Precision measurement of the top quark mass from dilepton events at CDF II

We report a measurement of the top quark mass, $M_t$, in the dilepton decay channel of $t \bar{t} \rightarrow b \ell^+ \nu \ell^− \bar{b}$ using an integrated luminosity of $1.0 \text{ fb}^{-1}$ of $p \bar{p}$ collisions collected with the CDF II detector. We apply a method that convolutes a leading-order matrix element with detector resolution functions to form event-by-event likelihoods; we have enhanced the leading-order description to describe the effects of initial-state radiation. The joint likelihood is the product of the likelihoods from 78 candidate events in this sample, which yields a measurement of $M_t = 164.5 \pm 3.9(\text{stat.}) \pm 3.9(\text{syst.}) \text{ GeV}/c^2$, the most precise measurement of $M_t$ in the dilepton channel.

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quark, is the most massive of the known fundamental particles. The top quark mass, $M_t$, is a fundamental parameter in the standard model. Precise measurements of $M_t$ along with those of other standard model parameters can be used to place constraints on the mass of the Higgs boson [1] and on particles in extensions to the standard model [2]. Currently, the Tevatron collider at Fermilab is the only accelerator capable of producing top quarks, where they are primarily produced in pairs. The dilepton channel, including decays with two charged leptons, has a small branching fraction but has the fewest jets in the final-state, giving a smaller dependence on the calibration of the jet energy scale and less ambiguity in jet-quark assignments. Nevertheless, discrepancies between measurements in different decay channels could indicate contributions from physics beyond the standard model [3]. Previous measurements of $M_t$ in the dilepton channel [4–6], while statistically limited, have yielded lower values than measurements in other decay channels [7–10]

The dilepton channel poses unique challenges in reconstructing the kinematics of $t\bar{t}$ events as two neutrinos from $W$ decays escape undetected. Measurements of $M_t$ in this channel made using Run I data [5, 6] and recent measurements made using Run II data [11] utilize methods that make a series of kinematic assumptions and integrate over the remaining unconstrained quantities. The greatest statistical precision, however, was achieved through the application of a matrix-element method [9, 12, 13] which makes minimal kinematic assumptions, instead integrating the leading-order matrix-element for $t\bar{t}$ production and decay over all unconstrained quantities. The first application of this method to the dilepton channel by the CDF collaboration [4, 14] used 340 pb$^{-1}$ of Run II data.

This Letter reports a measurement using an enhanced version [15] of the matrix-element method described in Ref. [14]. The enhanced method accounts for initial-state radiation from the incoming partons and has substantially improved statistical power. This measurement uses data collected by the CDF II detector between March 2002 and March 2006 corresponding to an integrated luminosity of 1.0 fb$^{-1}$ and includes the 340 pb$^{-1}$ used in Ref. [14].

The CDF II detector [16] is a general-purpose detector, designed to study $p\bar{p}$ collisions at the Tevatron collider. The charged particle tracking system consists of a silicon microstrip tracker and a drift chamber, both immersed in a 1.4 T magnetic field. Electromagnetic and hadronic calorimeters surround the tracking system and measure particle energies. Drift chambers located outside the calorimeters detect muons.

The data used in this measurement are collected using the same triggers as in Ref. [14]. After events passing the trigger requirement are reconstructed, we impose the selection criteria defined as “DIL” in Ref. [17] to isolate the dilepton candidates. These selection cuts yield 78 candidate events.

We express the probability density for $t\bar{t}$ decays as

$$P_s(x|M_t) = \frac{1}{N} \int d\Phi |\mathcal{M}_{t\bar{t}}(q_i,p_i;M_t)|^2 \prod_{k=1,2} W_{j\ell}(p_{k\ell},j_k) W_{pT}(p_{T\ell}^k,U) f_{P\bar{P}DF}(q_1)f_{P\bar{P}DF}(q_2), \quad (1)$$

where the integral $d\Phi$ is over the eight remaining unconstrained momenta of the initial and final-state par-

