Top Quark Mass Measurements from CDF

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On Behalf of the CDF Collaboration
The Top Mass

• The top quark mass is interesting because ...

• Applications at the LHC:
  - Calibrating energies of highly boosted jets

• It can teach us about the Higgs
  - The top quark and the Higgs both couple to the W boson
  - Top mass and W mass determine SM Higgs mass
    • Measure to constrain Higgs mass
    • Test of standard model

1-Sigma Constraint on Higgs mass (2006)

Higgs and top quark couplings to W boson
Identifying $t\bar{t}$

- Tops decay to $W$'s a $b$'s
- Three very different types of mass analyses depending on $W$ decay modes
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    - Tiny backgrounds, low stats
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- Identify $b$'s (reduces background)
  - Use long $b$-lifetime: may travel many mm before decay
  - Use tracking to locate displaced vertex
Determining the Mass

- Event reconstruction challenges:
  - Which partons came from which top and which $W$?

- Jet reconstruction challenges:
  - Have to measure energies of decay quarks to get top mass
    - But can't measure quarks directly, see spray of particles
    - Leads to many “Jet Energy Scale” (JES) uncertainties
      - Black: full uncertainty on quark energy
      - 3-4 GeV uncertainty on top mass
Controlling the JES Uncertainty

- **Option 1: Use hadronic W decays**
  - Assume all jets in event have same JES
  - Constrain JES to reconstruct proper W mass
  - Invariant top mass in simulation increases with JES
  - Obviously impossible in dilepton channel

![Expected top mass depends on JES](image)
Controlling the JES Uncertainty

- Option 1: Use hadronic W decays
  - Assume all jets in event have same JES
  - Constrain JES to reconstruct proper W mass
  - Invariant top mass in simulation increases with JES
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- Fit for JES and top mass simultaneously
  - JES uncertainty becomes statistical!
  - Caveat: left with residual systematics due to assuming same JES for all jets
  - Most top analysis do this, but JES still largest uncertainty on world average top mass

Expected top mass depends on JES
Controlling the JES Uncertainty

- Option 2: don't use jet energy!
  - Decay length of b-tagged jets
  - Transverse momentum of leptons

- Evaluate top mass from mean decay length and mean lepton momentum
  - Plenty of stats at LHC: systematics are what are important
  - Decay length systematics limited by calibration of simulation to data
  - Lepton systematics limited by background modeling, simulation calibration, QCD radiation
    - Actively working on them

Results with 1.9 fb⁻¹ in Lepton+Jets Channel:

\[
\begin{align*}
  m_t &= 176.7^{+10.0}_{-8.9} (\text{stat}) \pm 3.4(\text{syst}) \text{GeV}/c^2 \\
  \text{Lepton Transverse Momentum} \\
  m_t &= 173.5^{+8.9}_{-9.1} (\text{stat}) \pm 4.2(\text{syst}) \text{GeV}/c^2 \\
\end{align*}
\]

Combined decay length and lepton transverse momentum

\[
m_t = 175.3 \pm 6.2(\text{stat}) \pm 3.0(\text{syst}) \text{GeV}/c^2
\]
Template Based $m_t$

- One of two “standard” methods for measuring the top mass
  - Make probability distribution functions (templates) for signal and backgrounds
  - Fit data, integrating over all allowed jet and lepton combinations
  - Straightforward and reliable

Example: Signal chi$^2$ for CDF Lepton+Jets Template Fit

$$\chi^2 = \sum_{i=\ell,4 jets} \frac{(p_T^{i,fit} - p_T^{i,meas})^2}{\sigma_i^2} + \sum_{j=x,y} \frac{(U_j^{fit} - U_j^{meas})^2}{\sigma_j^2} + \frac{(M_{jj} - M_W)^2}{\Gamma_W^2} + \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2} + \frac{(M_{bjj} - m_t^{reco})^2}{\Gamma_t^2} + \frac{(M_{b\ell\nu} - m_t^{reco})^2}{\Gamma_t^2}$$

- $m_W$ Constraints
- $m_t$ Constraints
- Measurement Constraints
- Unclustered Energy Constraints
L+J Template Method Results

- Number of background events extracted from direct fit
  - W+jet, QCD backgrounds largest
  - Constrained within uncertainties determined by cross section measurements
- Assign non b-tagged jets to W decay
  - In manner which best reproduces the W mass
- Dominant systematics:
  - Residual jet energy scale, behavior of b-jet (fragmentation, semileptonic fractions, etc)

CDF L+J Results (1.9 fb$^{-1}$)

$$m_t = 171.8 \pm 1.9(stat + JES) \pm 1.0(syst)GeV/c^2$$

Example of a template for $t\bar{t}$ signal (in 2D to Constrain JES)
Dilepton Template Method Results

- Dileptons channel differences
  - Much lower statistics and even smaller backgrounds
  - Underconstrained: two neutrinos you can't measure!

