6.1) Show that for a quantum field $f(t)$

$$\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \; f(t_1) f(t_2) = \frac{1}{2} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \; T\left[f(t_1) f(t_2)\right]$$

where

$$T\left[f(t_1) f(t_2)\right] = \begin{cases} f(t_1) f(t_2) & \text{for } t_1 > t_2 \\ f(t_2) f(t_1) & \text{for } t_2 > t_1 \end{cases}$$

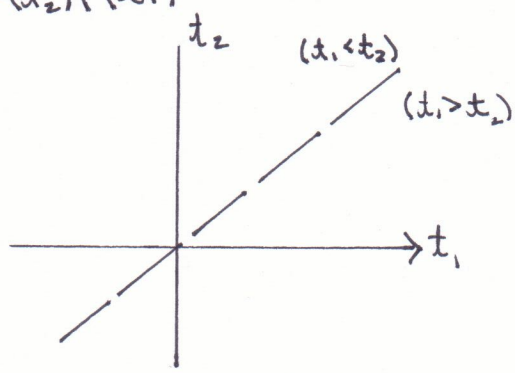
$$\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2\, T[f(t_1)f(t_2)] = \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2\, f(t_1)f(t_2) \qquad \text{for } t_1 > t_2$$

but if $t_1 > t_2$, then t_2 cannot be integrated past t_1. So the integral becomes

$$\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2\, T[f(t_1)f(t_2)] = \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2\, f(t_1)f(t_2) \qquad \text{for } t_1 > t_2$$

$$+ \int_{-\infty}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2\, f(t_2)f(t_1) \qquad \text{for } t_1 < t_2$$

$$= \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2\, f(t_1)f(t_2) + \int_{-\infty}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2\, f(t_2)f(t_1)$$



Change the order of integration of the 2nd integral

$$= \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2\, f(t_1)f(t_2) + \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{t_2} dt_1\, f(t_2)f(t_1)$$

And the only difference left is a matter of labelling.