Hadronic Moments in Semileptonic $B$ Decays

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(For the CDF II Collaboration)
Motivation (I)

Most precise determination of $V_{cb}$ comes from $\Gamma_{sl}$ (“inclusive” determination):

$$\Gamma_{sl}(b \rightarrow c\ell^-\bar{\nu}) = \frac{BR(b \rightarrow c\ell^-\bar{\nu}_\ell)}{\tau_b} = |V_{cb}|^2 \times F_{\text{theory}}$$

$\Upsilon(4S)$, LEP/SLD, CDF measurements.
Experimental $\Delta |V_{cb}| \sim 1\%$

Theory with pert. and non-pert. corrections. $\Delta |V_{cb}| \sim 2.5\%$

$F_{\text{theory}}$ evaluated using OPE in HQET: expansion in $\alpha_s$ and $1/m_B$ powers:

- $O(1/m_B) \rightarrow 1$ parameter: $\Lambda$ (Bauer et al., PRD 67 (2003) 071301)
- $O(1/m_B^2) \rightarrow 2$ more parameters: $\lambda_1, \lambda_2$
- $O(1/m_B^3) \rightarrow 6$ more parameters: $\rho_1, \rho_2, T_{1-4}$

$$\Gamma_{sl} = \frac{G_F^2|V_{cb}|^2}{192\pi^3} \frac{m_B^5}{c_1} \left[ 1 - c_2 \frac{\alpha_s}{\pi} + \frac{c_3}{m_B} \Lambda (1 - c_4 \frac{\alpha_s}{\pi}) + \frac{c_5}{m_B^2} (\Lambda^2 + c_6 \lambda_1 + c_7 \lambda_2) + O\left(\frac{1}{m_B^3}\right) + O\left(\frac{\alpha_s^2}{\pi}\right) \ldots \right]$$
Motivation (II)

Many inclusive observables can be written using the same expansion (same non-perturbative parameters) – the spectral moments:

- **Photonic moments:** Photon energy in $b \rightarrow s\gamma$  
  
- **Leptonic moments:** $B \rightarrow X_c l\nu$, lepton $E$ in $B$ rest frame  
  
- **Hadronic moments:** $B \rightarrow X_c l\nu$, recoil mass $M^2(X_c)$  

Many inclusive observables can be written using the same expansion (same non-perturbative parameters) – the spectral moments:

\[
M_1 = \int_{s_{H}^{\text{min}}}^{s_{H}^{\text{max}}} ds_{H} \left( s_{H} - m_{D}^{2} \right) \frac{1}{\Gamma_{sl}} \frac{d\Gamma_{sl}}{d s_{H}} = \langle s_{H} \rangle - m_{D}^{2}, \quad s_{H} \equiv M_{X_c}^{2}
\]

\[
M_2 = \int_{s_{H}^{\text{min}}}^{s_{H}^{\text{max}}} ds_{H} \left( s_{H} - \langle s_{H} \rangle \right)^{2} \frac{1}{\Gamma_{sl}} \frac{d\Gamma_{sl}}{d s_{H}} = \langle \left( s_{H} - m_{D}^{2} \right)^{2} \rangle - M_1^{2}
\]

Constrain unknown non-pert. parameters to reduce $|V_{cb}|$ uncertainty.

With enough measurements: test of underlying assumptions (duality…).
What are the $X_c$?

Semi-leptonic widths (PDG 03):

<table>
<thead>
<tr>
<th></th>
<th>BR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \rightarrow X_c \ell \nu$</td>
<td>$10.89 \pm 0.26$</td>
</tr>
<tr>
<td>$B^+ \rightarrow D^* \ell \nu$</td>
<td>$6.00 \pm 0.24$</td>
</tr>
<tr>
<td>$B^+ \rightarrow D \ell \nu$</td>
<td>$2.23 \pm 0.15$</td>
</tr>
</tbody>
</table>

($b/B^+/B^0$ combination, $b \rightarrow u$ subtracted)

$\Rightarrow \sim 25\%$ of semileptonic width poorly accounted for

Higher mass states: $D^{**}$

Possible $D' \rightarrow D(\ast)\pi\pi$ contributions neglected:

