A measurement of $\sin^2\theta_{\text{eff, lept}}^{\text{Mz}}$, $\sin^2\theta_w$ and indirect measurement of $M_w$ at CDF

Arie Bodek, University of Rochester
On behalf of the CDF Collaboration
ICHEP 2014:
Saturday, July 5, 2014  9:00-9:20  (17+3).  Room: Sala 8+9
Session: Top-quark and Electro Weak Physics

arXiv:1402.2239v2
2013: Long standing tension between LEP/SLD values of $\sin^2 \theta_{\text{eff}}^{\text{lept}} (M_Z)$

Tension between LEP and SLD

LEP SLD difference is 0.00122 New precision measurements of $\sin^2 \theta_{\text{eff}}$ would help resolve this diff.
$\sin^2 \theta_w = 1 - \frac{M_w^2}{M_z^2}$  
Is independent of mass or lepton or quark flavor

In hadron colliders: $A_{FB}$ for $e^+e^-$ or $\mu^+\mu^-$ pairs in the Z boson Region is sensitive to $\sin^2 \theta_{eff}^{lept} (M_{\mu+\mu})$ (which depends on mass and quark flavor)

Collinear, No dilepton PT

Born level polar angle distribution: $1 + \cos^2 \theta + A_4 \cos \theta$

We define for short: $\sin^2 \theta_{eff} \equiv \sin^2 \theta_{eff}^{lept} (M_z)$  
$at the Z pole!$

The following is an approximate relation

$\sin^2 \theta_{eff}^{lept} \approx 1.037 \cdot \sin^2 \theta_w \cdot \text{[ZFITTER $\kappa_e (\sin^2 \theta_w, M_z)$ form factor]}$
Direct and indirect measurements of $M_W$ in SM

The new key element in the indirect extraction or inference of $M_W$ from $A_{FB}$ in the Standard Model is that the Higgs mass is now known. Therefore we can measure both $\sin^2 \theta_{\text{eff}}$ AND the on-shell $\sin^2 \theta_w = 1 - M_w^2 / M_z^2$ (we use $m_H = 125$ GeV).

A indirect measurement of $M_W$ is done by measuring the on-shell $\sin^2 \theta_w$ and using the SM relation

$$\sin^2 \theta_w = 1 - M_w^2 / M_z^2$$

A error of $\pm 0.00030$ in $\sin^2 \theta_w$ is equivalent to an indirect measurement of $M_W$ to a precision of $\pm 15$ MeV.

$W$ mass provides a stringent test of the SM. Within SM we can measure the $W$ mass both directly and indirectly. They should agree.
Note that $A_{FB}$ is not Zero at the Z pole. Most of the sensitivity to $\sin^2\theta_{\text{eff}}$ is at the Z pole.

For finite dilepton $P_T$, the change in the $\cos\theta$ distribution in the Collins-Soper frame is well understood. CDF has measured it, and data agrees with POWHEG QCD prediction. It is accounted for in the analysis and does not have much impact to the results.
We report on the following CDF measurement published in 2014. Phys. Rev D. 89, 072005 (2014) full Run II data set 9 fb⁻¹ μ⁺μ⁻

CDF analysis uses three new innovations which are essential:

1st innovation: \( \sin^2 \theta_W \) is constant --> \( \sin^2 \theta_{\text{eff}}^{\text{lept}} (M_z, \text{flavor}) \)
Full ZFITTER EW radiative corrections, Enhanced Born Approximation (EBA), include full complex form factors
(implemented private versions of RESBOS, POWHEG, and LO)
arXiv:1307.0770v3 [hep-ex]

2nd innovation:
Precise lepton momentum/energy scale corrections using a new method
arXiv:1208.3710v3 [hep-ex]

3rd innovation: Event weighting method for \( A_{FB} \) analyses
(all systematic errors in acceptance and efficiencies cancel)
arXiv:0911.2850v4 [hep-ex]
1st innovation: \( \sin^2 \theta_W \) is constant --> \( \sin^2 \theta_{\text{eff}}^{\text{lept}} (M_Z, \text{flavor}) \)

Full FITTER EW radiative corrections Enhanced Born Approximation (EBA)


\[ g_V^f \gamma_\mu + g_A^f \gamma_\mu \gamma_5. \] The Born-level couplings are

\[ g_V^f = T_3^f - 2Q_f \sin^2 \theta_W \]
\[ g_A^f = T_3^f, \]

They are modified by ZFITTER 6.43 form factors (which are complex)

\[ g_V^f \rightarrow \sqrt{\rho_{eq}} (T_3^f - 2Q_f \kappa_f \sin^2 \theta_W), \] and
\[ g_A^f \rightarrow \sqrt{\rho_{eq}} T_3^f, \]

- \( T_3 \) and \( \sin^2 \theta_W \) \rightarrow \text{effective } T_3 \text{ and } \sin^2 \theta_W : 1-4\% \text{ multiplicative form factors}
- On-mass shell scheme: \( \sin^2 \theta_W \equiv 1 - M_W^2/M_Z^2 \) to all orders

\[ \sin^2 \theta_{\text{eff}}^{\text{lept}} \approx 1.037 \cdot \sin^2 \theta_W \] [ZFITTER \( \kappa_e (\sin^2 \theta_W M_Z) \) form factor]
This new technique is used in CDF and CMS (for muons and electrons). It is currently used in CMS to get a precise measurement of the Higgs mass.

