Field Contractions with State Vectors

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_p e^{-ip\cdot x} + a_p^\dagger e^{ip\cdot x} \right] \qquad \text{(Scalar Fields)}$$

$$\phi(x)|p'\rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_p e^{-ip\cdot x} + a_p^\dagger e^{ip\cdot x} \right] |p'\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_p e^{-ip\cdot x} + a_p^\dagger e^{ip\cdot x} \right] \sqrt{2E_{p'}}\, a_{p'}^\dagger |0\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} e^{-ip\cdot x} \sqrt{2E_{p'}}\, (2\pi)^3 \delta^3(p-p') |0\rangle$$

$$= e^{-ip'\cdot x} |0\rangle$$

$$\langle p'|\phi(x) = \langle p| \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_p e^{-ip\cdot x} + a_p^\dagger e^{ip\cdot x} \right]$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \langle 0| a_{p'} \sqrt{2E_{p'}} \left[a_p e^{-ip\cdot x} + a_p^\dagger e^{ip\cdot x} \right]$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \langle 0| (2\pi)^3 \delta^3(p-p') \sqrt{2E_{p'}}\, e^{ip\cdot x}$$

$$= \langle 0| e^{ip'\cdot x}$$

Now consider fermion fields, which expand as

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(a_p^s u^s(p) e^{-ip\cdot x} + b_p^{s\dagger} v^s(p) e^{ip\cdot x} \right)$$

$$\bar{\psi}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(b_p^s \bar{v}^s(p) e^{-ip\cdot x} + a_p^{s\dagger} \bar{u}^s(p) e^{ip\cdot x} \right)$$

$$\overline{\Psi(x)|p'\rangle} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(a_p^s u^s(p) e^{-ip\cdot x} + b_p^{s\dagger} v^s(p) e^{ip\cdot x} \right) |p'\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(a_p^s u^s(p) e^{-ip\cdot x} + b_p^{s\dagger} v^s(p) e^{ip\cdot x} \right) \sqrt{2E_{p'}}\, a_{p'}^{s\dagger} |0\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(u^s(p) e^{-ip\cdot x} \right) \sqrt{2E_{p'}}\, (2\pi)^3 \delta^3(p-p') |0\rangle$$

$$= u^s(p) e^{-ip\cdot x} |0\rangle$$

$$\overline{\langle p'|\Psi(x)} = \langle p'| \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(a_p^s u^s(p) e^{-ip\cdot x} + b_p^{s\dagger} v^s(p) e^{ip\cdot x} \right)$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \langle 0| b_{p'}^s \sqrt{2E_{p'}} \left(a_p^s u^s(p) e^{-ip\cdot x} + b_p^{s\dagger} v^s(p) e^{ip\cdot x} \right)$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \langle 0| \delta^3(p-p') (2\pi)^3 \sqrt{2E_{p'}} \left(v^s(p') e^{ip'\cdot x} \right)$$

$$= \langle 0| v^s(p) e^{ip'\cdot x}$$

$$\overline{\bar{\Psi}(x)|p'\rangle} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(b_p^s \bar{v}^s(p) e^{-ip\cdot x} + a_p^{s\dagger} \bar{u}^s(p) e^{ip\cdot x} \right) |p'\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(b_p^s \bar{v}^s(p) e^{-ip\cdot x} + a_p^{s\dagger} \bar{u}^s(p) e^{ip\cdot x} \right) \sqrt{2E_{p'}}\, b_{p'}^{s\dagger} |0\rangle$$

$$= \bar{v}^s(p') e^{-ip'\cdot x} |0\rangle$$

$$\overline{\langle p'|\bar{\Psi}(x)} = \langle 0| \bar{u}^s(p') e^{ip'\cdot x}$$

Now consider vector fields. I'll solve this for the W boson, but the photon is the same except for requiring the polarization vectors to take purely transverse components at the end.

$$W_\mu(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(a_p^s \epsilon_\mu^s(p) e^{-ip\cdot x} + a_p^{s\dagger} \epsilon_\mu^{s*}(p) e^{ip\cdot x} \right)$$

$$W_\mu(x)|p'\rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(a_p^s \epsilon_\mu^s(p) e^{-ip\cdot x} + a_p^{s\dagger} \epsilon_\mu^{s*}(p) e^{ip\cdot x} \right) |p'\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(a_p^s \epsilon_\mu^s(p) e^{-ip\cdot x} + a_p^{s\dagger} \epsilon_\mu^{s*}(p) e^{ip\cdot x} \right) \sqrt{2E_{p'}}\, a_{p'}^{s\dagger} |0\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(\epsilon_\mu^s(p) e^{-ip\cdot x} \right) \sqrt{2E_{p'}}\, (2\pi)^3 \delta^3(p-p') |0\rangle$$

$$= \epsilon_\mu^s(p') e^{-ip\cdot x} |0\rangle$$

$$\langle p'|W_\mu(x) = \langle p'| \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(a_p^s \epsilon_\mu^s(p) e^{-ip\cdot x} + a_p^{s\dagger} \epsilon_\mu^{s*}(p) e^{ip\cdot x} \right)$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \langle 0| \sqrt{2E_{p'}}\, a_{p'}^s \left(a_p^s \epsilon_\mu^s(p) e^{-ip\cdot x} + a_p^{s\dagger} \epsilon_\mu^{s*}(p) e^{ip\cdot x} \right)$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \langle 0| \sqrt{2E_{p'}}\, (2\pi)^3 \delta^3(p-p') \left(\epsilon_\mu^{s*}(p) e^{ip\cdot x} \right)$$

$$= \langle 0| \epsilon_\mu^{s*}(p') e^{ip'\cdot x}$$

$$\overline{W_\mu^\dagger(x)|p'\rangle} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(a_p^{s\dagger} \epsilon_\mu^{s*}(p) e^{ip\cdot x} + a_p^s \epsilon_\mu^s(p) e^{-ip\cdot x} \right) |p'\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(a_p^{s\dagger} \epsilon_\mu^{s*}(p) e^{ip\cdot x} + a_p^s \epsilon_\mu^s(p) e^{-ip\cdot x} \right) \sqrt{2E_{p'}}\, a_{p'}^{s\dagger} |0\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(\epsilon_\mu^s(p) e^{-ip\cdot x} \right) \sqrt{2E_{p'}}\, (2\pi)^3 \delta^3(p-p') \, |0\rangle$$

$$= \epsilon_\mu^s(p') e^{-ip'\cdot x} |0\rangle$$

$$\overline{\langle p'| W_\mu^\dagger(x)} = \langle 0| \epsilon_\mu^{s*}(p') e^{ip'\cdot x}$$