- No experimental evidence so far
- DELPHI limit:
  \[
  \begin{align*}
  BR(b \rightarrow D^+ \pi^+ \pi^- \ell^- \nu) < 0.18\% \text{ @ } 90\% \text{ CL} \\
  BR(b \rightarrow D^{*+} \pi^+ \pi^- \ell^- \nu) < 0.17\% \text{ @ } 90\% \text{ CL}
  \end{align*}
  \]

We assume no $D'$ contribution in our measurement
Analysis Strategy

Typical mass spectrum $M(X^0_c)$ (Monte Carlo):

- $D^0$ and $D^{*0}$ well-known
  - measure only $f^{**}$
  - only shape needed

1) Measure $f^{**}(s_H)$
2) Correct for background, acceptances, bias
   - moments of $D^{**}$
3) Add $D$ and $D^*$ $\rightarrow M_1, M_2$
4) Extract $\Lambda, \lambda_1$

$s_H \equiv M^2_{X_c}$

$$\frac{1}{\Gamma_{sl}} \frac{d\Gamma_{sl}}{ds_H} = \frac{\Gamma_0}{\Gamma_{sl}} \delta(s_H - m^2_{D^0}) + \frac{\Gamma^*}{\Gamma_{sl}} \delta(s_H - m^2_{D^{*0}}) + \left(1 - \frac{\Gamma_0}{\Gamma_{sl}} - \frac{\Gamma^*}{\Gamma_{sl}}\right) f^{**}(s_H)$$

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Channels

Must reconstruct all channels to get all the $D^{**}$ states. 

$\rightarrow$ However CDF has limited capability for neutrals

- $\bar{B}^0 \rightarrow D^{**+} l^- \nu$ always leads to neutral particles $\rightarrow$ ignore it

- $B^- \rightarrow D^{**0} l^- \nu$ doable, use isospin for missing channels:
  - $D^{**0}$
    (i) $\rightarrow D^+ \pi^- \text{ OK}$
    (ii) $\rightarrow D^0 \pi^0$ Not reconstructed: $\frac{1}{2}$ rate of $D^+ \pi^-$ (i)
      $\rightarrow D^{*+} \pi^-$
    (iii) $D^{*+} \rightarrow D^0 \pi^+$ OK
    (iv) $D^{*+} \rightarrow D^+ \pi^0$ Not reco’d: feed-down to $D^+ \pi^-$ (iii)
    (v) $\rightarrow D^{*0} \pi^0$ Not reconstructed: $\frac{1}{2}$ rate of $D^{*+} \pi^-$ (iii) & (iv)
Event Topology

Reconstruct $l+D^{(*)+}$,
Add $\pi^{**}$ track
→ get $l+D^{**0}$

Exclusive reconstruction of $D^{**}$:

$D^{**0} \rightarrow D^{+} \pi^{**-}$ (BR=9.1%)

$D^{**0} \rightarrow D^{*+} \pi^{**-}$

$D^{**0} \rightarrow D^{0} \pi^{*+}$ (BR=67.7%)

$D^{*+} \rightarrow K^{-} \pi^{+}$ (BR=3.8%)

$D^{*+} \rightarrow K^{-} \pi^{+} \pi^{-} \pi^{+}$ (BR=7.5%)

$D^{*+} \rightarrow K^{-} \pi^{+} \pi^{0}$ (BR=13.1%)
Backgrounds

Physics background:
\( B \rightarrow D(\ast)^+ D_s^- , \ D_s^- \rightarrow X l \nu \)
\( \rightarrow \) MC, subtracted

Combinatorial background under the D(\ast) peaks:
\( \rightarrow \) sideband subtraction

Prompt pions faking \( \pi^{**} \):
- fragmentation
- underlying event
\( \rightarrow \) separate B and primary vertices
  (also remove prompt charm)
\( \rightarrow \) use impact parameters to discriminate
\( \rightarrow \) model: wrong-sign \( \pi^{**} \ell^- \) combinations

Feed-down in signal:
\( D^{**0} \rightarrow D^{\ast\ast} [ \rightarrow D^+ \pi^0 ] \pi^- \)
irreducible background to \( D^{**0} \rightarrow D^+ \pi^- \).
\( \rightarrow \) subtracted using data:
\( \rightarrow \) shape from \( D^0 \pi^- \) in
\( D^{**0} \rightarrow D^{\ast\ast} [ \rightarrow D^0 \pi^+ ] \pi^- \)
\( \rightarrow \) rate:
\( \frac{1}{2} \) (isospin) \times eff. \times BR
**Lepton-$D^{(*)+}$ Reconstruction**

