Top Mass Measurements at CDF

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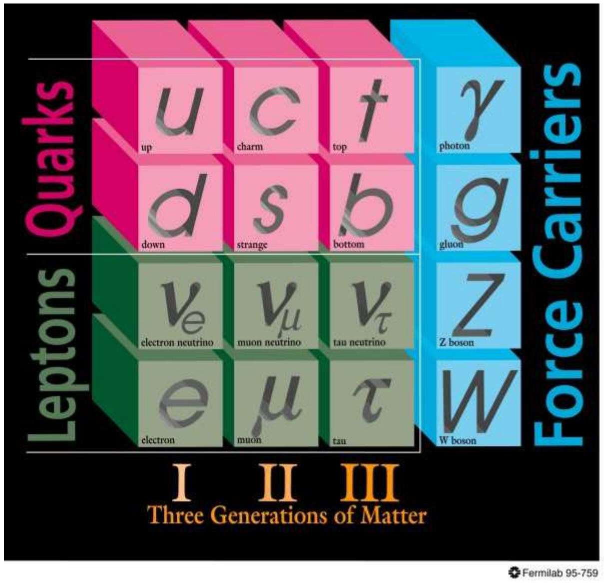
Where we're going

- Introduction to top physics and top mass
  - Run I world average: $M_{top}=178.0^{+4.3}_{-4.3}$ GeV/c^2
- Challenges in measuring top quark mass
- Description of the template method
- Current result: $M_{top}=173.5^{+4.1}_{-4.0}$ GeV/c^2
- Other techniques, results

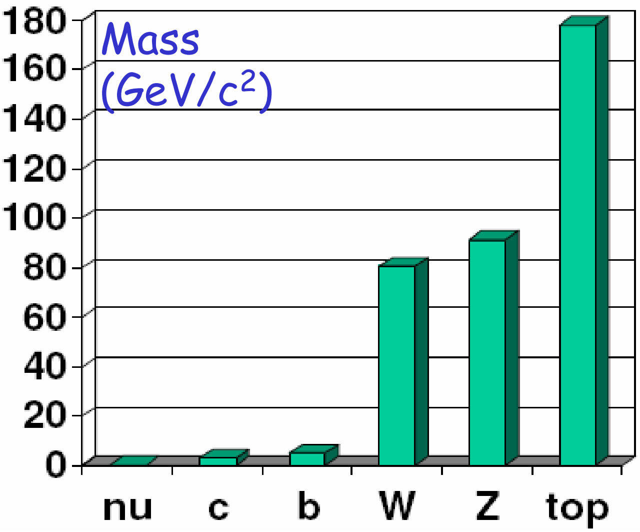
The Top Quark

- Feels strong, electroweak, gravitational forces.
- Short-lived—doesn't hadronize ($\tau=4\times10^{-25}$ s).
- Especially interesting due to its mass
  - Most massive particle at ~175 GeV/c^2.
  - More massive than b quark by factor of 35.
  - SM Yukawa coupling ~ 1 … Special role??

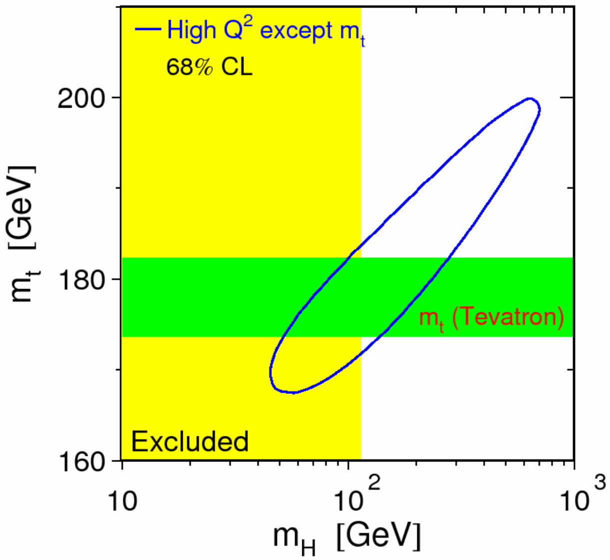


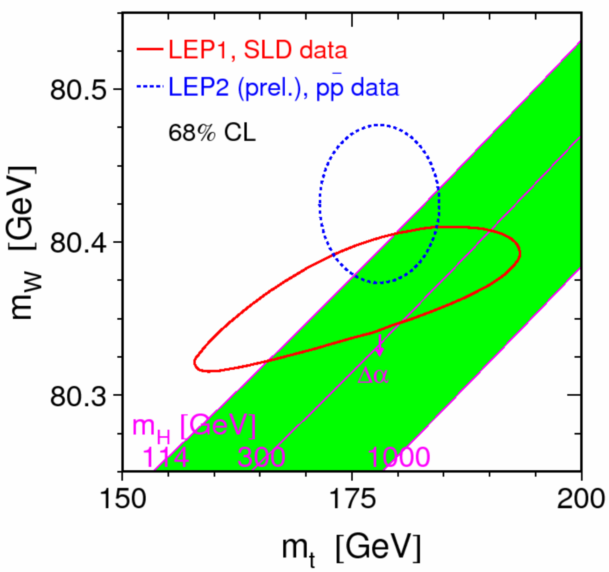
$$M_t = \frac{1}{\sqrt{2}} \lambda_t v$$

$$\Rightarrow \lambda_t = \frac{M_t}{173.9\,\mathrm{GeV}/c^2}$$

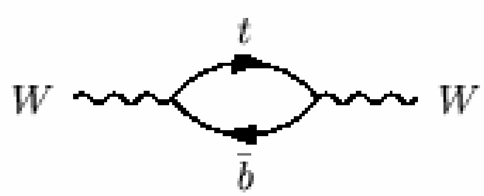


Why measure the top quark mass?



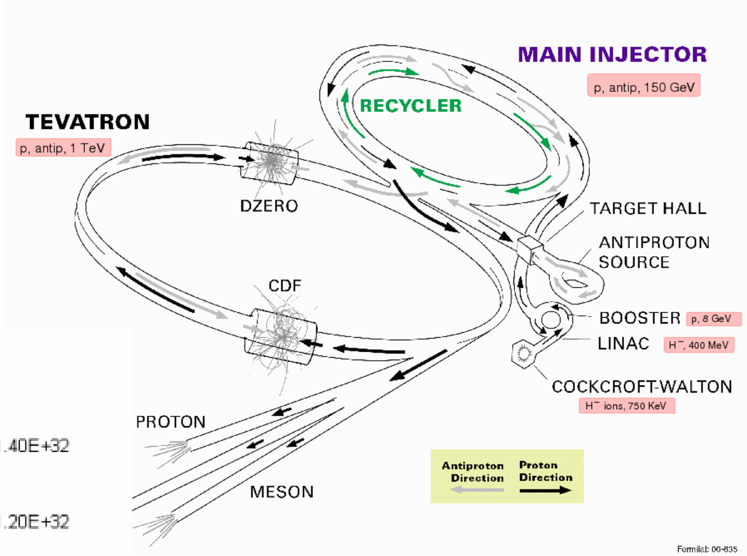


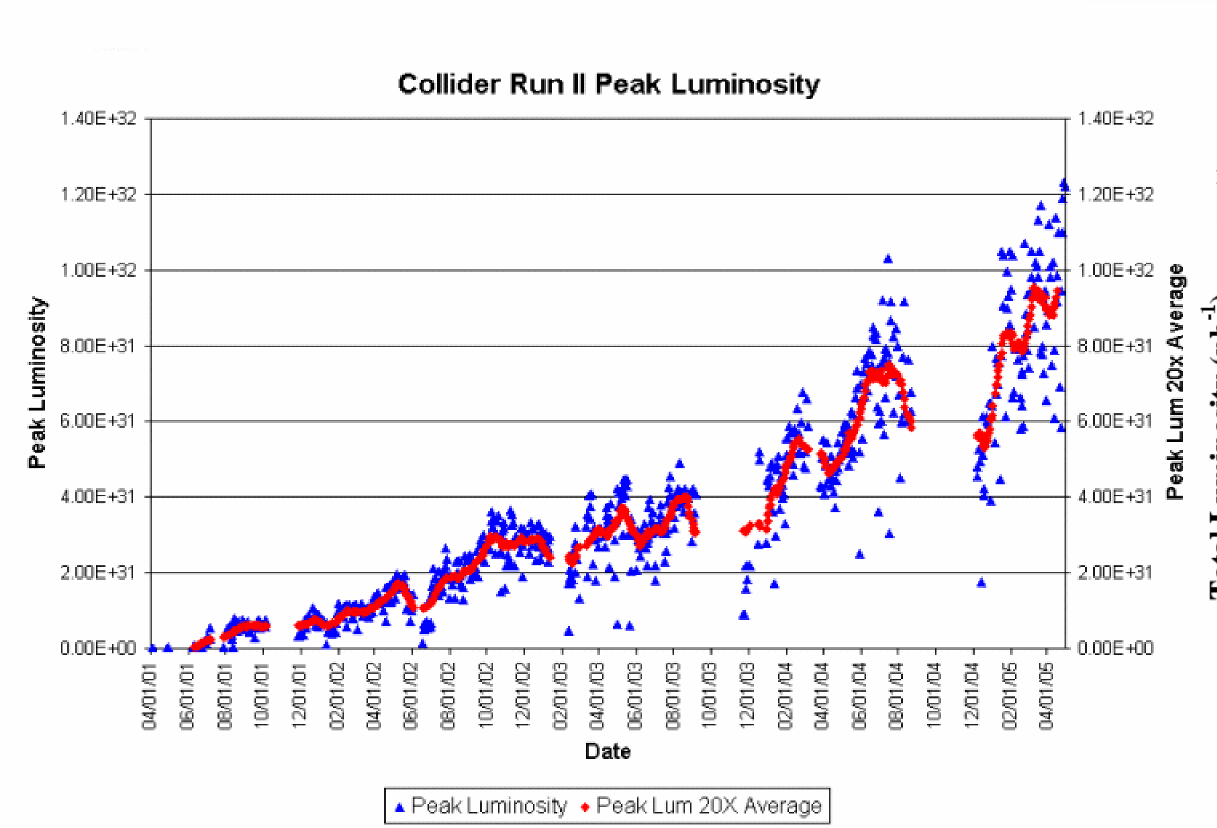
- Fundamental dimensionless parameter of SM close to 1.
- Related to other SM parameters and observables through loop diagrams.
  - Global fit (LEPEWWG) provides consistency check and predicts mass of putative Higgs particle.
  - M_t (and M_W) particularly poorly known in terms of effect on M_H prediction.

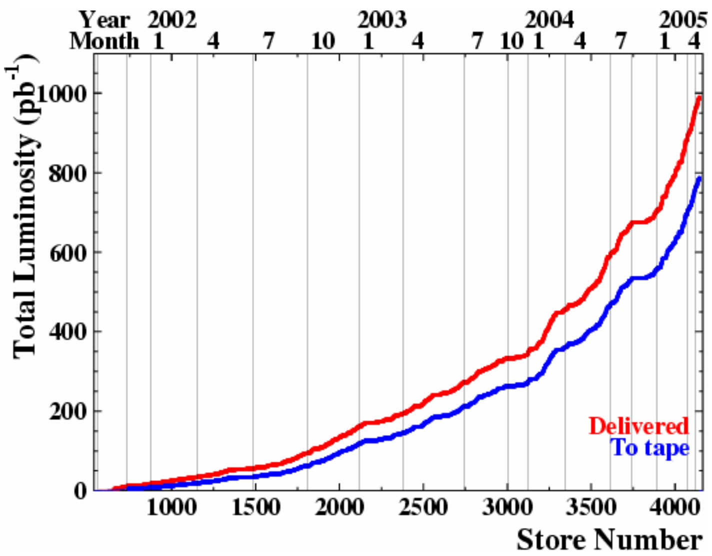


Obligatory accelerator slide

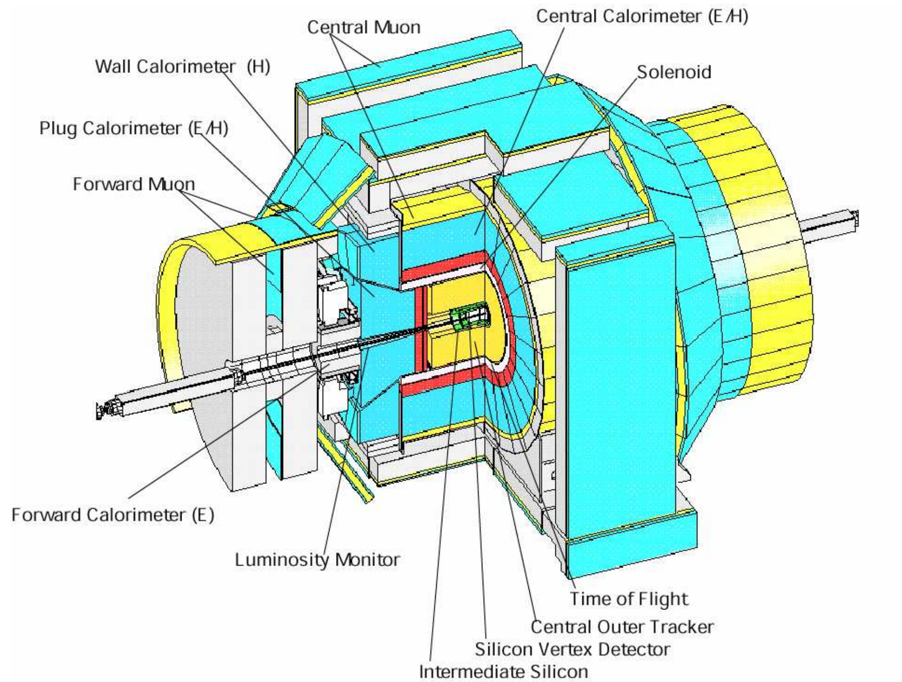
- Tevatron run II: √s = 1.96 TeV
- Peak luminosity broke 1.2 x 10^{32}!