- Integrate over all possible neutrino directions weighted by probability of consistency with observed objects

Dilepton Results (1.9 fb⁻¹)
\[ m_t = 171.6^{+3.4}_{-3.2} \text{(stat)} \pm 3.8 \text{(syst)} \text{GeV/c}^2 \]

Combined Dilepton & L+J Results (1.9 fb⁻¹)
\[ m_t = 171.9 \pm 1.7 \text{(stat + JES)} \pm 1.0 \text{(syst)} \text{GeV/c}^2 \]
• **Dileptons channel differences**
  
  - Much lower statistics and larger backgrounds
  
  - Underconstrained: two neutrinos you can't measure!

  • Integrate over all possible neutrino directions weighted by probability of consistency with observed objects

• **Examples of systematics shown**

  - Note large difference in jet energy scale sensitivity

<table>
<thead>
<tr>
<th>Systematic</th>
<th>LJ</th>
<th>DIL</th>
<th>Combination</th>
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<tr>
<td><strong>Combined</strong></td>
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<td>3.8</td>
<td>1.0</td>
</tr>
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</table>

**Systematics for these analyses**

**Dilepton Results (1.9 fb⁻¹)**

\[ m_t = 171.6^{+3.4}_{-3.2} (stat) \pm 3.8 (syst) \text{GeV/c}^2 \]

**Combined Dilepton & L+J Results (1.9 fb⁻¹)**

\[ m_t = 171.9 \pm 1.7 (stat + JES) \pm 1.0 (syst) \text{GeV/c}^2 \]
Matrix Element $m_t$

- Extract more information from each event
  - Find mass likelihood event by event based on theoretical Matrix Element calculation for signal/background

Signal Probability Proportional to:

$$L(\bar{y} | m_t, \Delta_{JES}) = |M(m_t, \bar{x})|^2$$

Top mass and JES Likelihood

“Matrix Element” (Probability Amplitude)
Matrix Element $m_t$

- Extract more information from each event
  - Find mass likelihood event by event based on theoretical Matrix Element calculation for signal/background
  - Based upon measured kinematics, $x$ (and hypothesized, $y$)

Signal Probability Proportional to:

$$L(\bar{y}|m_t, \Delta_{JES}) = \frac{TF(\bar{y}|\bar{x} \Delta_{JES})|M(m_t, \bar{x})|^2}{\text{Top mass and JES Likelihood}}$$

Probability of measured momenta, $y$, given $x$ and JES

“Matrix Element” (Probability Amplitude)
Matrix Element $m_t$

- Extract more information from each event
  - Find mass likelihood event by event based on theoretical Matrix Element calculation for signal/background
  - Based upon measured kinematics, $x$ (and hypothesized, $y$)

Signal Probability Proportional to:

$$L(y|m_t, \Delta_{JES}) = \frac{f(z_1) f(z_2)}{F F} \frac{TF(y|x \Delta_{JES})}{M(m_t, \bar{x})^2}$$

- Top mass and JES Likelihood
- Probability of measured momenta, $y$, given $x$ and JES
- Probabilities of incoming momenta
- "Matrix Element" (Probability Amplitude)
Matrix Element $m_t$

- Extract more information from each event
  - Find mass likelihood event by event based on theoretical Matrix Element calculation for signal/background
  - Based upon measured kinematics, $x$ (and hypothesized, $y$)
  - Integrate over unknowns, sum over probability weighted parton associations.

Signal Probability Proportional to:

$$L(y|m_t, \Delta_{JES}) = \sum_{i=1}^{24} \int w_i \frac{f(z_1) f(z_2)}{FF} TF(y|x\Delta_{JES}) |M(m_t, \bar{x})|^2 dx$$

Top mass and JES Likelihood

Sum over parton combinations

Probabilities of incoming momenta

“Matrix Element” (Probability Amplitude)
Matrix Element $m_t$

- Extract more information from each event
  - Find mass likelihood event by event based on theoretical Matrix Element calculation for signal/background
  - Based upon measured kinematics, $x$ (and hypothesized, $y$)
  - Integrate over unknowns, sum over probability weighted parton associations. And normalize.