**Data Sample:**
- $e/\mu +$ displaced track
- $\sim 180$ pb$^{-1}$
(→ Sep. 2003)

**Track Selection:**
- $e/\mu$: $p_T > 4$ GeV
- other: $p_T > 0.4$ GeV

**Lepton $+ D^{(*)+}$:**
- $D$ vertex:
  - 3D
- $l+D(+\pi^*)$ vertex ("B"):
  - 3D
  - $L_{XY}(B) > 500$ µm
  - $M(B) < 5.3$ GeV

Total: $\sim 28000$ events

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**Graphs:**
- $D^0 \rightarrow K^- \pi^+(\pi^0)$
- $D^0 \rightarrow K^- \pi^+$
- $D^0 \rightarrow K^- \pi^+(\pi^0)$
- $D^0 \rightarrow K^- \pi^+$
**π** **Selection**

Based on topology:
- impact parameter significances w.r.t. primary, B and D vertices

\[ \pi^* \text{ 2D IP signif. wrt} \ P V \]

\[ \pi^* \text{ 3D IP signif. wrt} \ B V \]

Cuts are optimized using MC and background data:
- \( p_T > 0.4 \text{ GeV} \)
- \( \Delta R < 1.0 \)
- \( |d_0^{PV}/\sigma| > 3.0 \)
- \( |d_0^{BV}/\sigma| < 2.5 \)

Additional cuts only for \( D^+ \):
- \( |d_0^{DV}/\sigma| > 0.8 \)
- \( L_{xy}^{B \rightarrow D} > 500 \mu m \)
Raw $m^{**}$ Distribution

CDF Run II Preliminary $L \approx 180$ pb$^{-1}$

- Data Right-Sign ($l^- \pi^-$)
- Data Wrong-Sign ($l^- \pi^+$)

Raw $m^{**}$ for $D^{*+}$ channels:

- $D^0 \rightarrow K^- \pi^+$
- $D^0 \rightarrow K^- \pi^+ \pi^+$
- $D^0 \rightarrow K^- \pi^+ \pi^0$

Feed-down

$D_1, D_1^*, D_2^*$

$D_2^*, D_0^*$
1) Correct the raw mass for any dependence of $\varepsilon_{\text{reco}}$ on $M(D^{**})$:
   - Possible dependence on the $D^{**}$ species (spin).
   - Monte Carlo for all $D^{**}$ (Goity-Roberts for non-resonant), cross-checked with pure phase space decays.

2) “Cut” on lepton energy in $B$ rest frame:
   - Theoretical predictions are given for well-defined $p_l^*$ cuts.
   - We cannot measure $p_l^*$, but we can correct our measurement to a given cut:
     $\rightarrow p_l^* > 700 \text{ MeV/c}$. 

Corrected Mass and D^{**} Moments

Procedure:

- Unbinned procedure using weighted events.
  - Assign negative weights to background samples.
  - Propagate efficiency corrections to weights.
  - Take care of the D^{+} / D^{**} relative normalization.
- Compute mean and sigma of distribution.

Results:

\[
m_1 = \left< m_{D^{**}}^2 \right> = (5.83 \pm 0.16_{stat}) \text{GeV}^2
\]
\[
m_2 = \left< (m_{D^{**}}^2 - m_1)^2 \right> = (1.30 \pm 0.69_{stat}) \text{GeV}^4
\]

No Fit!
Final Results

Pole mass scheme

\[ \Lambda = (0.390 \pm 0.075_{\text{stat}} \pm 0.026_{\text{exp}} \pm 0.064_{\text{BR}} \pm 0.058_{\text{theo}}) \text{ GeV} \]
\[ \lambda_1 = (-0.182 \pm 0.055_{\text{stat}} \pm 0.016_{\text{exp}} \pm 0.022_{\text{BR}} \pm 0.077_{\text{theo}}) \text{ GeV}^2 \]