Obligatory detector slide



- **C**ollider **D**etector at **F**ermilab
- Standard onion-like general-purpose particle physics detector
  - Tracking system
  - Calorimeters
  - Muon system

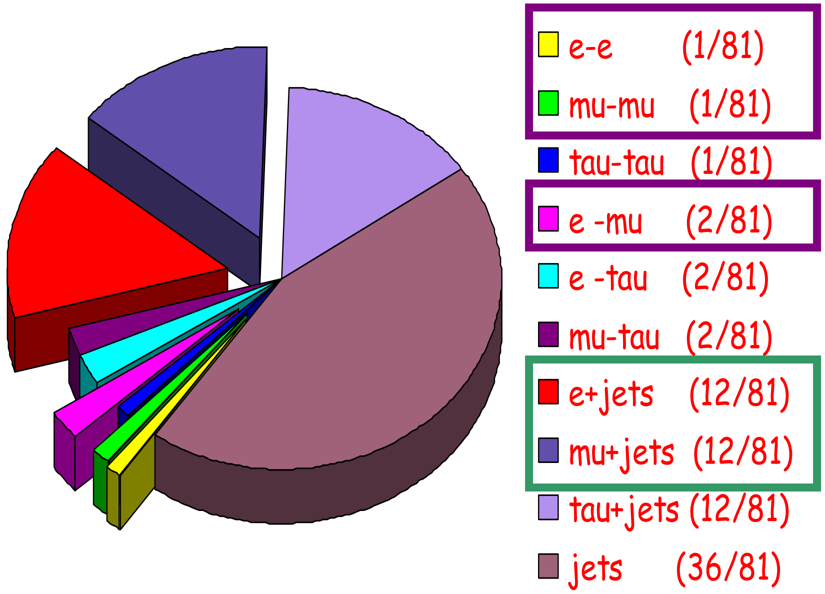
Silicon detector → *b* tagging

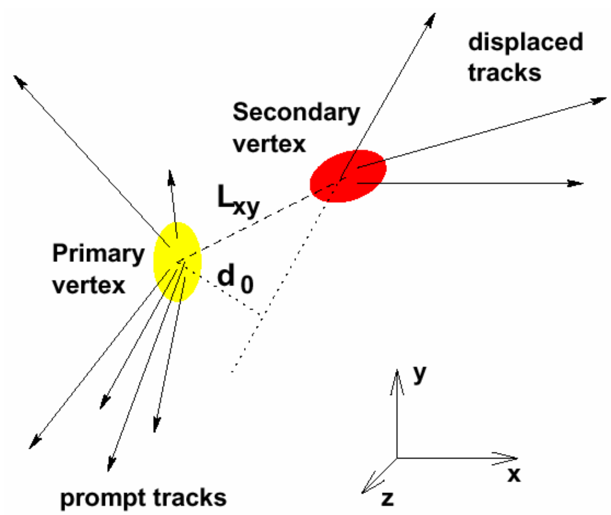
Excellent lepton ID and triggering

Coarse segmentation, non-linear response

Top phenomenology

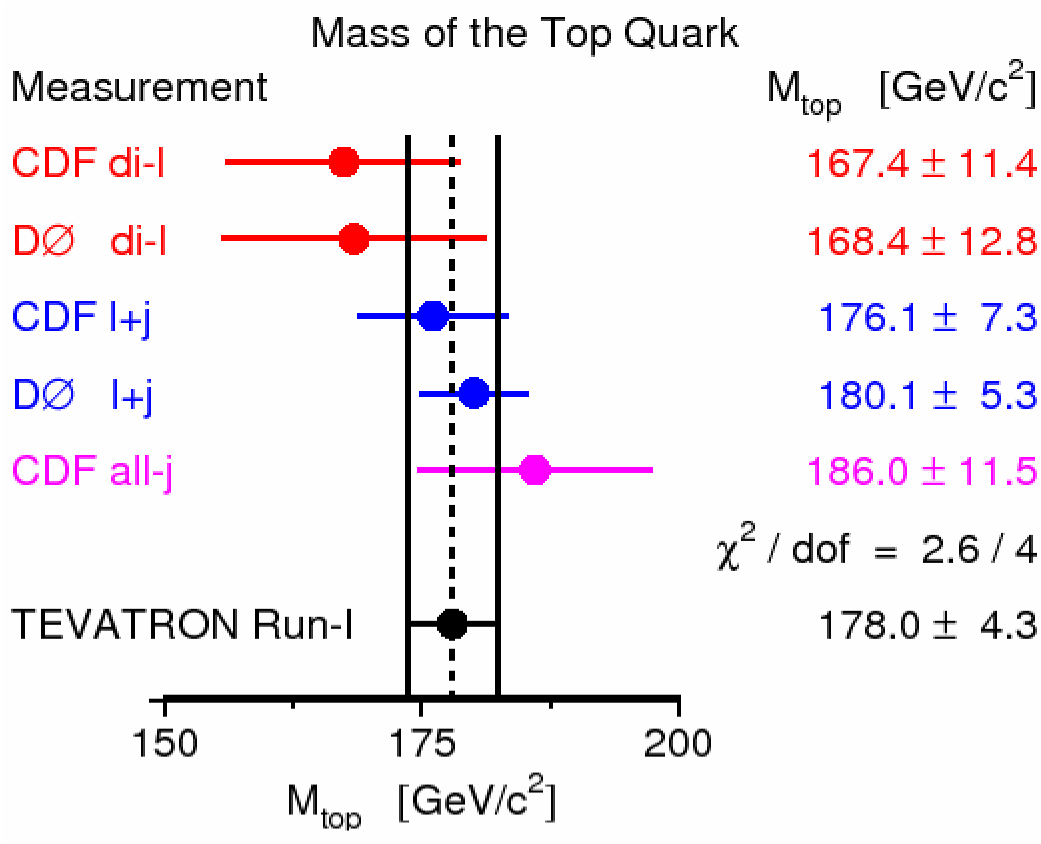
- Mass analyses use t-tbar pair events.
  - σ = 6.7 (5.7) pb @ M_t = 175 (180) GeV/c^2.
  - ~85% quark annihilation, ~15% gluon fusion.
- Top always decays to W boson and b quark.
  - Events classified by decay of W to leptons or quarks
  - Identifying b quark improves S/B ratio





Run I mass measurements

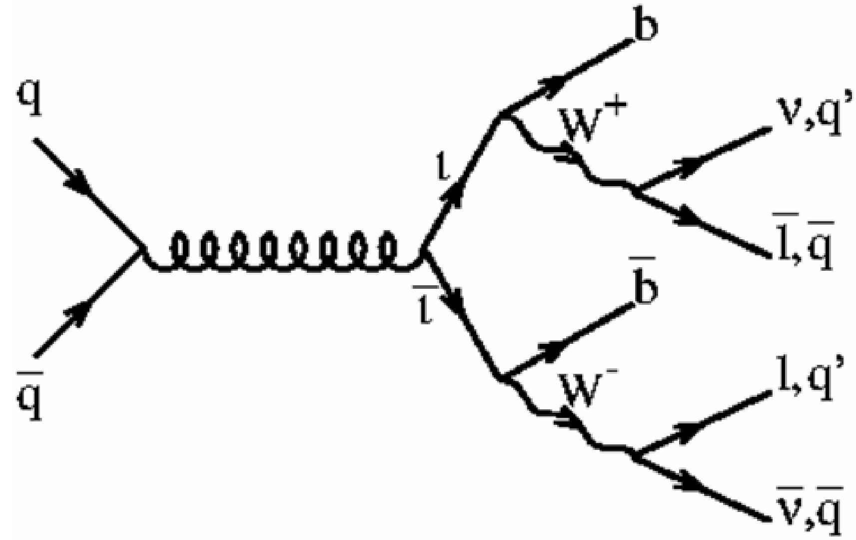
- 106-125 pb^{-1}.
- L+Jets most sensitive channel.
- World average 178.0 ± 4.3 dominated by D0 L+Jets result.
- Run II analyses using >300 pb^{-1}
  - Should do better!

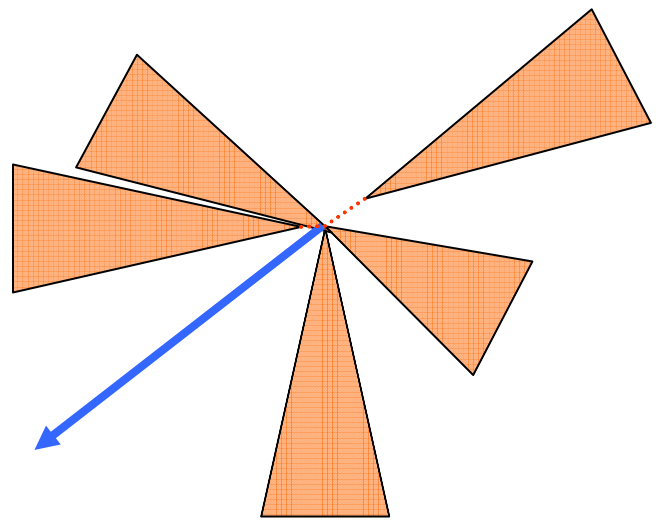


What's the big deal?

Events are complicated!

- Experimental observations are not as pretty as Feynman diagrams!
  - Additional jets from ISR, FSR.
  - Which jets go with which quarks?
  - Dileptons: 2 neutrinos, 1 $\not{E}_T$ measurement.





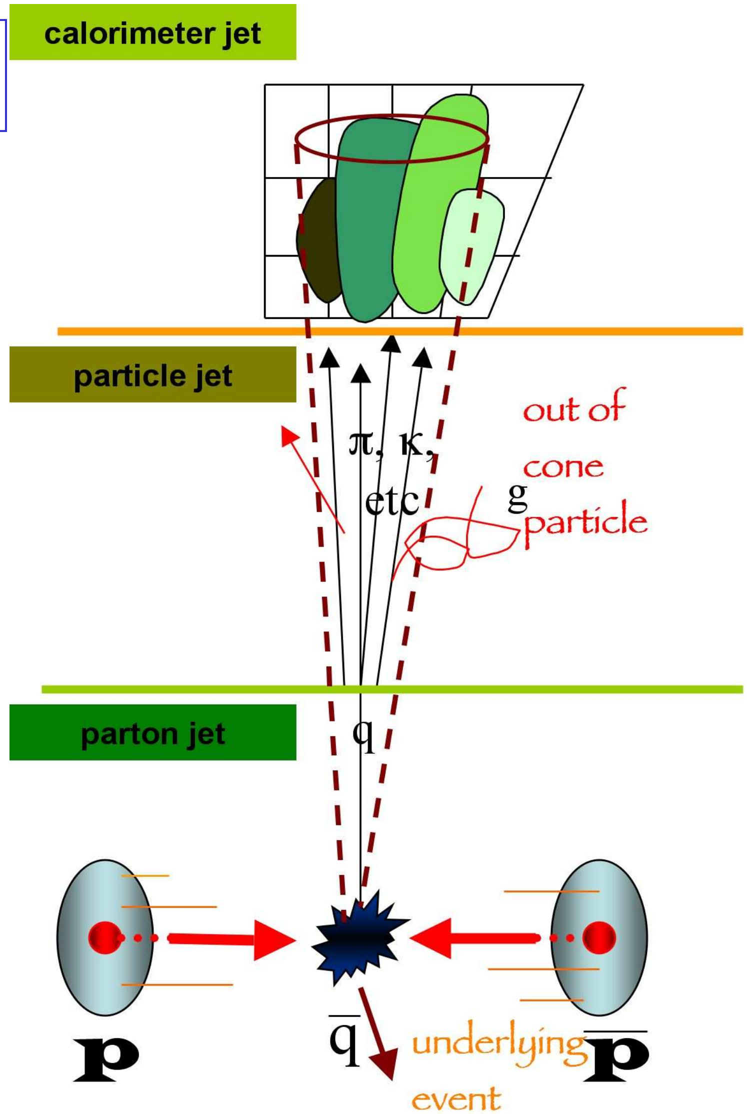
What’s the big deal, part II

Measurements are not perfect!

- Missing transverse energy ↔ Neutrino, but p_z^{ν} not measured.
- Jet energies poorly measured.
  - Large resolution → statistical error.
    - Fragmentation + calorimeter non-linearity
    - Particles leave jet cone
    - Underlying event
  - Uncertain scale → sytematic error.
    - Jets are hard to calibrate—no nice resonance.
    - See later for a novel approach to this problem.

$84\%/\sqrt{E_T}$

O(3%)



What's the big deal, part III

Background contamination!

- Top events: trade-off between sample size and purity.
- Presence of background events dilutes mass information from signal events.
- Effects of background must be treated properly to avoid bias.

Typical S:B Ratio			
L+Jets	0 tags	1 tag	2 tags
	<1:1	3:1	10:1
Dilep	2:1		

- Dominant backgrounds
  - W+jets (incl W+h.f.)
  - Non-W (QCD)
  - Z+jets
  - Fakes

How to Weigh Truth

TEMPLATES

1. Pick a test statistic (e.g. reconstructed mass).
2. Create "templates" using events simulated with different M_{top} values (+ background).
3. Perform maximum likelihood fit to extract measured mass.

DIRECT PROBABILITY

1. Build likelihood directly from PDFs, matrix element(s), and transfer functions that connect quarks and jets.
2. Integrate over unmeasured quantities (e.g. quark energies).
3. Calibrate measured mass and error using simulation.

Introduction to Templates

Template Analysis Overview

Datasets

Data

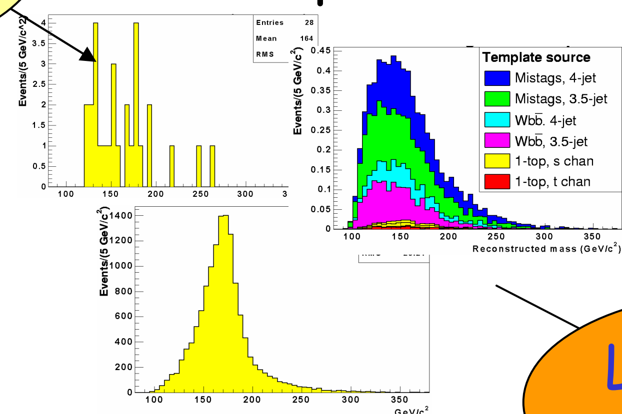
Wbb MC

tt MC

χ^2 mass fitter:
Finds best top mass and jet-parton assignment
One number per event
Additional selection cut on resulting χ^2

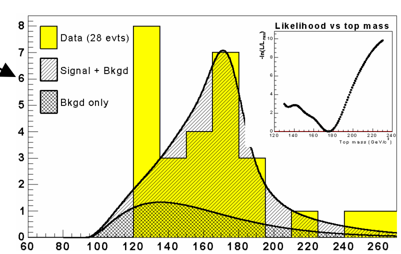
Mass fitter

Templates



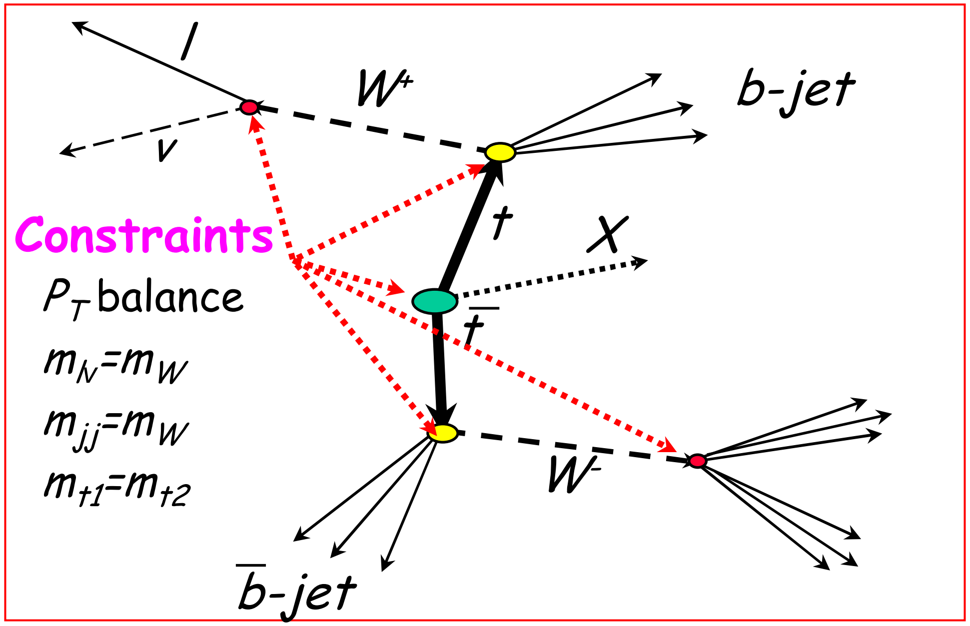
Likelihood fit

Result



Likelihood fit:
Best signal + bkgd templates to fit data
Compare to paramiz'n, not directly
Constraint on background normalization

Event-by-event Mass Fitter



- Distill all event information into one number (called reconstructed mass).
- Select most probable jet-parton assgnmt based on χ^2, after requiring b-tagged jets assigned to b partons.