Step 1: Remove the correlations between the scale for the two leptons by getting an initial calibration using $Z$ events and requiring that the mean $<1/p_T>$ of each lepton in bins of $\eta$, $\Phi$ and charge be correct.

Step 2: The $Z$ mass is used as a calibration. The method requires that the $Z$ mass as a function of $\eta$, $\Phi$, or charge of each lepton be correct. Extract fine tuned corrections in bins of $\eta$, $\Phi$ and charge.

After corrections, the $Z$ mass as a function of $\eta$, $\Phi$, charge for both the data and hit level MC agree with the generator level Monte Carlo (smeared by resolution, and with experimental acceptance cuts). All charge bias is removed.

<table>
<thead>
<tr>
<th></th>
<th>Stat. Error in $\sin^2\theta_{\text{eff}}$</th>
<th>Error in $\sin^2\theta_{\text{eff}}$ from momentum/energy scale:</th>
</tr>
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<tbody>
<tr>
<td>CMS (2011)</td>
<td>-0.00200</td>
<td>-0.00130 (prior to using EJC-2012)</td>
</tr>
<tr>
<td>ATLAS (2013)</td>
<td>-0.00040</td>
<td>-0.00050</td>
</tr>
<tr>
<td>CDF (2014)</td>
<td>-0.00090</td>
<td>-0.00005 (using EPJC-2012 method)</td>
</tr>
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Angular event weighting is equivalent to extraction of $A_4(M)$ in bins of $\cos \theta$, and averaging the results. It is done all at once using event weights. Events at large $\cos \theta$ provide better determination of $A_4$, so they are weighted more than events at small $\cos \theta$. (events $\cos \theta = 0$ have zero weight).

In this technique, all $\cos \theta$ acceptance and efficiencies cancel to first order and the statistical errors are 20% smaller. $A_{fb} = (3/8)A_4$

There are three kinds of event weighting can be used, (1) angular weighting (2) dilution weighting, (3) both. In the CDF analysis, angular event weighting is used (since dilution from antiquarks is small) (For LHC need to do both).
The error in $A_{fb}$ is reduced if we have more acceptance at large $\cos \theta$, however, $A_{fb}$ extracted with Angular event weighting is not sensitive to acceptance in $\cos \theta$.
Raw event weighted $A_{FB}$ (No corrections)

This event weighted $A_{FB}$ plot has no corrections.
Antiquark Dilution

The measured $A_{FB}$ depend on the coverage in rapidity. Sometimes the quark direction is not in the direction of the proton. This small dilution effect depends on the antiquark distributions i.e. on PDFs (we used CT10), and the rapidity range of the data.

If an additional dilution correction is included in the event weights than the extracted $A_{FB}$ is also independent of the acceptance in rapidity. (more important for the LHC)

PDF dilution error can be further reduced with better PDFs as LHC data is included in newer PDF sets.
QED FSR and detector resolution smear events between mass bins. We correct for this smearing using matrix unfolding.

(Here, the edge bins are underflow and overflow bins)

Raw $A_{FB} (y<1)$

This event weighted $A_{FB}$ plot has no corrections.

Effect of FSR
INPUT $M_W$ to RESBO- EBA or POWHEG-EBA

MH = 125 GeV

$$\sin^2\theta_w = 1 - M_W^2 / M_Z^2$$

EW Rad Corrections yield

$$\sin^2\theta_{eff}^{leptonic}(s)$$
$$\sin^2\theta_{eff}^{u-type}(s), \sin^2\theta_{eff}^{d-type}(s)$$

PDFs + QCD predict $A_4(s)$

Predicted $A_{FB}(s) = (3/8) A_4(s)$

CDF SM analysis with Full ZFITTER EBA rad corrections analogous to LEP analysis

Compare Predicted $A_{FB}(s)$ to $A_{FB}(s)$ experiment (unfolded for resolution and FSR) to extract the $\sin^2\theta_w$ that describes the data best

$$\sin^2\theta_w = 1 - M_W^2 / M_Z^2$$

is used to get a measured $M_W$ indirect

In addition

$$\sin^2\theta_{eff}^{lept}(M_Z) \equiv \text{Re } \kappa(M_Z, \sin^2\theta_w) \sin^2\theta_w = 1.037 \sin^2\theta_w$$