1S mass scheme

\[ m_b(1S) = (4.661 \pm 0.076_{\text{stat}} \pm 0.026_{\text{exp}} \pm 0.064_{\text{BR}} \pm 0.089_{\text{theo}}) \text{ GeV} \]
\[ \lambda_1 = (-0.276 \pm 0.047_{\text{stat}} \pm 0.016_{\text{exp}} \pm 0.022_{\text{BR}} \pm 0.094_{\text{theo}}) \text{ GeV}^2 \]
<table>
<thead>
<tr>
<th></th>
<th>$\Delta m_1$ (GeV$^2$)</th>
<th>$\Delta m_2$ (GeV$^4$)</th>
<th>$\Delta M_1$ (GeV$^2$)</th>
<th>$\Delta M_2$ (GeV$^4$)</th>
<th>$\Delta \Lambda$ (GeV)</th>
<th>$\Delta \lambda_1$ (GeV$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stat.</td>
<td>0.16</td>
<td>0.69</td>
<td>0.037</td>
<td>0.25</td>
<td>0.075</td>
<td>0.055</td>
</tr>
<tr>
<td>Syst.</td>
<td>0.08</td>
<td>0.20</td>
<td>0.065</td>
<td>0.12</td>
<td>0.090</td>
<td>0.082</td>
</tr>
<tr>
<td>Mass resolution</td>
<td>0.02</td>
<td>0.13</td>
<td>0.005</td>
<td>0.04</td>
<td>0.012</td>
<td>0.009</td>
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<tr>
<td>Eff. Corr. (data)</td>
<td>0.03</td>
<td>0.13</td>
<td>0.006</td>
<td>0.05</td>
<td>0.014</td>
<td>0.011</td>
</tr>
<tr>
<td>Eff. Corr. (MC)</td>
<td>0.06</td>
<td>0.05</td>
<td>0.016</td>
<td>0.03</td>
<td>0.017</td>
<td>0.006</td>
</tr>
<tr>
<td>Bkgd. (scale)</td>
<td>0.01</td>
<td>0.03</td>
<td>0.002</td>
<td>0.01</td>
<td>0.003</td>
<td>0.002</td>
</tr>
<tr>
<td>Physics bkgd.</td>
<td>0.01</td>
<td>0.02</td>
<td>0.002</td>
<td>0.01</td>
<td>0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>D$^+$ / D$^{*+}$ BR</td>
<td>0.01</td>
<td>0.02</td>
<td>0.002</td>
<td>0.01</td>
<td>0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>D$^+$ / D$^{*+}$ Eff.</td>
<td>0.02</td>
<td>0.03</td>
<td>0.004</td>
<td>0.01</td>
<td>0.005</td>
<td>0.002</td>
</tr>
<tr>
<td>Semileptonic BRs</td>
<td></td>
<td></td>
<td>0.062</td>
<td>0.10</td>
<td>0.064</td>
<td>0.022</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.041</td>
<td>0.069</td>
</tr>
<tr>
<td>$T_i$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.032</td>
<td>0.031</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.018</td>
<td>0.007</td>
</tr>
<tr>
<td>$m_b, m_c$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.001</td>
<td>0.008</td>
</tr>
<tr>
<td>Choice of $p_1^*$ cut</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.019</td>
<td>0.009</td>
</tr>
</tbody>
</table>
Main Systematics

• Semileptonic branching ratios when combining D** with D and D*

• Efficiency corrections from data
  − Use data-corrected efficiency. vs. pure MC efficiency

• Efficiency corrections from D** Monte-Carlo
  − D** states + NR Goity-Roberts vs. pure phase-space

• Mass resolution:
  − Dominated by satellite: ± 60 MeV

• Prompt background scale
  − Charge correlations WS / RS: ± 4%

• Other: D+/D* normalization, physics background, p_{T}\ cut, theory…
Comparison with Previous Measurements

Red bands represent HQET predictions using CDF measurement as input.

NB: measurements of two moments highly correlated

Pole mass scheme
Summary

• First measurement of hadronic moments in semileptonic B decays performed at a hadron collider.

• Good HQET agreement with previous determinations.

• Competitive with other experiments. Little model dependency. No assumptions on shape or rate of D** components.