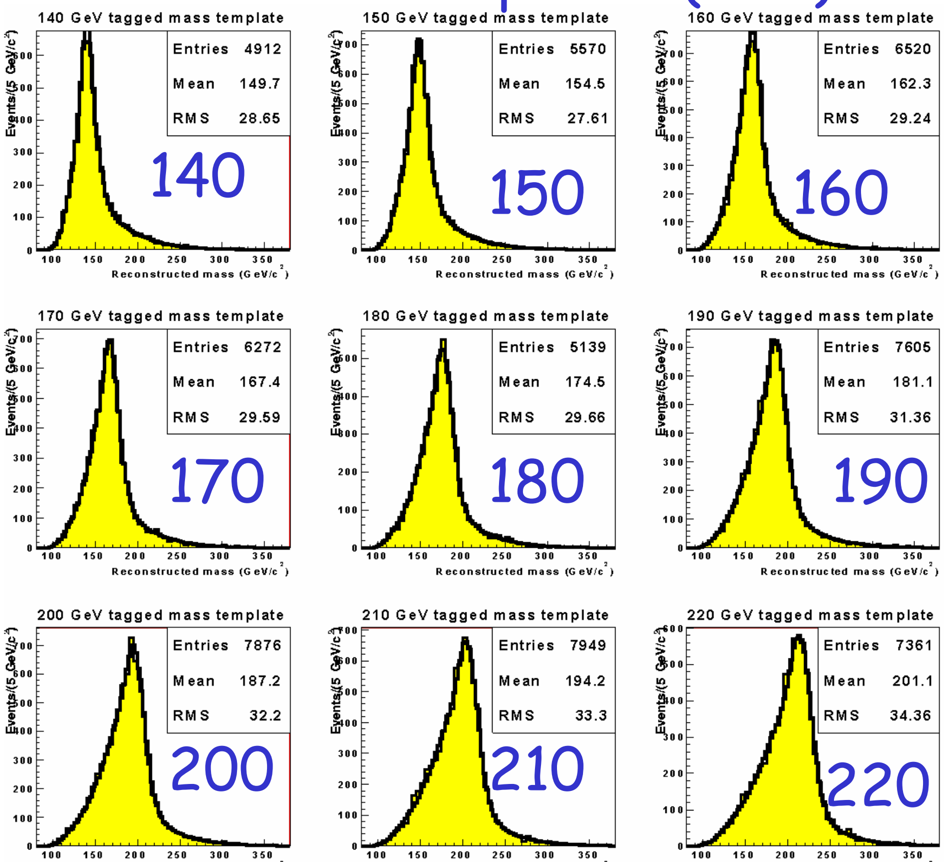
Reconstructed top mass is free parameter

$$\chi^2 = \sum_{i=\ell,4\text{ jets}} \frac{(p_T^{i,fit} - p_T^{i,meas})^2}{\sigma_i^2} + \sum_{j=x,y} \frac{(p_j^{UE,fit} - p_j^{UE,meas})^2}{\sigma_j^2}$$

$$+ \frac{(M_{jj} - M_W)^2}{\Gamma_W^2} + \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2} + \frac{(M_{bjj} - M_t)^2}{\Gamma_t^2} + \frac{(M_{b\ell\nu} - M_t)^2}{\Gamma_t^2}$$

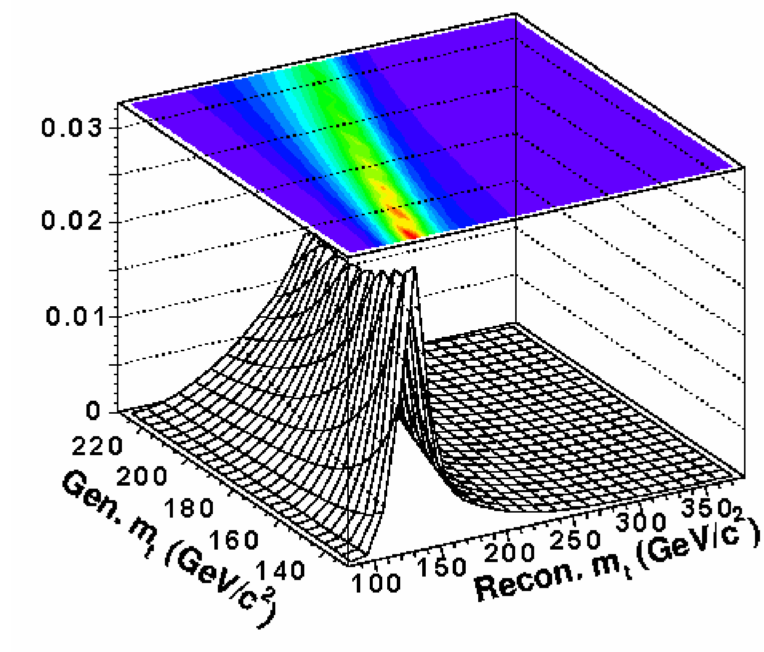
Signal templates

Selected templates (GeV)



Reconstructed Mass

Parameterization:
Build signal p.d.f. as a function of generated mass.



Background template

CDF Run II Preliminary (162 pb^{-1})

Mass Template Source	Background Source	# of events
W+jets (mistags)	Mistags, QCD	4.4 ± 1.0
Wbb	Wbb, Wcc, Wc, WW/WZ	2.1 ± 0.7
Single Top	Single Top	0.33 ± 0.04
Total		**6.8 ± 1.2**

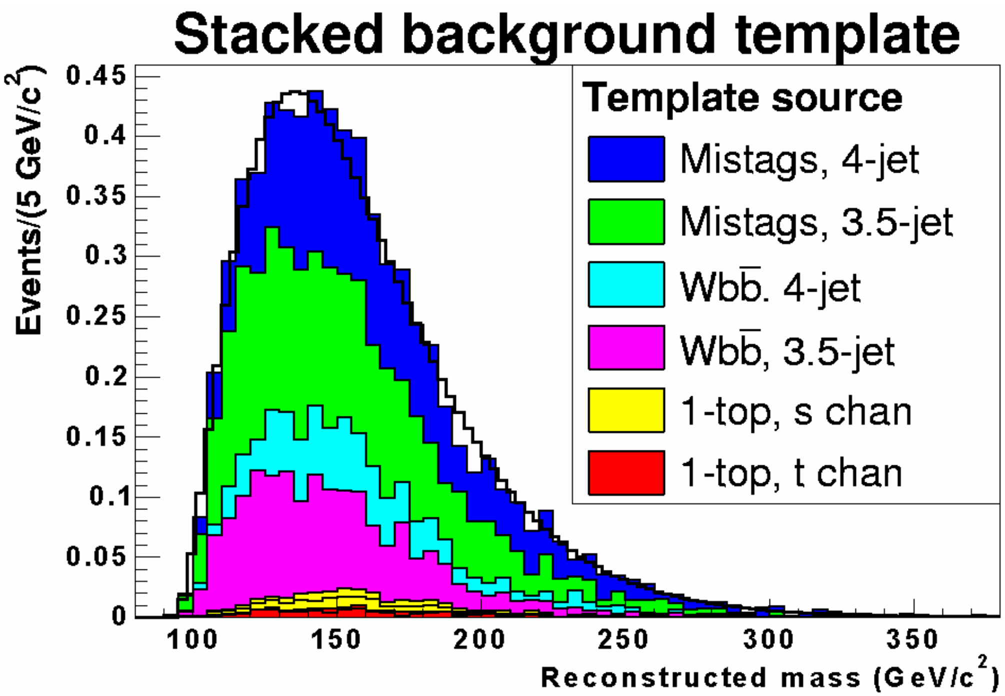
Constraint used in likelihood fit.

Major contributions:

W+heavy flavor

W+jets (mistag)

QCD



Unbinned likelihood fit

- Free parameters are M_{top}, n_s, and n_b.
  - Profile likelihood: minimize w.r.t. n_s,n_b, no integration.
- Fluctuations of n_b are a systematic effect. Allowing n_b to float in the fit means information in data is used to reduce the systematic uncertainty.

Interesting Parameter!

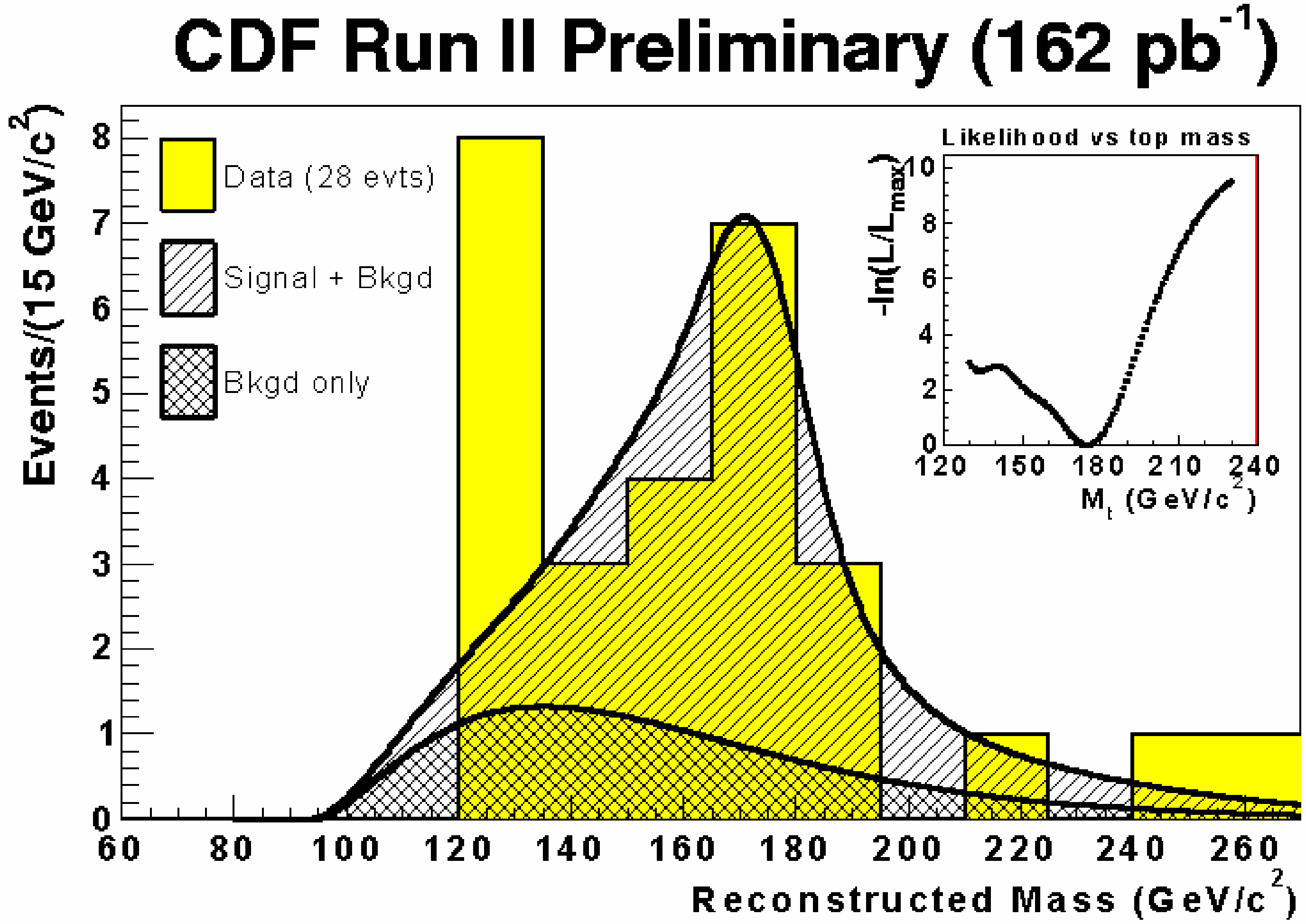
$$L = L_{\text{shape}} \times L_{\text{bg}}$$

$$L_{\text{shape}} = e^{-(n_s+n_b)}(n_s+n_b)^N \prod_{i=1}^{N} \frac{n_s P_{sig}(m_i; M_{\text{top}}) + n_b P_{bg}(m_i)}{n_s+n_b}$$

$$L_{\text{bg}} = e^{-\frac{(n_b^{fit}-n_b^{\exp})^2}{2\sigma_{n_b}^2}}$$

bkgd (mean) constraint

Data fit



More than 1 year ago!

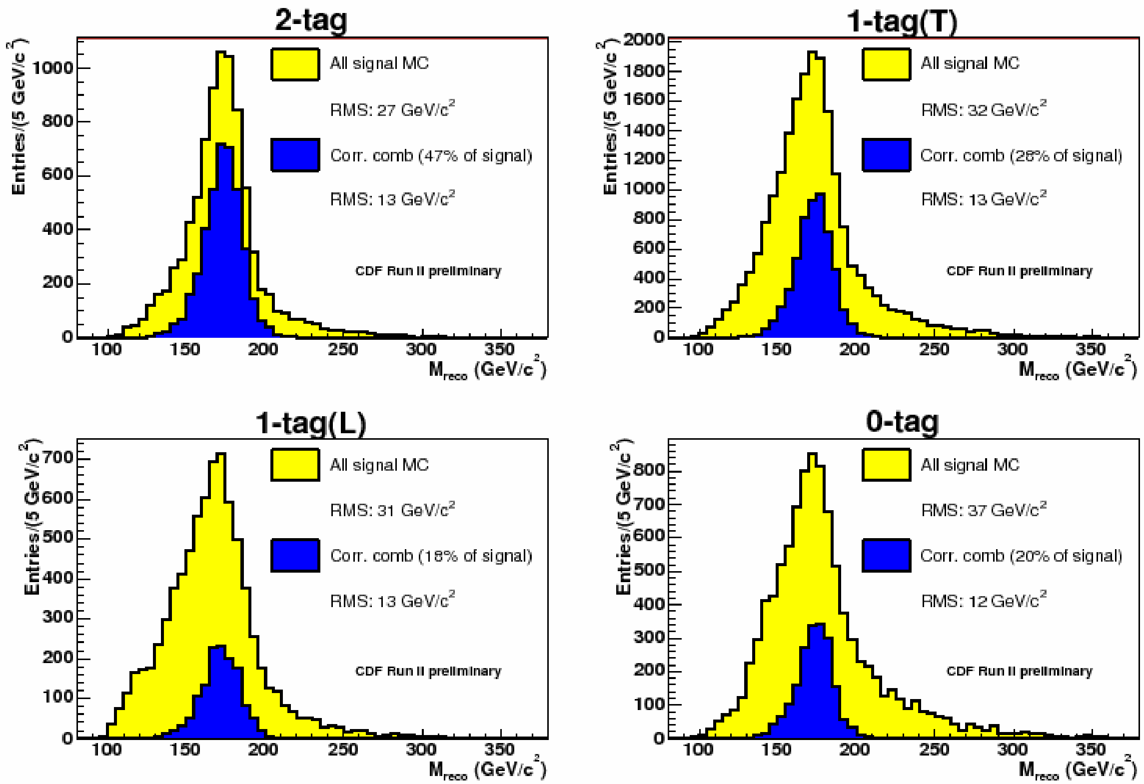
Best fit: 174.9 +7.1/-7.7 GeV/c^2

Updated template result

- More data!
- Subdivide sample.