Is used to compare to $\sin^2\theta_{eff}^{lept}(M_Z)$ at LEP

$\text{CDF}$
$\text{ResBos } \sin^2 \theta^\text{lept}_{\text{eff}} = 0.23150 \pm 0.00090 \pm 0.00011 \pm 0.00035 \text{ (CT10 PDFs)}$

$\sin^2 \theta^W = 0.22330 \pm 0.00080 \pm 0.00011 \pm 0.00035 \text{ CT10 (PDFs)}$

$M^W = 80.365 \pm 0.043 \pm 0.005 \pm 0.018 \text{ (CT10 PDFs)}$

<table>
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<tr>
<th>Template (measurement)</th>
<th>$\sin^2 \theta^\text{lept}_{\text{eff}}$</th>
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<td>RESBOS NLO Full ZFITTER EBA</td>
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<td>POWHEG-BOX NLO Full ZFITTER EBA</td>
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<td>PYTHIA No EW radiative cor. CT5L</td>
<td>0.2311 $\pm 0.0008$</td>
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<td>20.8</td>
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A factor of 2 reduction in errors is expected in Fall 2014 when the analysis of the CDF e+e- (9 fb\(^{-1}\)) data is completed.
Conclusion and more results in the near Future

- CDF (µ⁺μ⁻ 9 fb⁻¹) published 2014 EBA rad corr CT10 PDFs

  \[ M_{w(\text{indirect})} = 80.365 \pm 0.045 \text{ GeV (2014)} \]

  \[ \sin^2\theta_{\text{eff}} = 0.23150 \pm 0.00090(\text{stat}) \pm 0.00011(\text{sys}) \pm 0.00035 \text{ (PDF)} \]

Expected results in Fall 2014 CDF (e⁺e⁻) 10 fb⁻¹ (errors reduced by 1/2)

- \[ \sin^2\theta_{\text{eff}} = 0.23xxx \pm 0.00040(\text{stat}) \]
  \[ \pm 0.00005(\text{sys}) \]
  \[ \pm 0.00029 \text{ (PDF-CT10)} \]

  We also expect PDFs to improve 2014

- Versus

- LEP  \[ 0.23098 \pm 0.00026 \]
- SLD  \[ 0.23221 \pm 0.00029 \]

- End of 2014 CDF and D0 combined will match LEP/SLD errors.
  Indirect and direct measurements of Mw will have comparable errors.
Additional Slides
Conclusion: $\sin^2\theta_w$ and indirect measurements of $M_w$.

A: CDF  Phys. Rev D. 89, 072005 (2014)  Run II  9 fb$^{-1}$  $\mu^+\mu^-$
Reports three measurements with statistical errors of
$\sin^2\theta_{\text{eff}}$ ($\pm 0.00090$)  $\sin^2\theta_w$ ($\pm 0.00080$)  $M_w^{\text{indirect}}$ ($\pm 44$ MeV)

B: CDF: Expected Fall 2014  full Run II data set  9.7 fb$^{-1}$  $e^+e^-$
will have three measurements with statistical errors of
$\sin^2\theta_{\text{eff}}$ ($\pm 0.00044$),  $\sin^2\theta_w$ ($\pm 0.00040$) and  $M_w^{\text{indirect}}$ ($\pm 22$ MeV)
Which means that a measurement of $M_w^{\text{indirect}}$ with the combined CDF/D0
9 fb$^{-1}$ run II data would have a statistical error of $\pm 15$ MeV, which is equal to
the $\pm 15$ MeV error in average of all world measurements of  $M_w^{\text{direct}}$

In addition, it would address the LEP-SLD Difference. LEP SLD difference is 0.00122
Drell-Yan asymmetry is measured in the Collins-Soper frame. The Collins-Soper frame is the CM frame of the dilepton pair. It is also the q-qbar center of mass.

The dilepton pair can have $P_T$ in the laboratory frame. The $P_T$ could originate from gluon emission by the quark in the proton, or by the antiquark in the antiproton (and also from a $qG$ process). Therefore, unlike $e^+e^-$ collisions at LEP, the $q$ and $q\bar{q}$ are not collinear in the lab.

The $P_T=0$ Born level polar angle distribution: $1 + \cos^2\theta + A_4 \cos\theta$

Is replaced with $1+\cos^2\theta + A_0(M,P_T) (1- 3\cos^2\theta)/2 + A_4 \cos\theta$

For dileptons with a $P_T$, the small change in the $\cos\theta$ distribution in the Collins-Soper frame is well understood. CDF has measured this change and it agrees with POWHEG QCD prediction. It is accounted for in the analysis and does not have much impact to the results.
\[
\frac{dN}{d\Omega} \propto (1 + \cos^2 \theta) + A_4 \cos \theta + A_0(M, P_T) \left(1 - 3 \cos^2 \theta\right)/2
\]

\[A_0 = A_2 = \frac{k P_T^2}{k P_T^2 + M^2}\]

K=1.65 at the Tevatron (higher at the LHC since more qG)

The NLO QCD correction to the angular distribution are taken into account in POWHEG and RESBOS.

