8.4 Scattering From a Non-Pointlike Object

$e^-, \pi^+ \longrightarrow e^-, \pi^+$

In recent calculations, we have been considering interactions with a hypothetical (non-physical) s^+ particle. Other than to ease into Q.F.T. w/ a spinless particle, these exercises were meant to anticipate the non-pointlike π^+ particle.

Consider the interaction:

$$e^-(k,s) + \pi^+(p) \longrightarrow e^-(k',s') + \pi^+(p)$$

which is exactly like the $e^- + s^+ \rightarrow e^- + s^+$ process already treated, except π^+ is not pointlike. This new treatment introduces the "pion form-factor."

Recall that the Coulomb interaction of a point charge follows

$$\nabla^2 A^0(\vec{x}) = -Ze\,\delta(\vec{x}-\vec{x}')$$

for a point charge located at $\vec{x}'$.

Now ~~the~~ consider a charge distributed finitely over space according to density $\rho(\vec{x})$.

$$\nabla^2 A^0(\vec{x}) = -Ze\rho(\vec{x}) \qquad (8.128)$$

Recall equation (8.12), the scattering amplitude for s^+.

$$S_{s^+} = -iNN' \int d^4x \; e(p_\mu + p'_\mu) e^{-i(p-p')\cdot x} A'^\mu(x)$$

I need to figure out how to use (8.128) in (8.12).

$$\nabla^2 A^0(\vec{x}) = -Ze\rho(\vec{x})$$

$$\int e^{-i\vec{q}\cdot x} \nabla^2 A^0(\vec{x}) d^3\vec{x} = -Ze \underbrace{\int e^{-i\vec{q}\cdot\vec{x}} \rho(\vec{x}) d^3\vec{x}}_{\equiv F(\vec{q})} \qquad (\vec{q} = \vec{p} - \vec{p}')$$

We introduce the "form factor" $F(\vec{q})$ and define it as the integral that appears.

$$\boxed{F(\vec{q}) \equiv \int e^{-i\vec{q}\cdot\vec{x}} \rho(\vec{x}) d^3\vec{x}}$$

This $F(\vec{q})$ comes with the condition that

$$F(0) = \int \rho(\vec{x})\, d^3\vec{x} = 1$$

to ensure that the total charge present remains Ze.

$$\Rightarrow \int e^{-i\vec{q}\cdot\vec{x}} \nabla^2 A^0(\vec{x})\, d^3\vec{x} = -Ze\, F(\vec{q})$$

$$\int (\nabla^2 e^{-i\vec{q}\cdot\vec{x}}) A^0(\vec{x})\, d^3\vec{x} = -Ze\, F(\vec{q})$$

$$-\vec{q}^{\,2} \underbrace{\int e^{-i\vec{q}\cdot\vec{x}} A^0(\vec{x})\, d^3\vec{x}}_{\substack{\text{Fourier Transform} \\ \tilde{A}^0(\vec{q})}} = -Ze\, F(\vec{q})$$

$$\tilde{A}^0(\vec{q}) = \frac{F(\vec{q})}{\vec{q}^{\,2}} Ze$$

This is what needs to be used in the scattering amplitude.