Templates: subdivide sample

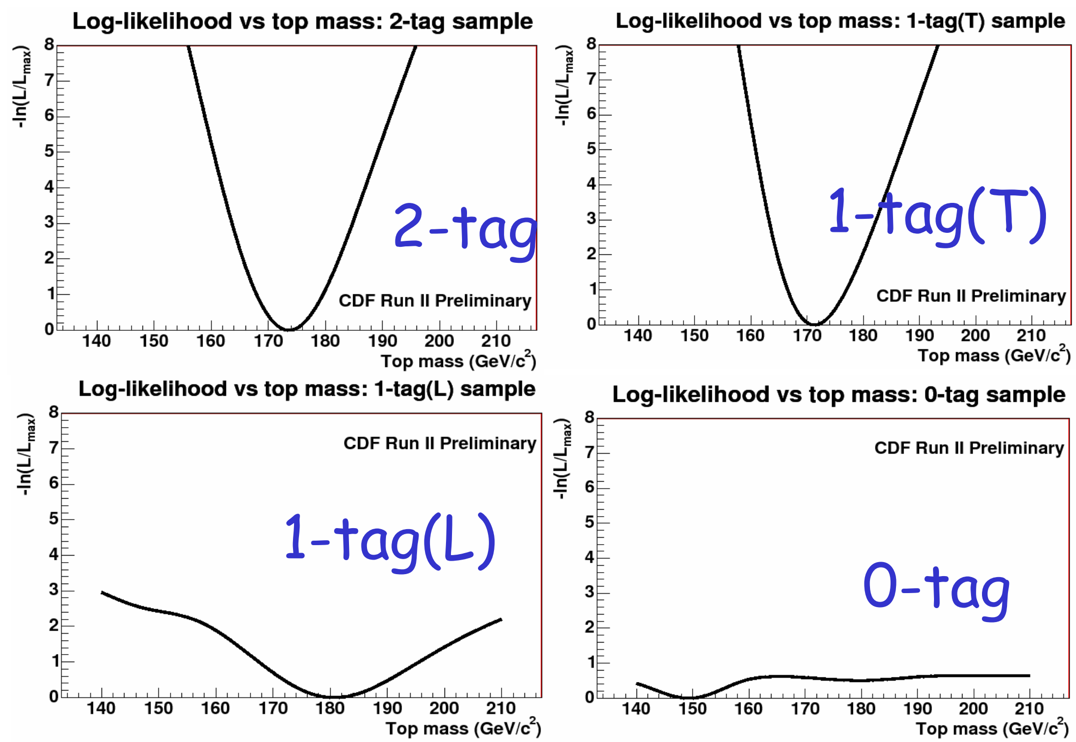
- Use 4 categories of events with different background content and reconstructed mass shape.
- More b tags are better
  - Increases S:B
  - More “golden” events, where correct jet-parton assignment is found.



Category	2-tag	1-tag(T)	1-tag(L)	0-tag
j1-j3	E_T>15	E_T>15	E_T>15	E_T>21
j4	E_T>8	E_T>15	15>E_T>8	E_T>21
S:B	18:1	4.2:1	1.2:1	0.9:1

Result with 318 pb^{-1}

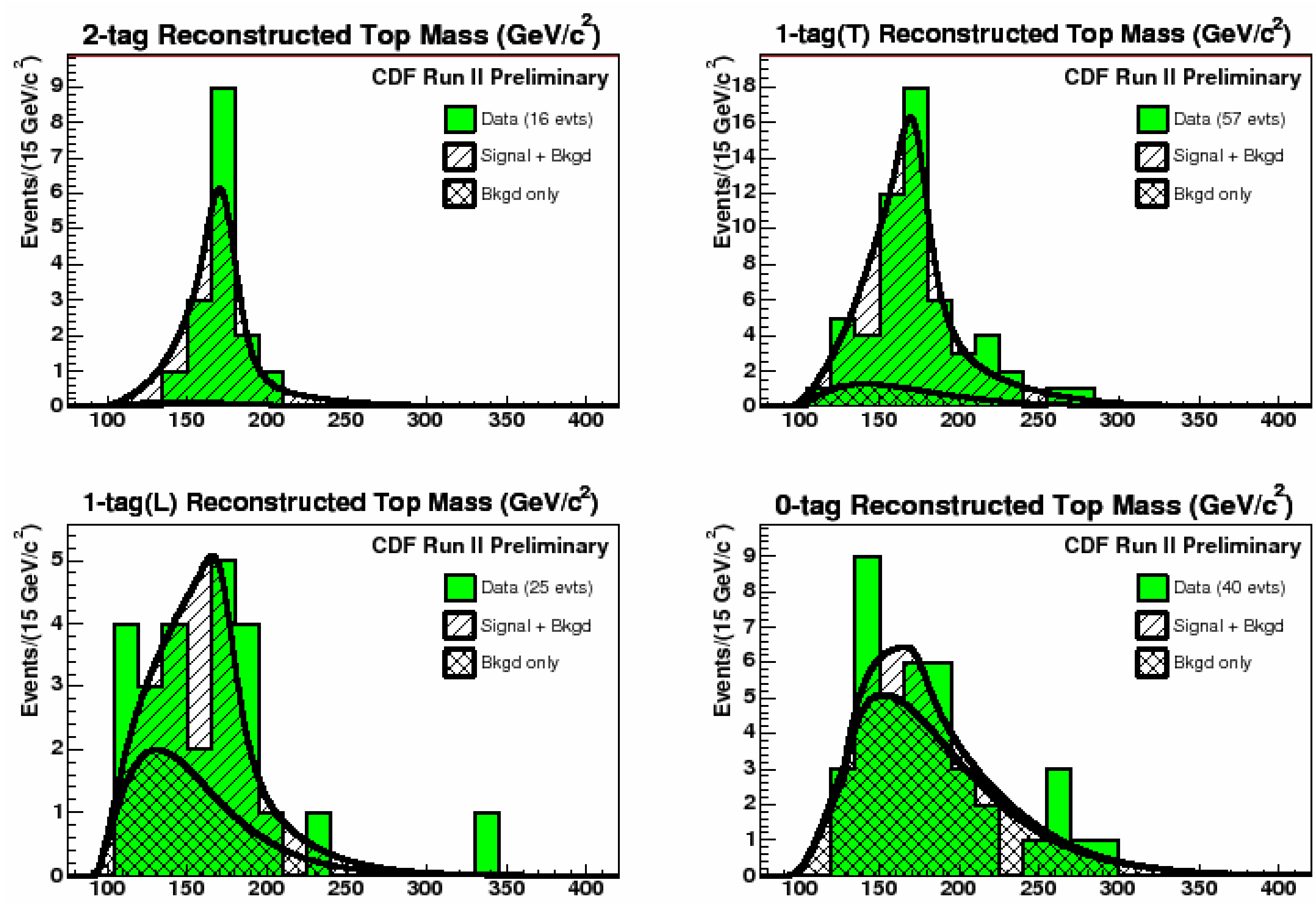
- Subdivision improves statistical uncertainty.
  - Pure and well reconstructed events contribute more to result.
  - Adds 0-tag events.
- Subdivision does *not* improve systematic uncertainty.
  - Most systematics, including jet energy scale, are highly correlated among the samples.



Expected Fraction of Sensitivity			
2-tag	1-tag(T)	1-tag(L)	0-tag
35%	45%	9%	11%

Result with 318 pb^{-1}

M_{top} = 173.2 $^{+2.9}_{-2.8}$ (stat) ± 3.4 (syst) GeV/c^2



Green histos: data distributions

Curves: expected signal and background from global best fit

Systematics Summary

CDF Run II Preliminary (318 pb^{-1})

Systematic Source	**Uncertainty (GeV/c^2)**
Jet Energy Scale	3.1
B-jet energy	0.6
Initial State Radiation	0.4
Final State Radiation	0.4
Parton Distribution Functions	0.4
Generators	0.3
Background Shape	1.0
MC statistics	0.4
B-tagging	0.2
Total	**3.4**

Was 6.8 !!
- Reduced double counting
- Tuned simulation to data

Systematics dominated by jet energy scale.

CDF Run II Preliminary (318 pb^{-1})

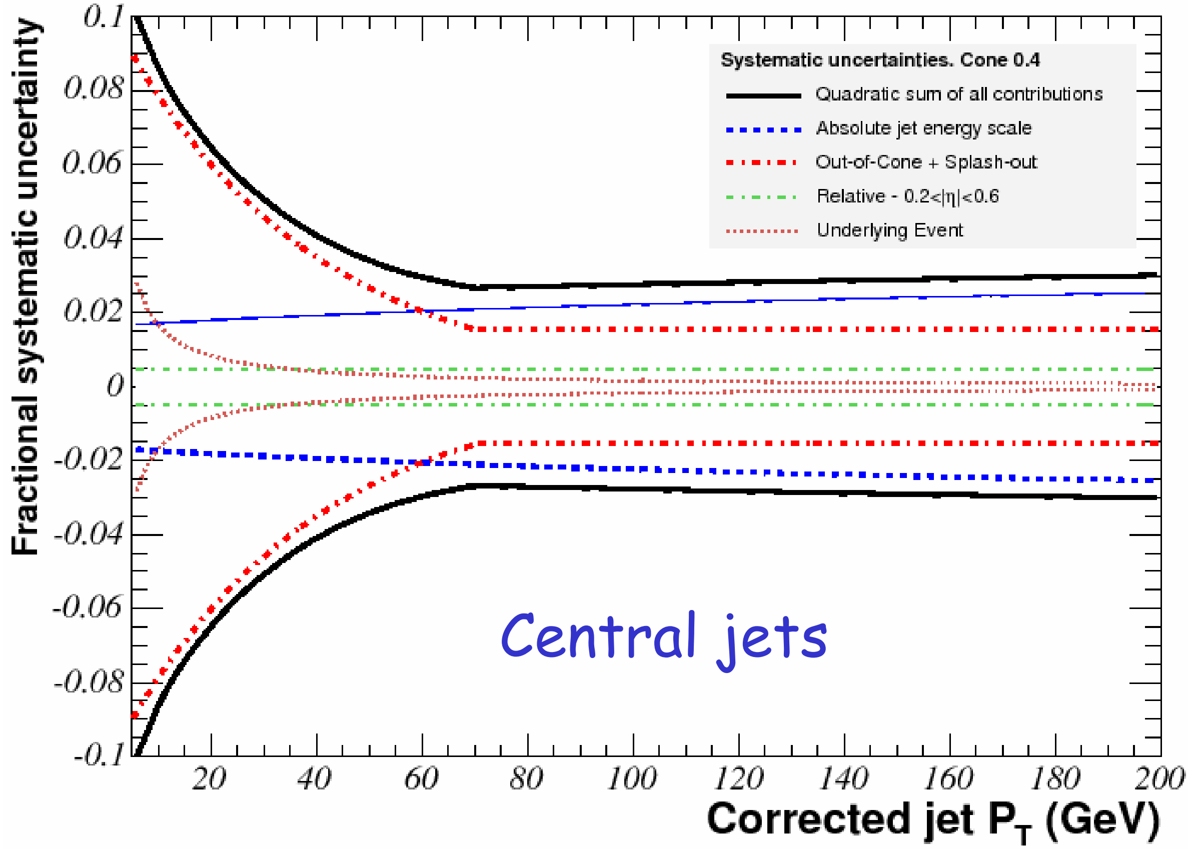
Jet Systematic Source	**Uncertainty (GeV/c^2)**
Relative to Central	0.6
Hadronic energy (Absolute Scale)	2.2
Parton energy (Out-of-Cone)	2.1
Total	**3.1**

World's Best Top Quark Mass: M_{top}+ JES simultaneous fit

- What is jet energy scale JES?
- Measure JES *in situ*.
- Perform simultaneous fit.

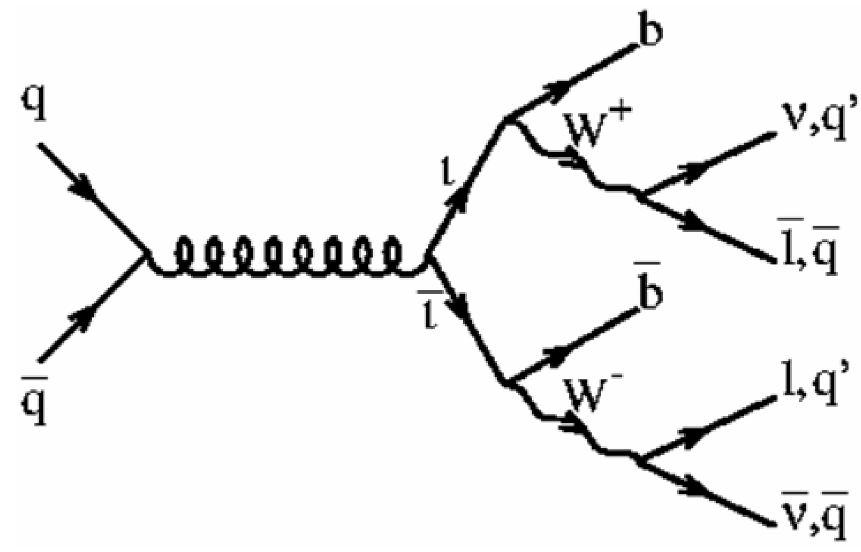
Jet systematics at CDF

- What is jet energy scale "JES"?
- Measures how incorrect is our nominal jet energy measurement.
- Units of σ: correspond to one s.d. of jet energy uncertainty
  - Accounts for p_T, η dependence.



W mass resonance in tt events!

- Can we use W→jj mass resonance to constrain JES?
- M_{top} measurement sensitive primarily to energy scale of b jets. (W mass constraint in χ^2.)
  - But studies show most uncertainty is shared by light quark, b jets.
  - Only 0.6 GeV/c^2 additional uncertainty on M_{top} due to b-jet-specific systematics.