The correction is fully taken into account in the CDF \(A_{fb}\) analysis. However it does not make a significant difference.

This is good because PYTHIA does not have the correct angular distribution (only q-qbar no qG)
In the CDF analysis, we calculate $A_4(M)$ for various values of the $\sin^2 \theta_W$ model parameter and compare it to the measurement

- ZFITTER EBA techniques and complex valued form factors ($\rho$, $\kappa$) are used
- Quark-loop corrections to the photon propagator ($1-\Delta a(s)$) are used
  Real part is the running EM coupling: $\text{Re } \Delta a(M_\gamma) \approx 0.06 \ (1/128)$
  Imaginary part is non-zero and is used: $\text{Im } \Delta a(M_\gamma) \approx -0.02$
- Complex valued corrections are incorporated into the Drell-Yan amplitude

$A_4$ is directly related to a mix of $\sin^2 \theta_{\text{eff}}$ from the lepton, d-, and u-type quarks

- The best fit value, $\sin^2 \theta_W$, is indirectly related to $A_4$ and model dependent
  Model is almost identical to the one derived from Z-pole fits at LEP
  We use $m_H = 125 \text{ GeV}$ (LHC value, but consistent with LEP fit value)
- $\sin^2 \theta_{\text{eff}}(M) = \kappa(M) \sin^2 \theta_W$: this product is model independent
- We provide the leptonic $\sin^2 \theta_{\text{eff, lept}}$ at the Z-pole for comparison with the LEP

$$\sin^2 \theta_{\text{eff, lept}} \equiv \text{Re } \kappa(M_\gamma, \sin^2 \theta_W) \sin^2 \theta_W = 1.037 \sin^2 \theta_W$$
$A_{f_{FB}}(M) = (3/8)A_4(M)$

- Vertical line is $M = M_Z$ where $\gamma/Z$ interference is zero
- $\gamma/Z$ interference $\propto (s - M_Z^2)$
  - gets large away from $Z$ peak and dominates
  - related to $g_A$ and no direct dependence on $\sin^2\theta_W$

The full EW ZFITTER modification (Enhanced Born Approx - EBA) were incorporated into two QCD calculation of $A_{FB}$ (e.g. POWHEG, RESBOS) with CT10 NLO PDFs, and also in a stand alone (LO) calculation.

The calculations have only one parameter, i.e. on shell $\sin^2 \theta_w$. We find the $\sin^2 \theta_w$ value which the model fits the data for $A_4(M)$.

We then have $\sin^2 \theta_{\text{eff lept}}(M_z) = 1.037 \sin^2 \theta_w$

Without ZFITTER EBA corrections, the input to POWHEG, RESBOS is just $\sin^2 \theta_{\text{eff lept}}$ which is assumed to be independent of $M$. In this case, since no EW radiative corrections are applied, the $\sin^2 \theta_{\text{eff lept}}$ which fits the data is an average which depends on the range of $M$ that is being used.

With the Full ZFITTER EBA radiative corrections the extracted value of $\sin^2 \theta_{\text{eff lept}}$ is larger by the following amounts.

RESBOS NLO +EBA template - by 0.00031
POWHEG-BOX NLO +EBA template - by 0.00021
LO template +EBA - by 0.00047
**Full ZFITTER EBA EW radiative corrections**

TABLE III. Extracted values of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ and $\sin^2 \theta_{W}$ for the EBA-based QCD templates. The PYTHIA entry is the value from the scan over non-EBA templates calculated by PYTHIA 6.4 with CTEQ5L PDFs. The uncertainties of the template scans are the measurement uncertainties ($\bar{\sigma}$). Other measurements are listed in parentheses.

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RESBOS NLO templates with full ZFITTER EBA EW rad correction yield a value of $\sin^2 \theta_{\text{eff}}$ which is **0.00040** larger than the values extracted using PYTHIA templates with CTEQ5L PDFs with no radiative corrections.
Investigating EW radiative corrections,

POWHEG+full ZFITTER EBA rad cor (private CDF version)

RESBOS+full ZFITTER EBA rad cor (private CDF version)

POWHEG (which is a MC) has a new version with EW radiative corrections - Eur.Phys.J. C73 (2013) 2474, arXiv:1302.4606. (we are currently testing this version)

HORRACE and Zgrad (not full EBA)

FEWZ3.1 also has EW radiative corrections (it is not a MC)