evolved to include a detailed breakdown of the various sources of uncertainty and aim to lump together sources of systematic uncertainty that share the same or similar origin. For example, the ‘‘Signal’’ category discussed below includes the uncertainties from ISR, FSR, and PDF - all of which affect the modeling of the  $t\bar{t}$  signal. The dominant systematic, the jet energy scale (JES) uncertainty, is sub-divided into several components in order to more accurately accommodate our best estimate of the relevant correlations [11]. Additional categories have been added in order to accommodate specific types of correlations. Each error category is discussed below.

**Statistical:** The statistical uncertainty associated with the  $M_{top}$  determination.

**iJES:** The statistical uncertainty on the JES arising from the in-situ  $W \rightarrow qq'$  calibration alone. Residual JES uncertainties, which arise from effects not considered in the in-situ calibration, are included in the Method category below.

**aJES:** This is specific to DØ Run II and is only included here in order to be consistent with reference [?].

**bJES:** The systematic uncertainty specific to the modeling of b-jets. This includes uncertainties arising from variations in the semi-leptonic branching fraction, b-fragmentation modeling, and differences in the color flow between b-quark jets and light-quark. This is usually labeled ‘‘B Jet’’ for CDF Run-II analyses.

**cJES:** The systematic uncertainty on the JES arising from the modeling of the out-of-cone corrections. This is the quadrature sum of the L7 and L8 JES uncertainties for CDF Run-II analyses.

**dJES:** The systematic uncertainty on the JES arising from the relative corrections. This is the L1 JES uncertainties for CDF Run-II analyses.

**rJES:** The systematic uncertainty on the JES arising from the modeling of the calorimeter response, the underlying event, and the multiple interaction corrections. This is the quadrature sum of the L4, L5, and L6 JES uncertainties for CDF Run-II analyses.

**Lepton  $P_T$ :** The systematic uncertainty arising from uncertainties in the scale of the lepton transverse momentum measurements. This is an important uncertainty in the analysis with minimal dependence on the jet energy scale. It was not considered a source of systematic uncertainty in the Run-I measurements.

**Signal:** The systematic uncertainty arising from uncertainties in the modeling of the  $t\bar{t}$  signal including variations in the ISR, FSR, and PDF descriptions used to generate the  $t\bar{t}$  Monte Carlo samples that calibrate each method.

**Generator:** The systematic uncertainty associated with variations observed when substituting Pythia (Run I and Run II) or ISAJET (Run I) for HERWIG when modeling the  $t\bar{t}$  signal.

**UN/MI:** This is specific to DØ Run I and is only included here in order to be consistent with reference [?].

**Background:** The systematic uncertainty arising from uncertainties in modeling the dominant background sources, including  $q^2$  variations. This is the quadrature sum of the ‘‘Background Shape’’ and ‘‘Background normalization’’ uncertainties for most CDF Run-II analyses.

**Method:** The systematic uncertainty arising from any source specific to a particular fit method, including the variations in B-tagging efficiency and the finite Monte Carlo statistics available to calibrate each method. This is the quadrature sum of the "Method", "B-tag", and "MC Statistics" categories for most CDF Run-II analyses.

**Color Reconnections (CR):** The systematic uncertainty arising from a variation of the phenomenological description of color reconnection between final state particles. This systematic source was not considered in the previous measurements and is added here for the first time.

**Multiple Hadron Interactions (MHI):** The systematic uncertainty arising from a mismodeling of the distribution of number of collision per bunch crossing due to the change in the collider instantaneous luminosity during data-taking. It has been separated from other sources to account for the fact that it is uncorrelated with  $D_0$ 's measurements.

These categories represent the current preliminary understanding of the various error categories and their correlations. We expect these to evolve as we continue to probe each method's sensitivity to the various systematic sources with ever improving precision. Small variations in the assignment of uncertainties to the error categories and in the correlations assumed negligibly affect the combination.

### III. INPUT MEASUREMENTS

For this combination we use seven measurements: three published Run-I [1-3] results and four Run-II results [4-7]. They are summarized in Table I. The correlations between the various inputs are described in the next section. Based on studies described in reference [12] the statistical correlation between the  $L_{xy} + P_T^{lep}$  and LJT inputs is set to 0 in the combination. Variations of  $\pm 0.05$ , which cover the full range observed in these studies, negligibly affect the combination reported here.