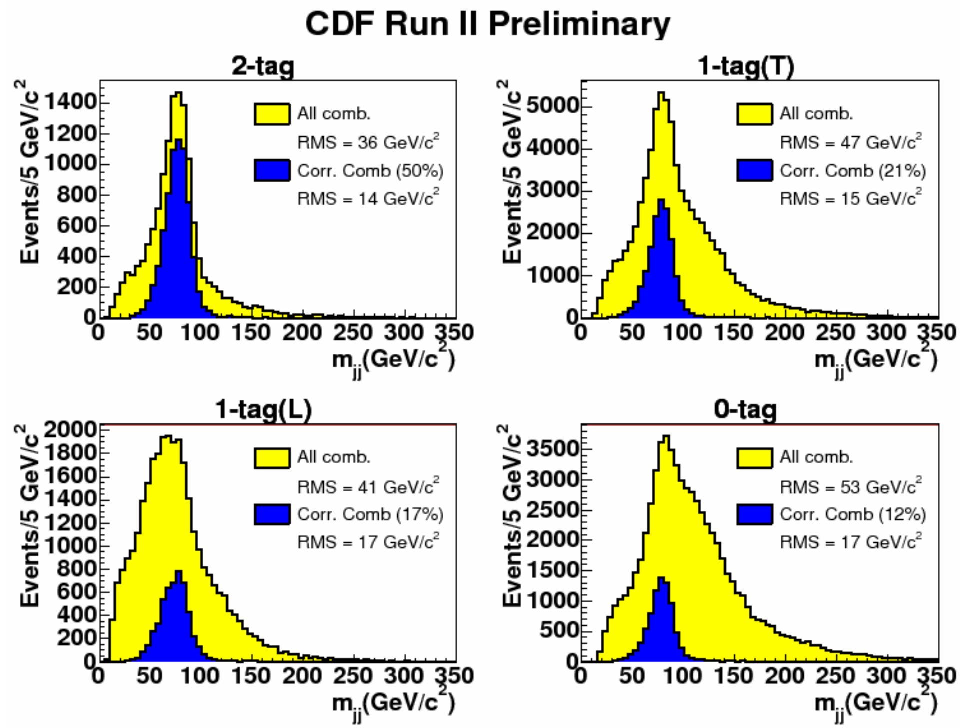


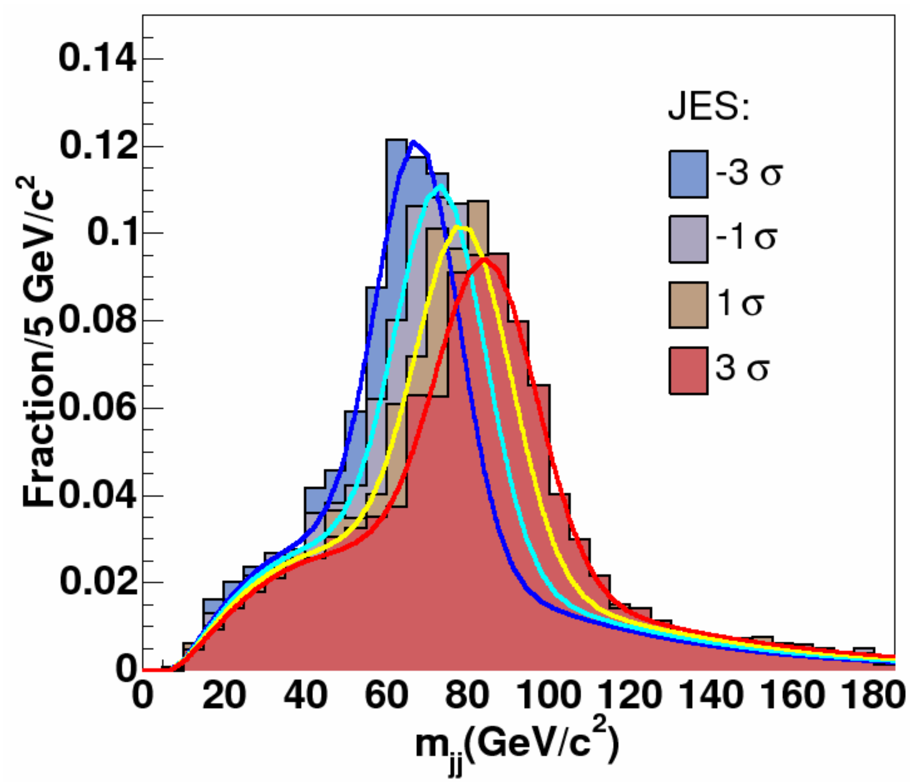
So use W→jj to improve understanding of q jets, therefore b jets, therefore M_{top}.

This constraint will only improve with statistics!

Measure JES using dijet mass

- Build templates using invariant mass m_{jj} of all non-tagged jet pairs.





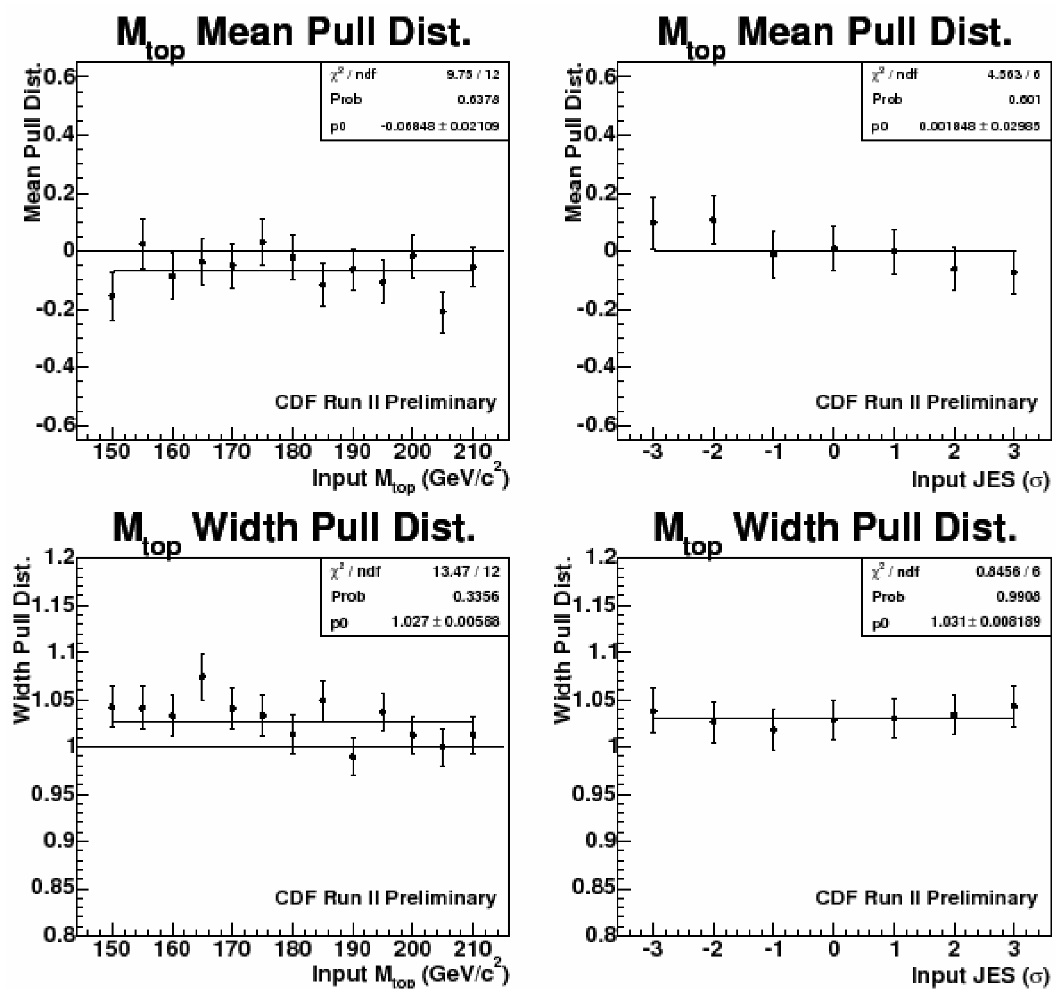
- Rather than assuming JES and measuring M_W...
- Assume M_W and measure JES
- Parameterize $P(m_{jj};JES)$ same as $P(m_t^{reco};M_{top})$

The "2D" measurement

- How do we use this bonanza of JES information?
- Too many correlations to treat this as an independent measurement of JES.
- Take the plunge and fit for M_{top} and JES simultaneously…
  - Need "2D" templates: $P(m_t^{reco};M_{top},JES)$ and $P(m_{jj};M_{top},JES)$.
  - More complex, but still tractable.
  - Constrain to prior knowledge: JES = 0 ± 1.
- Advantages:
  - Improve uncertainty on JES (dominant systematic)→improve uncertainty on M_{top}.
  - With this method, JES uncertainty begins to scale directly with statistics!

Method checks

- Prove to ourselves that parameterizations and likelihood machinery work: measure the top quark mass in MC samples.
- M_{top} fit unbiased across input top mass and jet energy scales.
- Reported uncertainty scaled by ~1.03 as shown (effect of non-Gaussian likelihood).

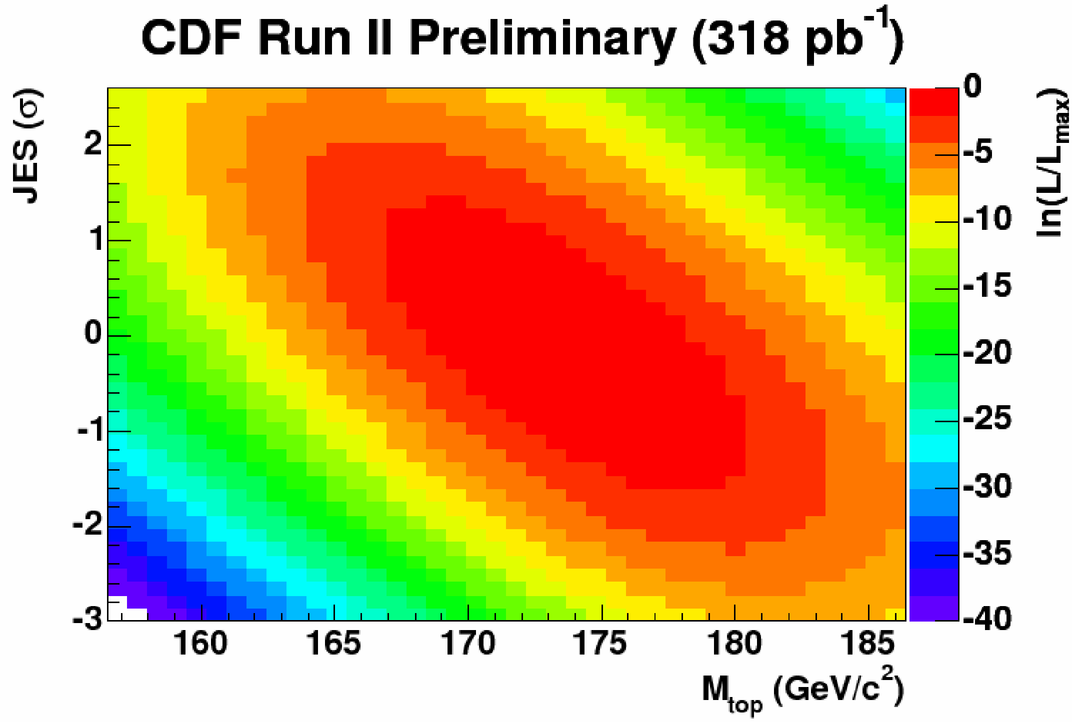


Apply 2D fit to the data

JES = -0.10 $^{+0.78}_{-0.80}$ (stat only) σ

$$M_{top} = 173.5^{+3.7}_{-3.6}\,(\text{stat.} + \text{JES}) \pm 1.7\,(\text{syst.})\,\text{GeV}/c^2$$

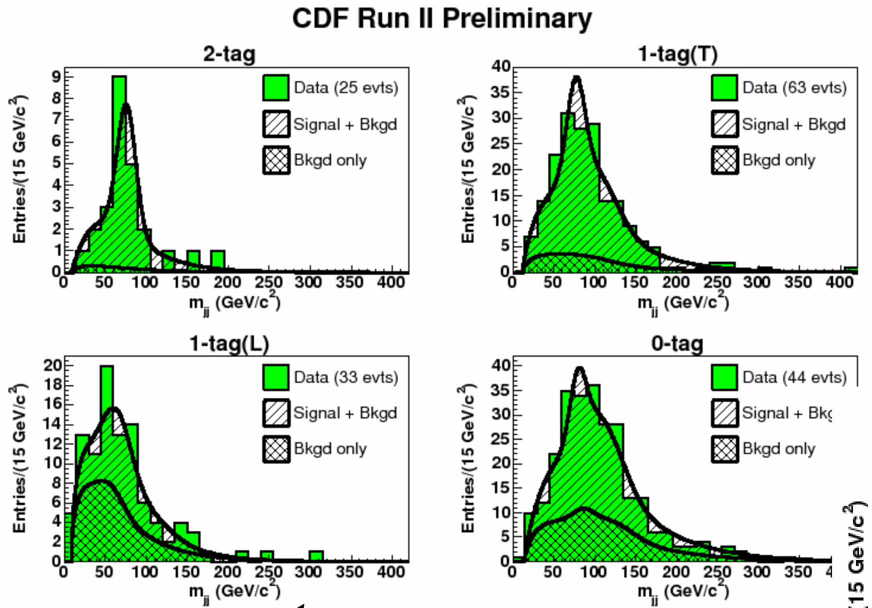
- Reported error includes both "pure statistics" and (reduced) JES systematic.
- Breaks down to $^{+2.7}_{-2.6}$ (stat) ± 2.5 (JES)
- 20% improvement in uncertainty due to JES!



1D result was:

$$M_{top} = 173.2^{+2.9}_{-2.8}\,(\text{stat.}) \pm 3.1\,(\text{JES}) \pm 1.5\,(\text{syst.})\,\text{GeV}/c^2$$