• Increased statistics and improved tracking will lead to substantially more precise results in the near future.
BACKUP SLIDES
CKM and $V_{cb}$

$|V_{cb}| = A\lambda^2$

(defines the scale of UT)
$V_{cb}$ measurements

$|V_{cb}|$ from exclusive B decays

- Large statistics on $B_d^{0} \rightarrow D(\ast)\ell^{-}\nu$ available and new measurements are coming
- Present precision (5%) is systematics limited:
  - Experiments: $D^{**}$ states, $D$'s BR
  - Theory: form factor extrapolation, corrections to $F(1)=1$
  - can be reduced in the future

$$|V_{cb}|^{\text{excl}} = (42.1 \pm 1.1_{\text{exp}} \pm 1.9_{\text{theo}}) \times 10^{-3}$$

(PDG 2002, $V_{cb}$ review)

$|V_{cb}|$ from inclusive B decays

- Experiment: large statistics on $\text{BR}(B \rightarrow X_c \ell^{-}\nu)$ and $t_B$ and small systematics

$$|V_{cb}|^{\text{incl}} = (40.4 \pm 0.5_{\text{exp}} \pm 0.5_{\Lambda,\lambda} \pm 0.8_{\text{theo}}) \times 10^{-3}$$

(PDG 2002, $V_{cb}$ review)
Lepton + D Reconstruction

Data Sample:
• $e/\mu +$ displaced track
• $\sim 180$ pb$^{-1}$
(→ Sep 2003)

Track Selection:
• $e/\mu$: $p_T > 4$ GeV
• other: $p_T > 0.4$ GeV

Lepton + $D^{(*)+}$:
• $D$ vertex:
  • 3D
• $l+D(+\pi^*)$ vertex (“B”):
  • 3D
  • $L_{XY}(B) > 500$ $\mu$m
  • $M(B) < 5.3$ GeV
D*+ Reconstruction and Yields

D*+ channels: \[ \Delta m^* \equiv M(D^0\pi^*) - M(D^0) \]

D*(*)+ l^- (+cc) yields:

<table>
<thead>
<tr>
<th>( K^-\pi^+ )</th>
<th>( K^-\pi^+\pi^-\pi^+ )</th>
<th>( K^-\pi^+\pi^0 )</th>
<th>( D^+ ) channel ( K^-\pi^+\pi^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Electrons</strong></td>
<td>1723 ± 42</td>
<td>1200 ± 38</td>
<td>3037 ± 66</td>
</tr>
<tr>
<td><strong>Muons</strong></td>
<td>2168 ± 47</td>
<td>1695 ± 43</td>
<td>3611 ± 72</td>
</tr>
<tr>
<td><strong>Combined</strong></td>
<td>3890 ± 63</td>
<td>2994 ± 57</td>
<td>6638 ± 98</td>
</tr>
</tbody>
</table>

~ 28000 events
Monte-Carlo Validation (I)

MC vs. semileptonic sample:

Matching $\chi^2$ probability for those plots:

<table>
<thead>
<tr>
<th>$K\pi\pi, e$</th>
<th>$K\pi\pi\pi, e$</th>
<th>$K\pi, \mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>67%</td>
<td>74%</td>
<td>23%</td>
</tr>
<tr>
<td>43%</td>
<td>69%</td>
<td>87%</td>
</tr>
</tbody>
</table>

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Relative yield prediction $K_{3\pi}/K\pi$: (cross-check)

$$R_{K_{3\pi}/K\pi} \equiv \frac{N(D^{*+}\bar{\nu}X, D^{*+} \rightarrow D^{0}\pi^{+}, D^{0} \rightarrow K^{-}\pi^{+}\pi^{-}\pi^{+})}{N(D^{*+}\bar{\nu}X, D^{*+} \rightarrow D^{0}\pi^{+}, D^{0} \rightarrow K^{-}\pi^{+})}.$$

- efficiency for adding tracks understood

$R_{\text{data}} = 0.77\pm0.02$

$R_{\text{pred}} = 0.80\pm0.04$

$R_{\text{pred}}/R_{\text{data}} = 1.04\pm0.06$

Relative yield prediction $K_{2\pi}/K\pi$: (needed for $D^{+}/D^{*}$ normalization)

$$R_{D^{+}/K\pi} \equiv \frac{N(B \rightarrow D^{*+}\bar{\nu}X, D^{+} \rightarrow K^{-}\pi^{+}\pi^{+})}{N(B \rightarrow D^{*+}\bar{\nu}X, D^{*+} \rightarrow D^{0}\pi^{+}, D^{0} \rightarrow K^{-}\pi^{+})},$$