There are other CDF measurements in the LJT, DIL, HAD, and minimal-JES-dependence channels. We have chosen to include in this combination the analysis methods which yielded the best *expected* sensitivity for each channel. The channels themselves are statistically independent of each other and are treated as such.

| Input                        | Run II Preliminary |       |       |                      | Run I Published |       |       |
|------------------------------|--------------------|-------|-------|----------------------|-----------------|-------|-------|
|                              | LJT                | DIL   | HAD   | $L_{xy} + P_T^{lep}$ | LJT             | DIL   | HAD   |
| $M_{top}$                    | 172.1              | 171.2 | 174.8 | 175.3                | 176.1           | 167.4 | 186.0 |
| Statistical                  | 0.9                | 2.7   | 1.7   | 6.2                  | 5.1             | 10.3  | 10.0  |
| iJES                         | 0.7                | 0.0   | 1.6   | 0.0                  | -               | -     | -     |
| aJES                         | -                  | -     | -     | -                    | -               | -     | -     |
| bJES                         | 0.4                | 0.4   | 0.2   | 0.0                  | 0.6             | 0.5   | 0.6   |
| cJES                         | 0.3                | 1.7   | 0.5   | 0.0                  | 2.7             | 2.6   | 3.0   |
| dJES                         | 0.1                | 0.1   | 0.1   | 0.0                  | 0.7             | 0.6   | 0.3   |
| rJES                         | 0.4                | 1.9   | 0.2   | 0.1                  | 3.4             | 2.8   | 4.0   |
| Lepton $P_T$                 | 0.2                | 0.1   | -     | 1.1                  | -               | -     | -     |
| Signal                       | 0.3                | 0.8   | 0.2   | 1.6                  | 2.6             | 2.8   | 1.8   |
| Generator                    | 0.5                | 0.9   | 0.3   | 0.6                  | 0.1             | 0.6   | 0.8   |
| UN/MI                        | -                  | -     | -     | -                    | -               | -     | -     |
| Background                   | 0.5                | 0.4   | 0.4   | 1.6                  | 1.3             | 0.3   | 1.7   |
| Method                       | 0.2                | 0.6   | 0.7   | 1.4                  | 0.0             | 0.7   | 0.6   |
| Color Reconnections          | 0.4                | 0.4   | 0.4   | 0.4                  | -               | -     | -     |
| Multiple Hadron Interactions | 0.1                | 0.2   | 0.2   | 0.7                  | -               | -     | -     |
| Statistical                  | 0.9                | 2.7   | 1.7   | 6.2                  | 5.1             | 10.3  | 10.0  |
| Systematics-Total            | 1.3                | 3.0   | 1.9   | 3.1                  | 5.3             | 4.9   | 5.7   |
| Total                        | 1.6                | 4.0   | 2.6   | 6.9                  | 7.3             | 11.4  | 11.5  |

TABLE I: The measurements used to determine the CDF combined top-quark mass. All numbers are in units of  $\text{GeV}/c^2$ . The error categories and their correlations are defined in the text. Some of the systematic sources were not accounted for in RunI measurements, due to them being negligible with respect to the actual measurements.

For the  $L_{xy} + P_T^{lep}$  measurement, we include the systematic uncertainty associated with the potential mis-modeling of the background decay length distribution in the "Method" category, since it's a source of uncertainty unique to this method.

For the Run-I measurements, we back propagate the systematic uncertainty specific to B-jets (bJES) as determined in Run II and then correct the Run-I absolute corrections (rJES) to keep the total JES uncertainty constant; we attempted using the estimation of the color reconnection systematic in Run II and apply it to Run I measurements as well, finding no noticeable difference. Same attempt was done with the Lepton  $P_T$  systematic, and again no noticeable difference was found.

Variations of the above assumptions were explored and found to negligibly affect the combination.

#### IV. CORRELATIONS

The following correlations are used when making the combination:

- The uncertainties in the Statistical, Method, and iJES categories are taken to be uncorrelated among the measurements.
- The uncertainties in the aJES, dJES, Lepton  $P_T$  and Multiple Hadron Interaction categories are taken to be 100% correlated among all Run I and all Run II measurements, but uncorrelated between Run I and Run II.
- The uncertainties in the Background category are taken to be 100% correlated among all measurements in the same channel.
- The uncertainties in the bJES, cJES, rJES, Signal, Generator and Color Reconnection categories are taken to be 100% correlated among all measurements.