Reconstructed masses w/ overlaid fits

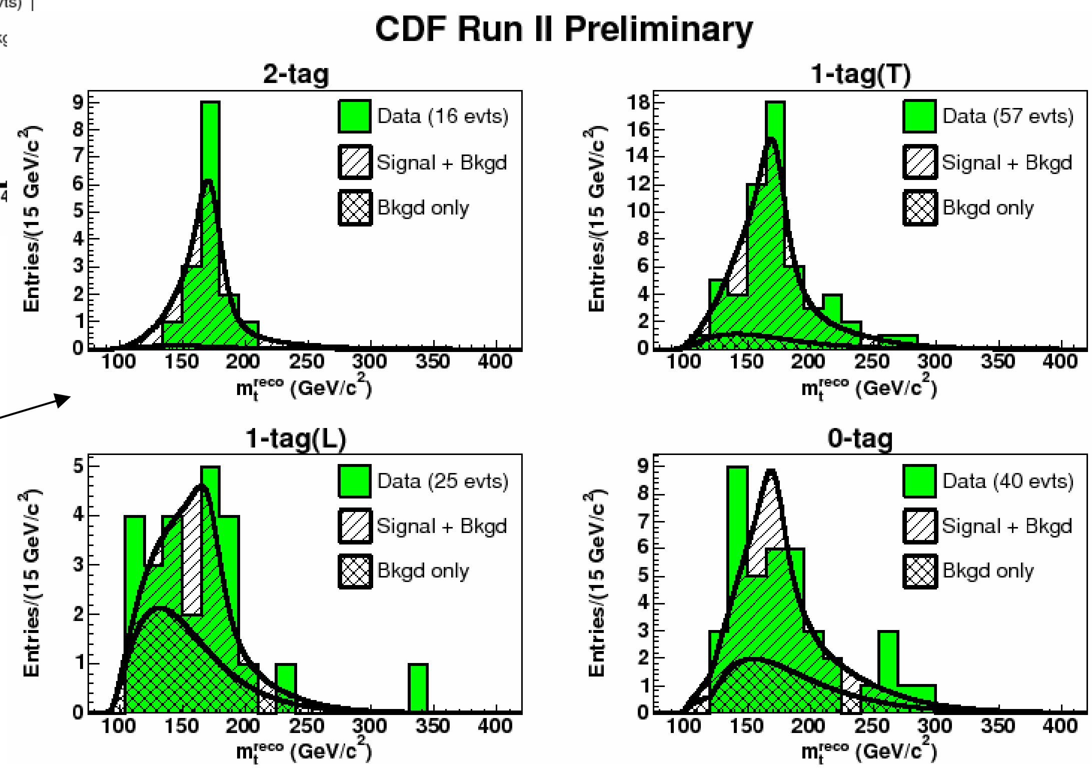


Green histos: data distributions

Curves: expected signal and background from global best fit

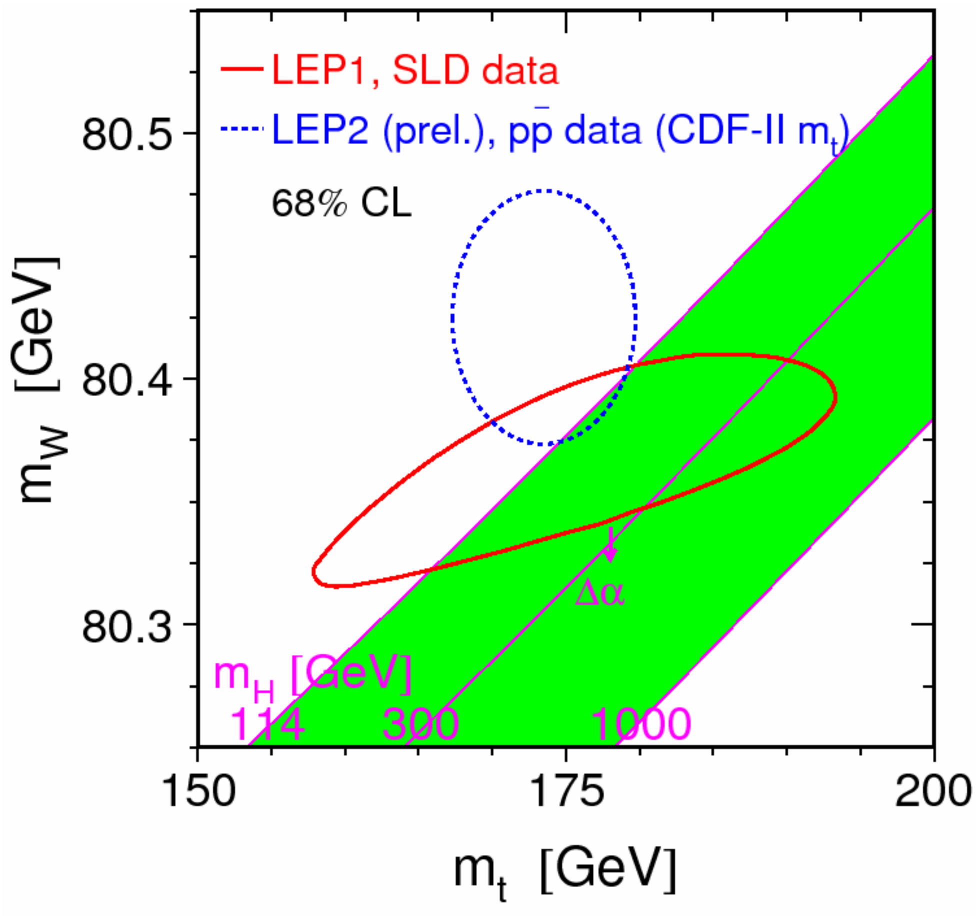
m_{jj}

m_t^{reco}



What if this were the only M_{top} result?

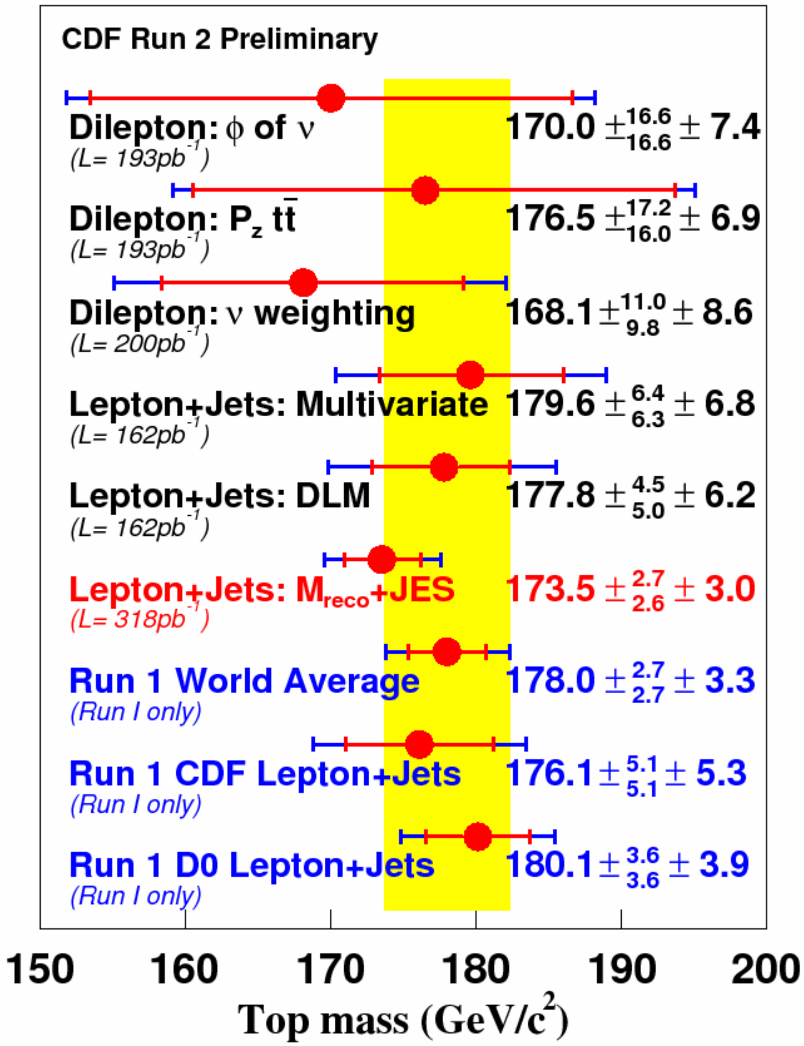
- Electroweak fit using *only* this result as top quark mass.
- A full combination of CDF/D0, Run I/II is months away…



Other Techniques and Results

- Dynamical Likelihood Method
- Multivariate Template Method
- Neutrino (Eta) Weighting
- Kinematic Method
- Neutrino (Phi) Weighting

Current CDF Measurements



Dynamical Likelihood Method

- Maximum likelihood method, where likelihood is built up for each event i as below.

Sum over all possible jet-parton assnmts

Quadratic eqns give multiple solns for ν: sum over them.

Parton Distribution Functions

Transfer functions connect jets to partons

Matrix element provides complete dynamical event information

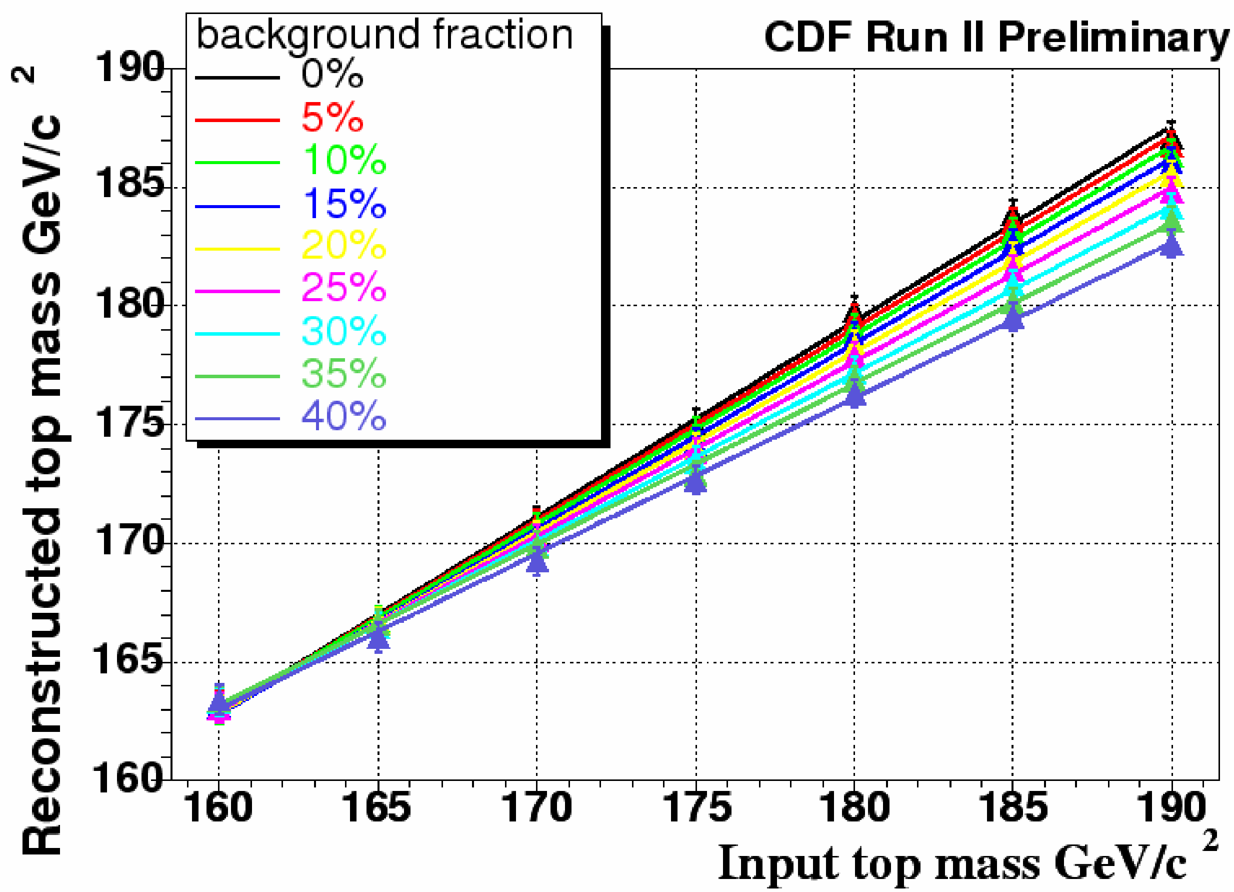
Ad hoc treatment of ISR

Integrate over z_1, z_2, $\mathbf{y}$ (partons)

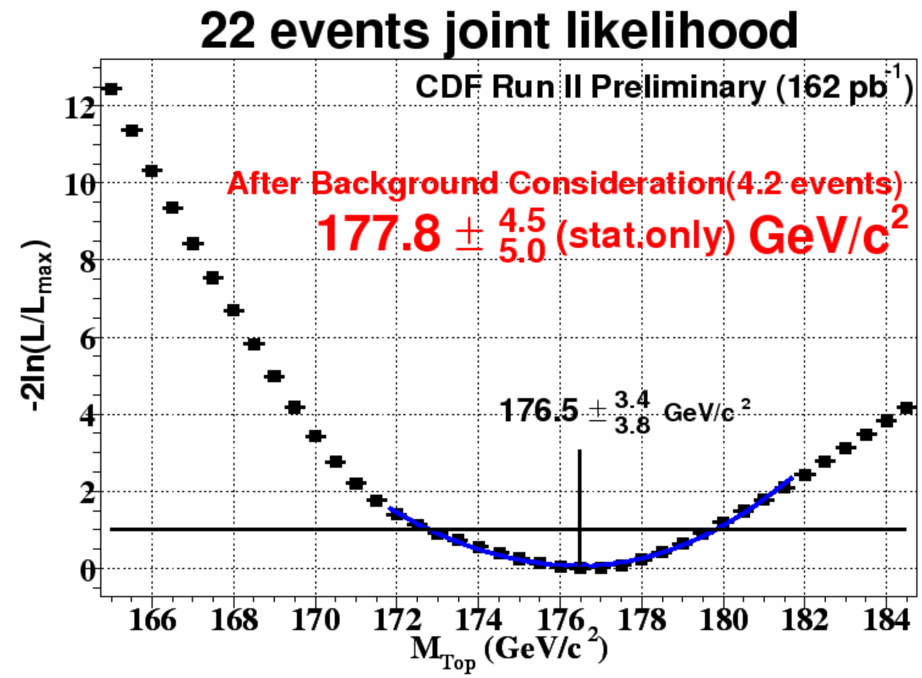
$$L^i(M_{top}) = \int \sum_{comb} \sum_{nsol} \frac{2\pi^4}{Flux} |M|^2 F(z_1, z_2) f(p_t) w(\mathbf{x}, \mathbf{y}_t; \alpha) d\mathbf{x}$$

DLM background

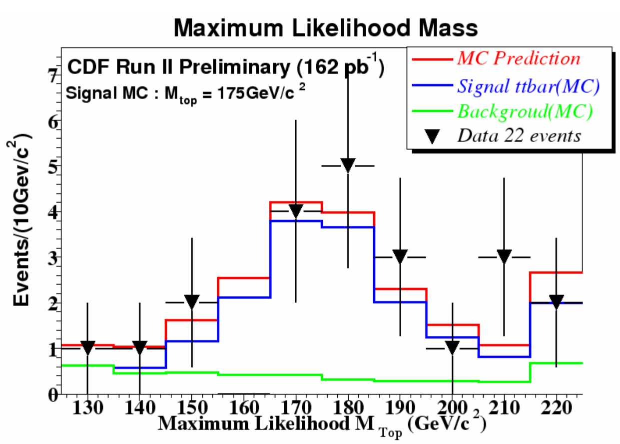
- More difficult to treat background than in template analyses.
- Ideally, need matrix element for background.
- Instead, DLM uses a mapping function: background dilutes mass information in a known manner, so correct for it.



DLM results



Source	ΔM_{top} GeV/c^2
Jet Energy Corrections	5.3
ISR	0.5
FSR	0.5
PDFs	2.0
Generator	0.6
Spin correlation	0.4
NLO effect	0.4
Transfer Function	2.0
Background fraction($\pm$5%)	0.5
Background modeling	0.5
Monte Carlo modeling	0.6
Total	6.2



Jet systematics smaller than template methods.

Effect of transfer functions, integration over partons?

Use more event information?