**Signal Probability Proportional to:**

\[
L(y|m_t, \Delta_{JES}) = \frac{1}{N(m_t)} \frac{1}{A(m_t, \Delta_{JES})} \sum_{i=1}^{24} \int w_i \frac{f(z_1)f(z_2)}{FF} TF(y|x, \Delta_{JES}) M(m_t, \bar{x})^2 dx
\]

- Top mass and JES Likelihood
- Sum over parton combinations
- Integrate over kinematics
- Normalizations
- Probability of measured momenta, $y$, given $x$ and JES
- Probability of incoming momenta
- "Matrix Element" (Probability Amplitude)

Leads to very precise results (2.7 fb$^{-1}$): $m_t = 172.2 \pm 1.0(\text{stat.}) \pm 0.9(JES) \pm 1.0(\text{syst})$ GeV/c$^2$

**Dominant Systematics:** Residual jet energy scale, generator uncertainties
All Hadronic

- Special challenges in this channel
  - 1/400 S/B from base event selection
  - 6 factorial combinations of parton assignments
- Solutions:
  - Require two b-tags in event
  - Use neural network event shape selection (S/B: close to 1/1)
- Fit mass with signal, mismatched signal, background shapes

CDF Run II Preliminary (1.9 fb⁻¹)

Final Mass Distribution
Results (1.9 fb⁻¹)

\[ m_t = 165.2 \pm 4.4\text{(stat)} \pm 1.9\text{(syst)} GeV/c^2 \]

Residual jet energy scale, Pileup, Generator, QCD radiation, Background Modeling systematics all play a modest role
D0+CDF Combination

- Using Best Linear Unbiased Estimator technique
  - Correlations estimated between 12 types of uncertainties
  \[ m_t = 172.4 \pm 0.7 \text{(stat)} \pm 1.0 \text{(syst)} \, \text{GeV/c}^2 \]
- Electroweak fits: SM Higgs mass now < 154 GeV/c^2 at 95% confidence level!
  - Counting LEP lower limit of \( M_H > 114 \, \text{GeV/c}^2 \), upper limit rises to 185 GeV/c^2
Summary and Outlook

- CDF Top group has performed many high precision top mass measurements
  - Not enough time to talk about all of them
  - Some others in backup slides. For full details, see: http://www-cdf.fnal.gov/physics/new/top/public_mass.html

- Dominant jet energy scale systematic is coming under control
  - Using hadronic W mass calibration
  - Using alternate variables the LHC can ~completely eliminate it

- Work to be done
  - Must be especially careful with systematics in a high precision era
  - More sophisticated combination procedures
  - Limitations of Leading Order simulation must be properly considered
Backups
CDF

- Tevatron collides $p\bar{p}$ at world's highest energies

- Beam luminosity has been steadily improving
  - Total of $\sim 5 \text{ fb}^{-1}$ has been delivered to each detector
    - About 80% data acquisition efficiency
    - Recent analyses use about $3 \text{ fb}^{-1}$
Lepton + Jets Matrix Element

- Makes very few assumptions about kinematics
  - Integrates over 19 parameters representing probabilistic kinematic spreads
    - Top and W masses, boost of system, directions and masses of each jet
      - Specialized integration techniques to make this possible
  - Neural network trained to distinguish signal and background
    - Background count determined from this output

Light quark angular transfer function, \( \eta = 0, m = 5 \)

Example integration variables: discrepancies in measured jet direction

Results (2.7 fb\(^{-1}\))

\[ m_t = 172.2 \pm 1.0(\text{stat.}) \pm 0.9(\text{JES}) \pm 1.0(\text{syst}) \text{GeV/c}^2 \]

Dominant Systematics: Residual jet energy scale, generator uncertainties
Dilepton Matrix Element

- Mass likelihood evaluated for signal and bkg hypotheses simultaneously
  - Based on tagging information, priors (small p's), kinematic information (x), for signal and backgrounds, k

\[ L(\text{#tag}, \vec{x}, m_t) = P_{\text{sig}}(\vec{x}, m_t)p_{\text{sig}}(\text{#tag}) + \sum_k P_{bg}^k(\vec{x})p_{bg}^k(\text{#tag}) \]

- Key feature: finds best neural network
  - Optimize NN for mass resolution, not signal purity. 20% improvement in statistical uncertainty.

  • First dilepton analysis to be limited by systematics instead of statistics!

Results (1.9 fb\(^{-1}\))

\[ m_t = 171.2 \pm 2.7(\text{stat}) \pm 2.9(\text{syst}) \text{GeV}/c^2 \]

Dominant Systematic: Jet energy scale
Future Improvements

- Mass results are now more systematically limited
  
  - But even without systematic improvements will have better than 1% precision at CDF
  
  - Work on improving systematics still ongoing
    
    - Already far ahead of where we projected we would be at this luminosity!

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CDF Top Mass Uncertainty
(all channels combined)

\[ \Delta M_{\text{total}} \text{ (GeV/c}^2) \]

- CDF Results
- Run IIa LJ goal (TDR 1996)

Scale \( \Delta(\text{stat}) / \sqrt{L} \), Fix \( \Delta(\text{syst}) \)
(assumes no improvements)

Scale \( \Delta(\text{total}) / \sqrt{L} \)
(improvements required)

Past Expectations and Future \( M_T \) Projections at CDF