Two methods (a,b) to derive this BR

<table>
<thead>
<tr>
<th></th>
<th>$R_{\text{data}}$</th>
<th>$R_{\text{pred}}$</th>
<th>$R_{\text{pred}}/R_{\text{data}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>3.71±0.08</td>
<td>3.31±0.58</td>
<td>0.89±0.16</td>
</tr>
<tr>
<td>b)</td>
<td>3.71±0.08</td>
<td>3.23±0.29</td>
<td>0.87±0.08</td>
</tr>
</tbody>
</table>

- uncertainties in Br (incl.) and $D^{**}$ spectroscopy (excl.) compromise prediction
- $1. - 0.87 = 13\%$ used as systematics
Impact Parameters in MC

Comparison data/MC for IP: (worst case)

- \( K\pi \)
- \( \pi^* \)
- non-SVT D daughters \((p_T > 1.5 \text{ GeV})\)
- corrections from double ratios
  - in \( p_T \)
  - in \( m^* \)

Residual corrections:
- derived from data:
  - \( \pi^* \)

\[ \chi^2 / \text{ndf} = 94.82 / 39 \]
\[ \text{Prob} = 1.501 \times 10^{-6} \]
\[ p_0 = 0.8432 \pm 0.01066 \]
\[ p_1 = 0.02898 \pm 0.00244 \]
Background Subtraction

- Use mass side-bands to subtract combinatorial background.
- Use $D^{*+} \rightarrow D^0 \pi^+ \pi^-$ to subtract feed-down from $D^{*+} \rightarrow D^+ \pi^0 \pi^-$ to $D^+ \pi^- \pi^-$.  
- Use wrong-sign $\pi^{**+} l^-$ combinations to subtract prompt background to $\pi^{**}$.
  - Possible charge asymmetry of prompt background studied with fully reconstructed B’s: 4% contribution at most.
- Possible $D' \rightarrow D(\ast)\pi\pi$ contributions neglected:
  - No experimental evidence so far.
  - DELPHI limit:
    \[
    \begin{align*}
    BR(b \rightarrow D^+ \pi^+ \pi^- \ell^- \nu) &< 0.18\% @ 90\% CL \\
    BR(b \rightarrow D^{*+} \pi^+ \pi^- \ell^- \nu) &< 0.17\% @ 90\% CL
    \end{align*}
    \]
  - We assume no $D'$ contribution in our sample
Combination with $D^0, \ D^{*0}$

$$\frac{1}{\Gamma_{sl}} \frac{d\Gamma_{sl}}{ds_H} = \frac{\Gamma_0}{\Gamma_{sl}} \cdot \delta(s_H - m_{D^0}^2) + \frac{\Gamma^*}{\Gamma_{sl}} \cdot \delta(s_H - m_{D^{*0}}^2) + \left(1 - \frac{\Gamma_0}{\Gamma_{sl}} - \frac{\Gamma^*}{\Gamma_{sl}}\right) \cdot f^{**}(s_H)$$

Take $M(D^0), \ M(D^{*0}), \ \Gamma_{sl}, \ \Gamma_0, \ \Gamma^*$ from PDG 2003:

- $\Gamma_{sl}, \ \Gamma_0, \ \Gamma^*$ are obtained combining BR’s for $B^-, \ B^0$ and admixture, assuming the widths are identical (not the BR’s themselves), and using
  
  $$\frac{f_-}{f_0} = 1.04 \pm 0.08$$
  $$\frac{\tau(B^-)}{\tau(B^0)} = 1.085 \pm 0.017$$

- Results:
  
  $BR(B^+ \rightarrow X^0_c \ l^+ \ \nu_\ell) = 0.1089 \pm 0.0026$
  $BR(B^+ \rightarrow D^0 \ l^+ \ \nu_\ell) = 0.0223 \pm 0.0015$
  $BR(B^+ \rightarrow D^{*0} \ l^+ \ \nu_\ell) = 0.0600 \pm 0.0024$