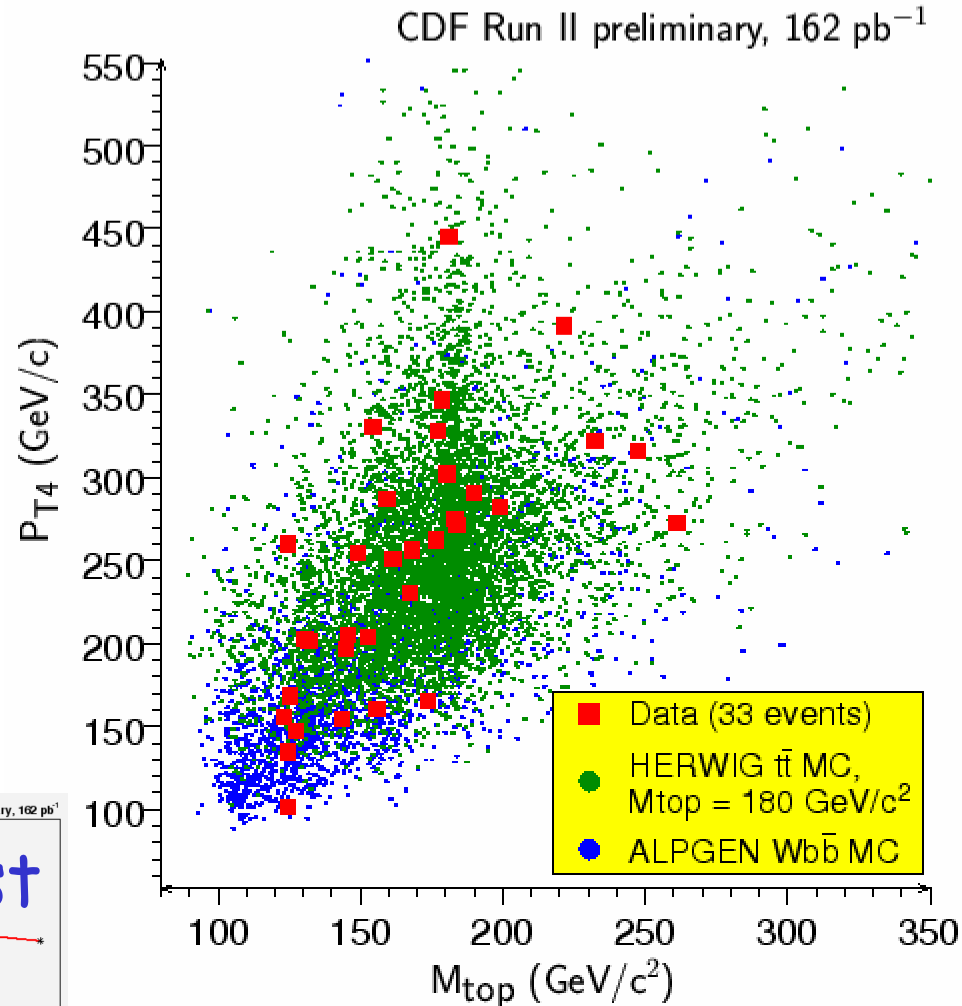
Template/DLM comparison

Method	**Template**	**DLM**
163 pb^{-1} result	174.9 ± 9.9 (“1D”)	177.8 ± 7.8
318 pb^{-1} result	173.5 ± 4.1 (“2D”)	??
Selection	≥ 4 jets	= 4 jets
Combinatorics	Best χ^2	Use all
JES	$W \to jj$	None yet
Background	Template	Mapping

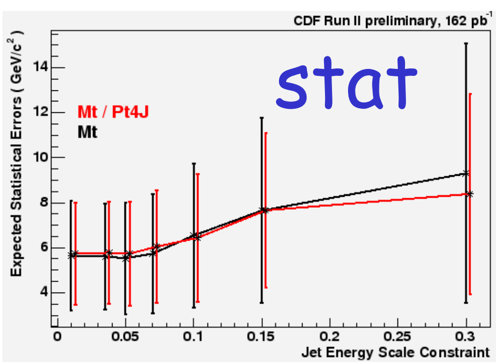
- Complementary methods.
  - Different sensitivity to details of production and decay.
- Within a few weeks, DLM should have a result comparable to 2D template analysis.

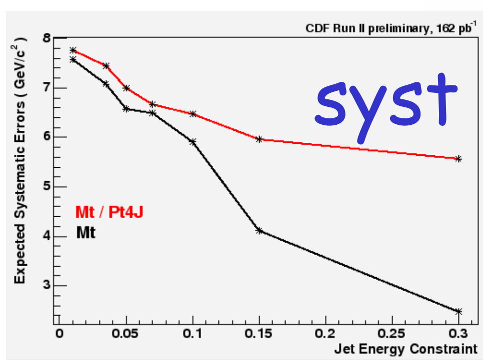
Multivariate template method

- Add second test statistic: Σp_T^{4j}—discriminates signal vs background events.
- Fit jet energy scale in every event using W mass—trades statistical error for systematic.



$$M_{top} = 179.6^{+6.4}_{-6.3} \pm 6.8\ \mathrm{GeV}/c^2$$





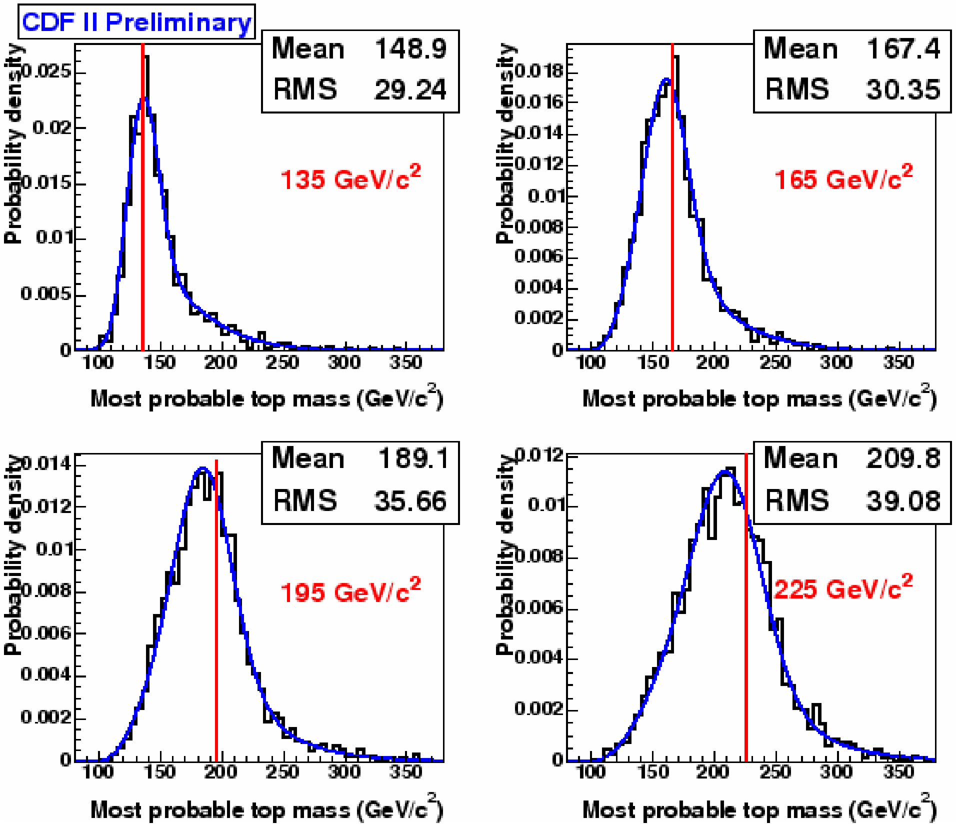
Dilepton analyses

- Under-constrained kinematic system.
- Must always make extra assumptions.
  - η_1, η_2 of neutrinos
  - ϕ_1, ϕ_2 of neutrinos
  - P^z_{ttbar}
- So far, all analyses in this channel use the template approach.

Neutrino weighting approach

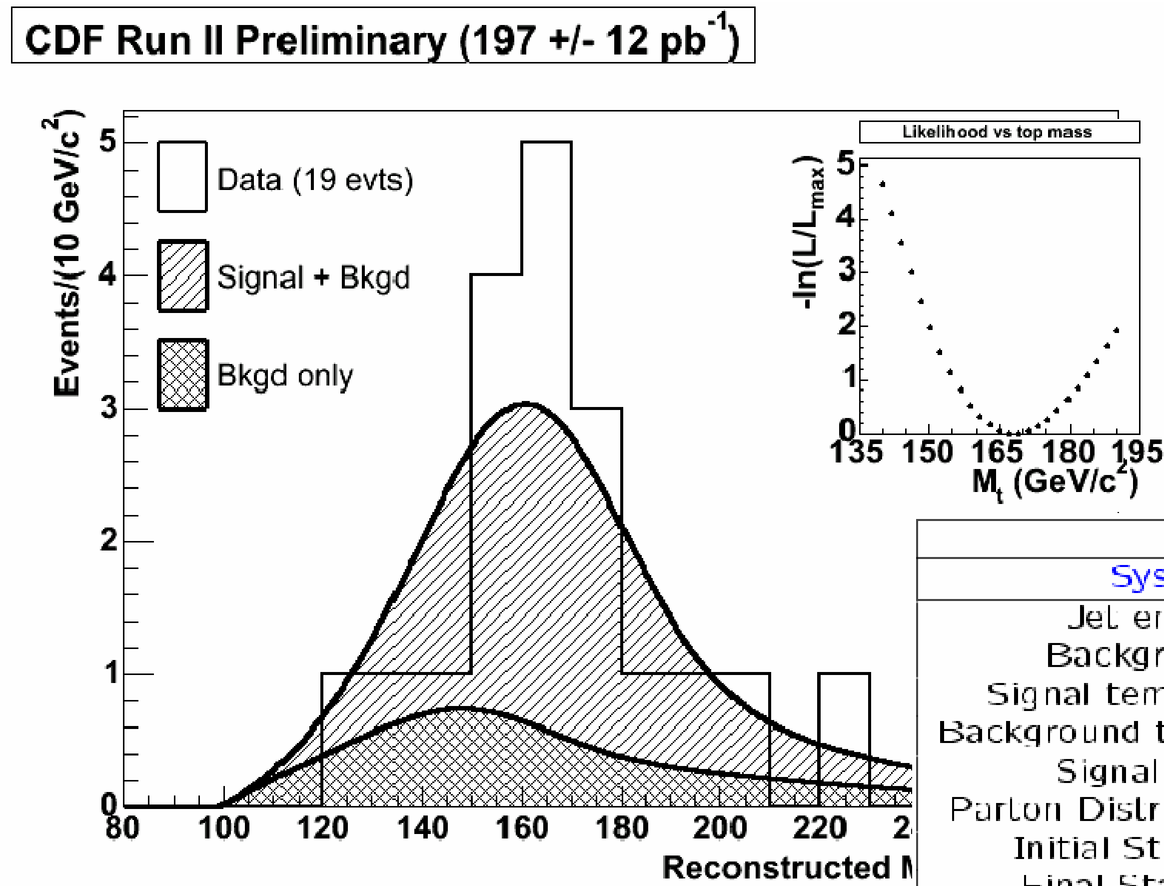
$$W(M_t, M_W = 80.5\,\mathrm{GeV/c^2}) = \sum_{\text{l-jet assn}} \iint_{\eta_\nu,\eta_{\bar\nu}} P(\eta_\nu,\eta_{\bar\nu}) \sum_{\nu\ \text{solns}} e^{-\frac{(\not{E}_x - p_x^\nu - p_x^{\bar\nu})^2}{2\sigma_x^2}} \cdot e^{-\frac{(\not{E}_y - p_y^\nu - p_y^{\bar\nu})^2}{2\sigma_y^2}}$$

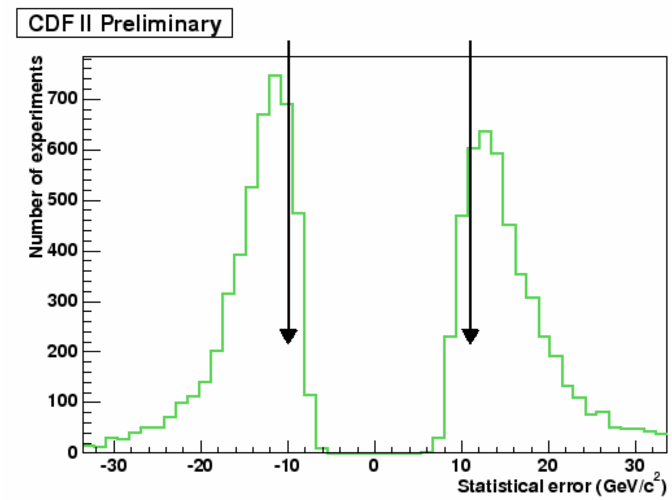
- Assume top mass, W mass, determine probability of event.
- Integrate over unknowns.
  - Lepton-jet pairing
  - Neutrino η
  - Missing energy solutions
- M_t for which event is most likely → M_{reco}.



NWA results

$$M_{top} = 168.1^{+11.0}_{-9.8}\,(\text{stat.}) \pm 8.6\,(\text{syst.})\ \text{GeV}/c^2$$

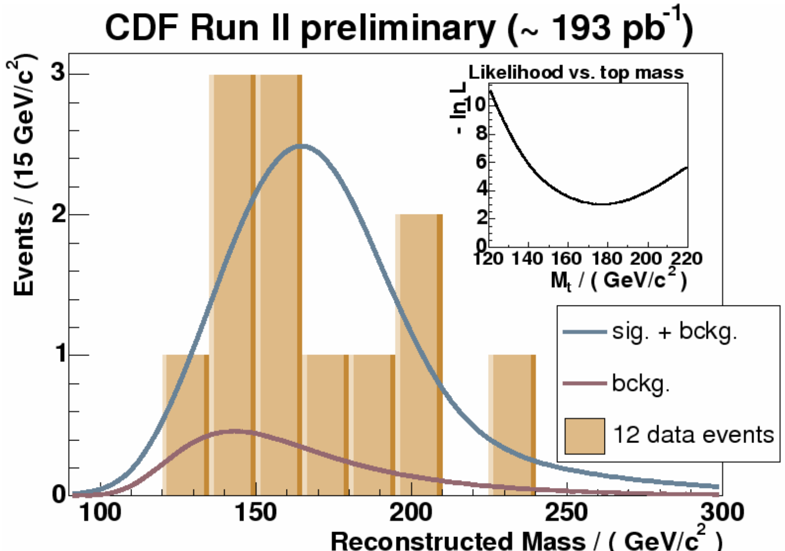


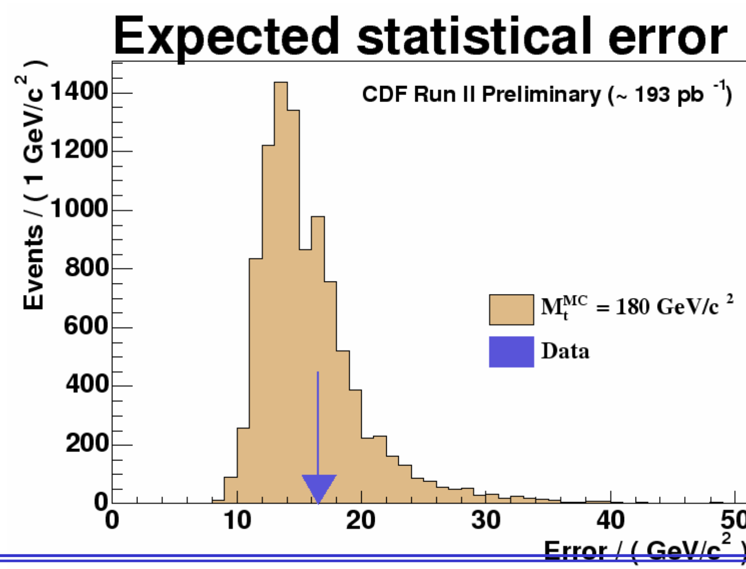


CDF II Preliminary	
Systematic	Uncertainty (GeV/c²)
Jet energy scale	7.4
Background shape	2.8
Signal template statistics	0.3
Background template statistics	1.3
Signal generators	0.6
Parton Distribution Functions	0.8
Initial State Radiation	2.5
Final State Radiation	1.3
$\not{E}_T$ resolution	0.3
Total	8.6

Kinematic reconstruction assuming P^z_{tt}

- Assume $P^z_{tt} = 0 \pm 180$ GeV/c
- Scan over P^z_{tt} and parton energies, perform kinematic reconstruction at each point.
- Test statistic is the top mass that contains the most likely point in this phase space (no integration).