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In addition to $t\bar{t}$ production, we calculate the probability for dominant background processes. The final event-by-event probability is then $P(x|M_t) = P_s(x|M_t)p_s + P_b(x)p_b + P_b(x)p_b \cdots$, where $p_s$ and $p_b$ are determined from the expected fractions of signal and background events (see Table I). To determine the $p_b$, we numerically evaluate background matrix elements using algorithms adopted from the ALCHEMY [26] generator. We calculate probabilities for the following background processes: $Z/\gamma^* \rightarrow e\nu, \mu\nu$ plus associated jets, $W + 3 \text{ jets}$ where one jet is incorrectly identified as a lepton, and $W W$ plus associated jets. We do not calculate probabilities for $Z \rightarrow \tau\tau$ or $W Z$, comprising 11% of the expected background. Studies indicate that use of the background probabilities improves the expected statistical uncertainty by 10%. The posterior probability for the sample is the product of the event-by-event probabilities. The mean of the posterior probability, $P(M_t)$, is the raw measured mass, $M_t^{raw}$, and its standard deviation is the raw measured statistical uncertainty, $\Delta M_t^{raw}$. Both are subject to corrections, described below.

### TABLE I: Expected numbers of signal and background events

For a data sample of integrated luminosity of 1.0 fb$^{-1}$. The number of expected $t\bar{t}$ is given for $M_t = 165$ GeV/c$^2$. Other backgrounds are negligible; expected signal and background numbers have an additional correlated uncertainty of 6% from uncertainty in the sample luminosity.

<table>
<thead>
<tr>
<th>Source</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected $t\bar{t}$ ($M_t = 165$ GeV/c$^2$)</td>
<td>63.4 ± 1.7</td>
</tr>
<tr>
<td>Expected Background</td>
<td>26.9 ± 4.8</td>
</tr>
<tr>
<td>Drell-Yan ($Z/\gamma^* \rightarrow \ell\ell$)</td>
<td>13.1 ± 4.4</td>
</tr>
<tr>
<td>Misidentified Lepton</td>
<td>8.7 ± 1.5</td>
</tr>
<tr>
<td>Diboson (WW/WZ)</td>
<td>5.1 ± 1.0</td>
</tr>
<tr>
<td>Total Expected ($M_t = 165$ GeV/c$^2$)</td>
<td>90.3 ± 5.1</td>
</tr>
<tr>
<td>Run II Observed</td>
<td>78</td>
</tr>
</tbody>
</table>

To test the performance of our method we perform Monte Carlo experiments of signal and background events. Signal events are generated using HERWIG for top quark masses ranging from 155 GeV/c$^2$ to 195 GeV/c$^2$. Background events are modeled using observed events in the case of background due to misidentified leptons, ALCHEMY-simulated events in the case of $Z/\gamma^* \rightarrow e\nu, \mu\nu$, and PYTHIA-simulated [28] events in the case of $Z/\gamma^* \rightarrow \tau\tau, WW, WZ, ZZ$. The numbers of signal and background events in each Monte Carlo experiment are Poisson-fluctuated values around the mean values given in Table I. The estimate for the $t\bar{t}$ signal at varying masses is evolved to account for the variation of cross-section and acceptance. The response of the method for these Monte Carlo experiments is shown in Fig. 1 (left). While the response is consistent with a linear dependence on the top quark mass, its slope is less than unity due to the presence in the sample of background events for which probabilities are not calculated. Corrections, $M_t = 178\, \text{GeV}/c^2 + (M_t^{raw} - 176.4\, \text{GeV}/c^2)/0.83$ and $\Delta M_t = \Delta M_t^{raw}/0.83$, are derived from this response and applied to values measured in the data.

The width of the pull distributions in these Monte Carlo experiments, shown in Fig. 1 (right), where pull is defined as $(M_t - M_t^{raw})/\Delta M_t$, indicates that the statistical uncertainty is underestimated by a factor of 1.17, after applying the corrections described above. This results from the simplifying assumptions described above, made to ensure the computational tractability of the integrals in Eq. 1. The largest effects [14] are the leading two jets in an event not resulting from $b$-quark hadronization, imperfect lepton momentum resolution, imperfect jet angle resolution, and unmodeled backgrounds. Correcting by this factor of 1.17, we estimate the mean statistical uncertainty to be 5.0 GeV/c$^2$ if $M_t = 175\, \text{GeV}/c^2$ or 4.2 GeV/c$^2$ if $M_t = 165\, \text{GeV}/c^2$.

![FIG. 1: Left: Mean measured $M_t$ in Monte Carlo experiments of signal and background events at varying top quark mass. The solid line is a linear fit to the points. Right: Pull widths of Monte Carlo experiments of signal and background events at varying top quark mass. The solid line is the average of all points, 1.17 ± 0.02.](image)

Applying the method and corrections described above to the 78 candidate events observed in the data, we measure $M_t = 164.5 ± 3.9$ (stat.) GeV/c$^2$. Figure 2 shows the joint probability density, without systematic uncertainty, for the events in our data set.