Using the inputs from Table I and the correlations specified here, the resulting matrix of total correlation co-efficients is given in Table II. To help quantify how sensitive the combination is to these assumptions we vary these correlations by 10% and find that the combination is negligibly affected.

|                    | Run II Preliminary |      |      |                    | Run I Published |      |     |
|--------------------|--------------------|------|------|--------------------|-----------------|------|-----|
|                    | LJ                 | DIL  | HAD  | $L_{xy}+P_T^{lep}$ | LJT             | DIL  | HAD |
| LJT                | 1                  |      |      |                    |                 |      |     |
| DIL                | 0.36               | 1    |      |                    |                 |      |     |
| HAD                | 0.17               | 0.19 | 1    |                    |                 |      |     |
| $L_{xy}+P_T^{lep}$ | 0.20               | 0.12 | 0.06 | 1                  |                 |      |     |
| LJT                | 0.35               | 0.46 | 0.15 | 0.17               | 1               |      |     |
| DIL                | 0.19               | 0.28 | 0.10 | 0.08               | 0.29            | 1    |     |
| HAD                | 0.21               | 0.33 | 0.13 | 0.07               | 0.32            | 0.19 | 1   |

TABLE II: The resulting matrix of total correlation coefficients used to determine the CDF combined top quark mass.

## V. RESULTS

Using the measurements of Table I and the correlations of Section IV the CDF combined top mass is

$$M_{top} = 172.6 \pm 0.9 \text{ (stat)} \pm 1.2 \text{ (syst)} \text{ GeV}/c^2 \quad (1)$$

$$= 172.6 \pm 1.5 \text{ GeV}/c^2 \quad (2)$$

with  $\chi^2/dof = 3.6/6$ , which corresponds to a chi-squared probability of about 73%, indicating good agreement among all the input measurements. The method of reference [8] decomposes the total uncertainty into the contributions from the various error categories; the results can be found in Table III.

| Uncertainty source        | Stat. | iJES | aJES | bJES | cJES | dJES | rJES | Lepton $P_T$ | Signal | Generator | UM  | Background | Fit | CR  | MHI |
|---------------------------|-------|------|------|------|------|------|------|--------------|--------|-----------|-----|------------|-----|-----|-----|
| Size ( $\text{GeV}/c^2$ ) | 0.9   | 0.7  | 0.0  | 0.3  | 0.3  | 0.1  | 0.2  | 0.2          | 0.2    | 0.5       | 0.0 | 0.4        | 0.2 | 0.4 | 0.1 |

TABLE III: The statistical and systematic uncertainty from the combination, broken down into each category. All numbers are in units of  $\text{GeV}/c^2$ .

The corresponding pull and weight for each of the inputs are listed in Table IV. The input measurements and the resulting CDF combined  $M_{top}$  are summarized in Figure 1.

|        | Run II Preliminary |       |        |          | Run I Published |       |       |
|--------|--------------------|-------|--------|----------|-----------------|-------|-------|
|        | LJT                | DIL   | HAD    | $L_{xy}$ | LJT             | DIL   | HAD   |
| Pull   | -0.58              | -0.38 | +1.03  | +0.40    | +0.49           | -0.46 | +1.18 |
| Weight | +76.5%             | +2.7% | +24.4% | +0.8%    | -3.1%           | -0.5% | -0.9% |

TABLE IV: The pull and weight for each of the inputs used to determine the CDF combined top quark mass.

With respect to [9], the weight of the all-hadronic channel increased largely due to improvements in the measurement technique.

Although the chi-squared from the combination of all measurements indicates that there is good agreement among them, and no input has an anomalously large pull, it is still interesting to also fit for the top mass in the LJT, DIL, and HAD channels separately. We use the same methodology and include the systematic correlations among the measurements as described in Section IV. The results are shown in Table V. Using the expression in reference [13] we calculate the following chi-squares  $\chi^2(LJT - DIL) = 0.54/1$ ,  $\chi^2(LJT - HAD) = 1.24/1$ , and  $\chi^2(DIL - HAD) = 1.96/1$ . These correspond to chi-squared probabilities of 46%, 27%, and 16%, respectively, and indicate that all channels are reasonably consistent with each other. Moreover, the agreement between different channel has improved over the previous CDF combination [9].

|        | fit value ( $\text{GeV}/c^2$ ) | correlations |        |        |
|--------|--------------------------------|--------------|--------|--------|
|        |                                | M(LJT)       | M(DIL) | M(HAD) |
| M(LJT) | $171.9 \pm 1.6$                | 1            |        |        |
| M(DIL) | $169.4 \pm 3.6$                | 0.34         | 1      |        |
| M(HAD) | $175.0 \pm 2.6$                | 0.19         | 0.20   | 1      |

TABLE V: The results of a fit to determine the top-quark mass in the three final states separately.

## VI. CONCLUSION

We have combined CDF Run-I and Run-II top-quark mass measurements from all three final states, HAD, LJT, and DIL to get a CDF combined top-quark mass of  $172.6 \pm 1.5 \text{ GeV}/c^2$ . This CDF-only combination has thus a relative precision of  $\Delta M_{top}/M_{top} = 0.85\%$ , to be compared with 0.93% from the previous CDF combination [9].