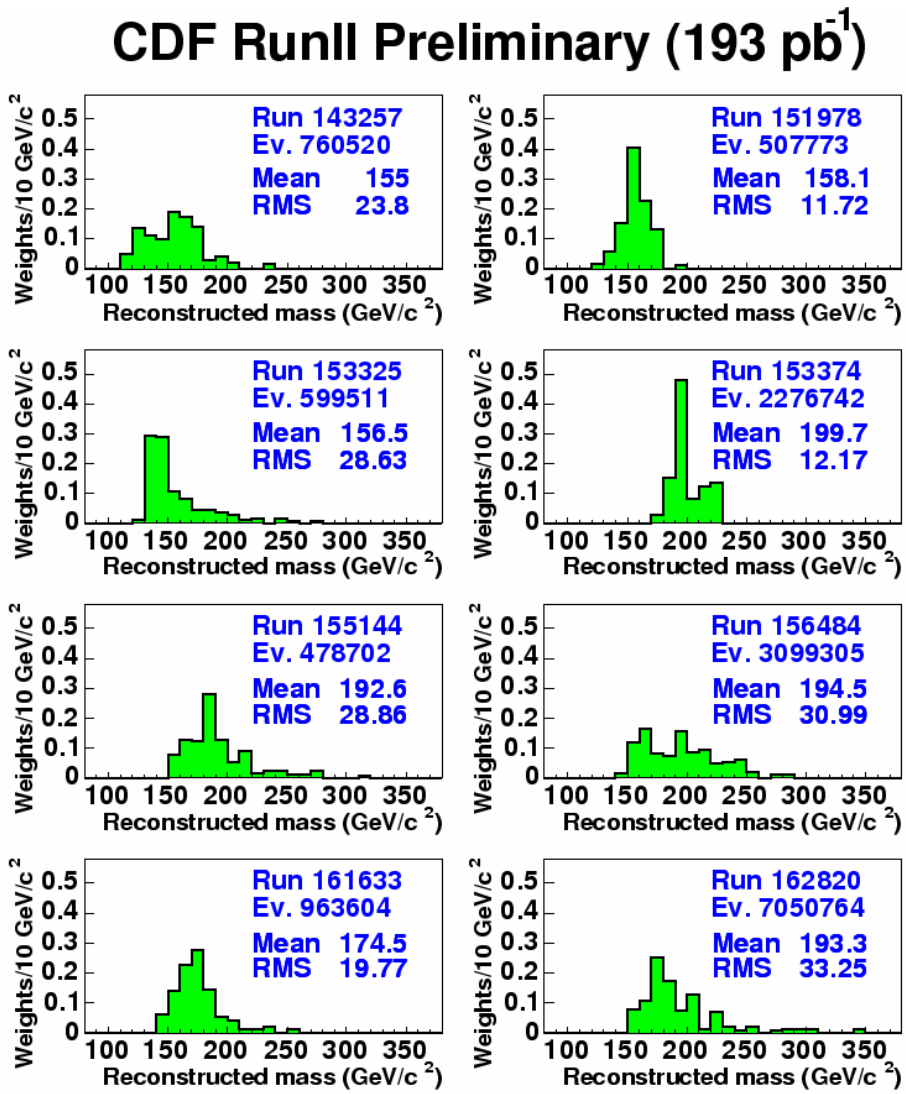




$$M_{top} = 176.5^{+17.2}_{-16.0}\,(\text{stat.}) \pm 6.9\,(\text{syst.})\ \text{GeV}/c^2$$

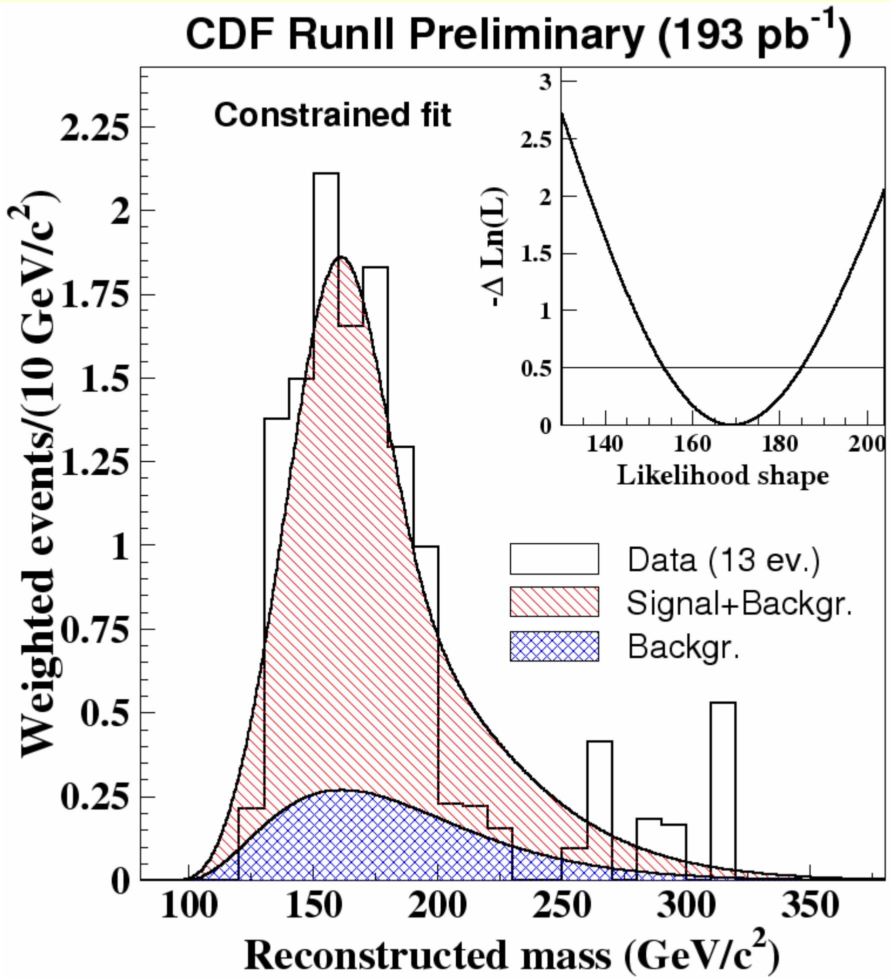
Dilepton Reconstructed Mass

- Use χ^2 from the lepton + jets analysis, slightly modified.
  - Assume ϕ_1, ϕ_2 of the two neutrinos (scan over plane).
  - Weight each point in ϕ_1-ϕ_2 space by $\exp(-\chi^2/2)$.
  - All points contribute to templates and to data distribution.



Dilepton reconstructed mass results

- Tighter selection than NWA gives fewer events, but smoother distrubution due to weighting solutions.
- Background peaks near signal—dilutes information in likelihood.



$$M_{top} = 170.0 \pm 16.6\,(\text{stat.}) \pm 7.4\,(\text{syst.})\ \text{GeV}/c^2$$

What's coming?

Maturing Analyses

- Matrix element techniques
  - Full background matrix element treatment
  - Apply to dilepton channel
- All-hadronic channel
  - Several algorithms in progress
  - Large background, more jets, even harder combinatorics!

General efforts

- Combine measurements across channels, techniques
  - Hard problem. Highly correlated systematics, non-Gaussian stat uncertainties.

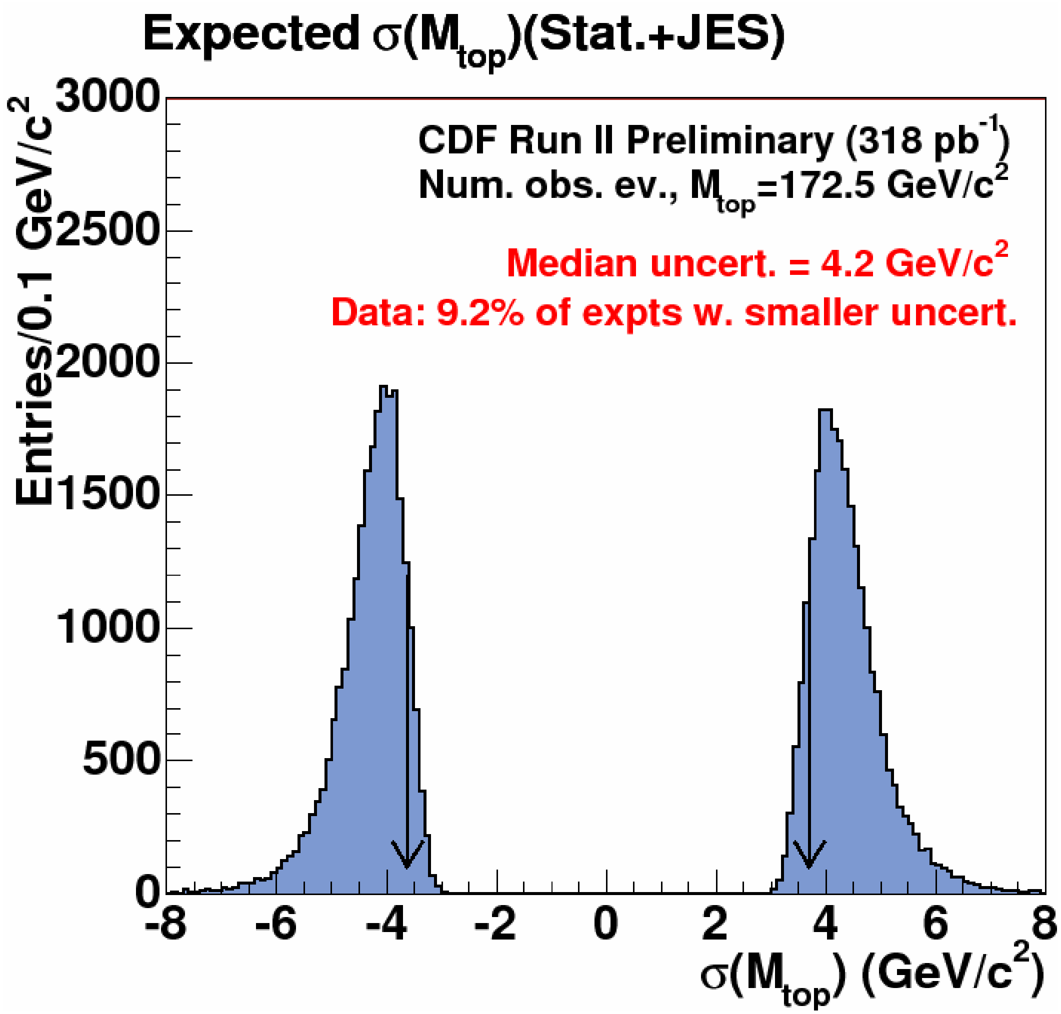
Keep an eye on this page in the coming weeks…

http://www-cdf.fnal.gov/physics/new/top/top.html

CDF top quark mass: $M_{top}=173.5^{+4.1}_{-4.0}$ GeV/c^2

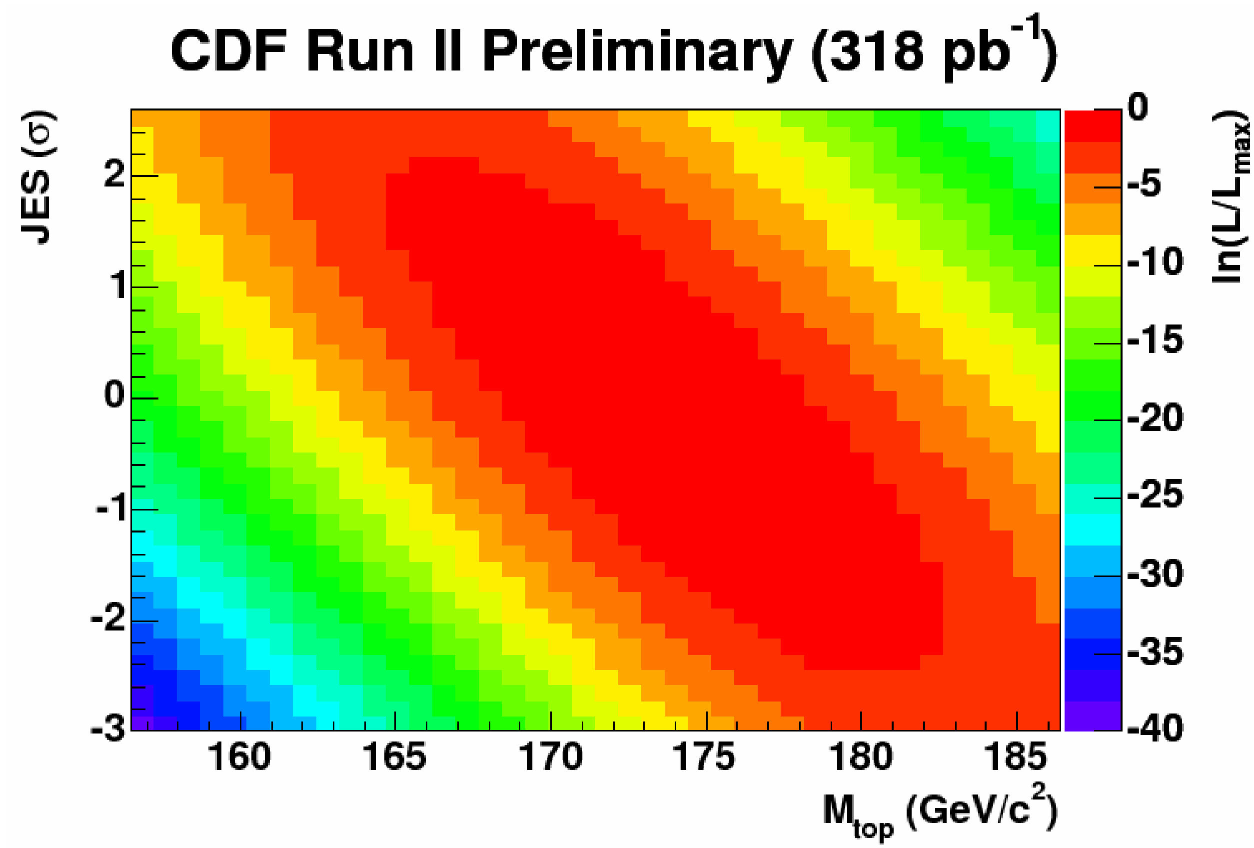
Backups

Expected Statistical Uncertainty



Fit without JES constraint

- Remove constraint JES = 0 ± 1 σ in likelihood.
- So all information about jet energy scale comes from *in situ* W resonance!



$$M_{top} = 174.0 \pm 4.5\,(\text{stat.} + \text{JES}) \pm ??\,(\text{syst.})\ \text{GeV}/c^2$$

Sample Composition

- Number of jets w/ E_T>15 GeV in W+jets events.
- Contributions from each background process + tt.