The measured statistical uncertainty is consistent with the distribution of statistical uncertainties in Monte Carlo experiments where signal events with $M_t = 165\, \text{GeV}/c^2$ are chosen according to a Poisson distribution with mean $N_t = 63.4$ events. This number of
events corresponds to the cross section and acceptance at $M_t = 165 \text{ GeV}/c^2$. Of these Monte Carlo experiments, 31% yielded a statistical uncertainty less than 3.9 GeV/$c^2$.

A summary of systematic uncertainties in this measurement is shown in Table II. The largest source of systematic uncertainty in our measurement is due to uncertainty in the jet energy scale [29], which we estimate at 3.5 GeV/$c^2$ by varying the scale within its uncertainty, including effects of high instantaneous luminosity (which have been found to contribute an uncertainty of 0.2 GeV/$c^2$). This is necessarily larger than in the previous application of this method [14], as we have included additional jets measurements in our calculation; future measurements would benefit from a direct calibration of the $b$-jet energy scale from $Z \rightarrow b\bar{b}$ decays. We estimate the uncertainty due to the limited number of background events available for Monte Carlo experiments to be 0.7 GeV/$c^2$. Uncertainties due to PDFs are estimated using different PDF sets (CTEQ5L [22] vs. MRST72 [30]), different values of $\Lambda_{QCD}$ and varying the eigenvectors of the CTEQ6M [22] set; the quadrature sum of these uncertainties is 0.8 GeV/$c^2$. Uncertainty due to showering model in the Monte Carlo generator used for $tt$ events is estimated as the difference in the extracted top quark mass from PYTHIA events and HERWIG events and amounts to 0.9 GeV/$c^2$. We estimate the uncertainty coming from modeling of the two largest sources of background, $Z/\gamma^*$ and events with a misidentified lepton, to be 0.2 GeV/$c^2$. Uncertainty due to imperfect modeling of initial-state (ISR) and final state (FSR) QCD radiation is estimated by varying the amounts of ISR and FSR in simulated events [31], giving 0.3 GeV/$c^2$ for FSR and 0.3 GeV/$c^2$ for ISR. The uncertainty in the mass due to uncertainties in the response correction shown in Fig. 1 is 0.6 GeV/$c^2$. The contribution from uncertainties in background composition is estimated by varying the background estimates from Table I within their uncertainties and amounts to 0.7 GeV/$c^2$. The uncertainty in the lepton energy scale contributes an uncertainty of 0.1 GeV/$c^2$ to our measurement. Adding all of these contributions together in quadrature yields a total systematic uncertainty of 3.9 GeV/$c^2$.

In summary, we have presented a new measurement of the top quark mass in the dilepton channel, $M_t = 164.5 \pm 3.9(\text{stat.}) \pm 3.9(\text{syst.}) \text{ GeV}/c^2$. This is the most precise measurement of $M_t$ in this channel with an approximately 35% improvement in statistical precision over the previous best measurement [14]. The systematic uncertainty, while 15% larger, is nearly completely correlated with systematic uncertainties in measurements in other channels and so does not impact the global combination nor an analysis of measurements in different channels. Previous measurements yielded smaller values of $M_t$ in the dilepton channel [4–6] than in the single lepton [7] and all-hadronic [32] decay channels, though the discrepancy was not statistically significant. Our measurement continues that trend with substantially increased statistical precision. A global combination [33], however, shows that these variations are consistent with statistical fluctuations.

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[18] Missing transverse energy, $E_T$, is defined as the magnitude of the vector, $-\sum_i E_T \vec{n}_i$, where $E_T$ are the magnitudes of transverse energy contained in each calorimeter tower $i$, and $\vec{n}_i$ is the unit vector from the interaction vertex to the tower in the transverse ($x$, $y$) plane.
[19] The unclustered transverse energy in an event is the total transverse energy in the event that is measured in the calorimeter but not clustered into a lepton or jet.
[25] While up to 15% of $t\bar{t}$ pairs at the Tevatron are produced by gluon-gluon fusion ($gg \rightarrow t\bar{t}$), this term can be excluded from the matrix element with negligible effect on the precision of the measurement.