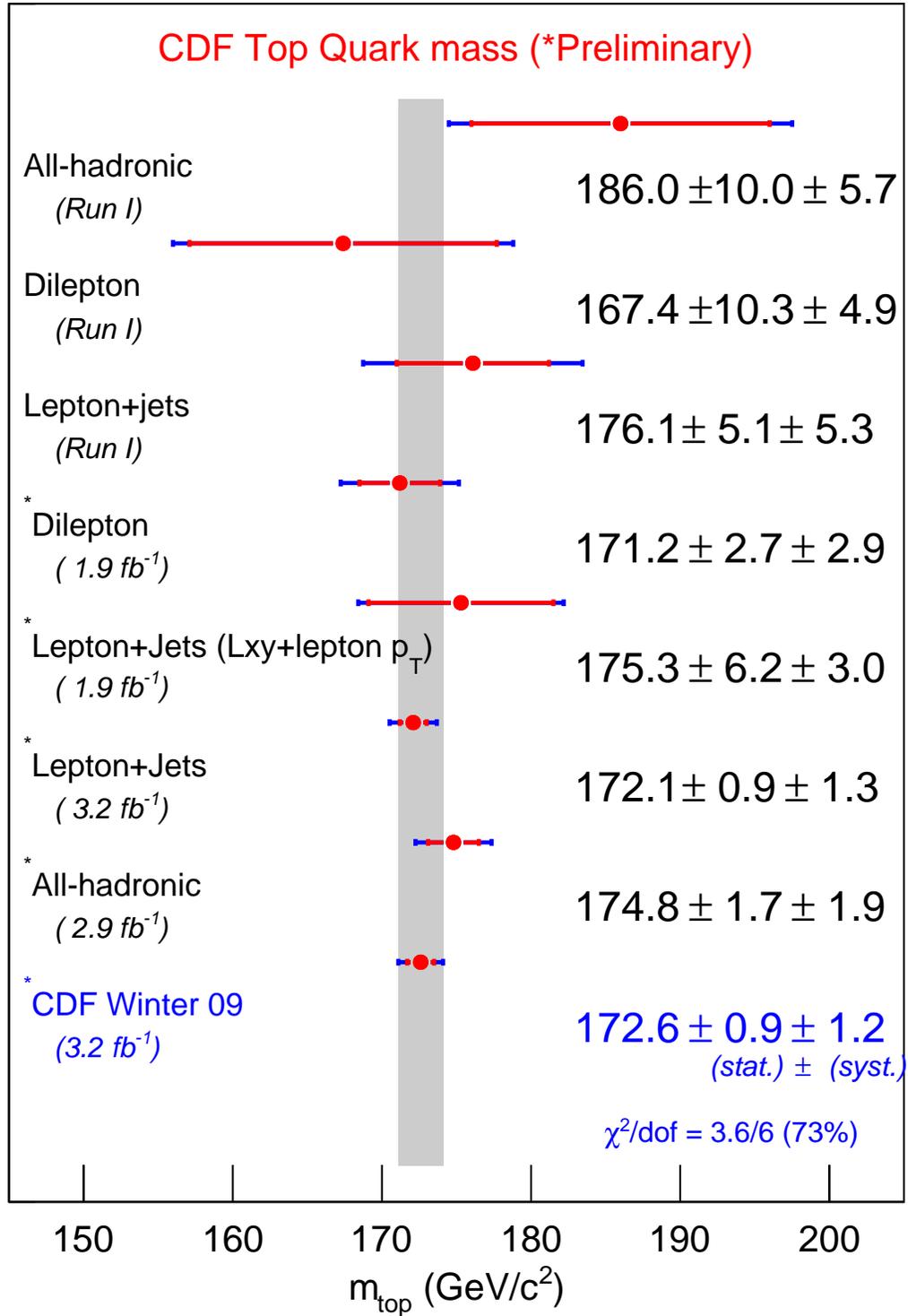


FIG. 1: A summary of the input measurements and resulting CDF combined top-quark mass.

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  - [13] For two measurements,  $x$  and  $y$ , we calculate their consistency using  $\chi^2 = (x-y)^2/\sigma_{x-y}^2$ , where  $\sigma_{x-y}^2 = \sigma_x^2 + \sigma_y^2 - 2\rho_{xy}\sigma_x\sigma_y$ , where  $\rho_{xy}$  is the  $(x, y)$  correlation coefficient.