The Higgs Mechanism and Fermions' Masses

The kinetic part of a free Dirac fermion does not mix the L and R components of the field

$$\bar{\psi}\gamma_\mu\partial^\mu\psi = \bar{\psi}_R\gamma_\mu\partial^\mu\psi_R + \bar{\psi}_L\gamma_\mu\partial^\mu\psi_L .$$

Because of this, we can gauge the L and R components differently. Weak interactions are parity violating and the $SU(2)_L$ covariant derivative acts only on the left-handed term. However, a Dirac mass term has the form

$$-m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

when left and right components are written separately. So the components are coupled, meaning any such mass term breaks $SU(2)_L$ gauge invariance.

In a theory with spontaneous symmetry breaking, there is a way of giving mass to fermions without explicitly introducing a mass term in the Lagrangian. Consider the electron $SU(2)_L$ doublet $\begin{bmatrix}\nu\\ e\end{bmatrix}_L$, the Higgs doublet

$$\phi = \begin{bmatrix}\phi^+\\ \phi^0\end{bmatrix} \qquad \phi^+ = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2)$$
$$\phi^0 = \frac{1}{\sqrt{2}}(\phi_3 - i\phi_4)$$

and the right-handed electron singlet in a Yukawa model.

$$\mathcal{L}_e = -g_e \bar{l}_L \phi e_R - g_e \bar{e}_R \phi^\dagger l_L$$

Notice that the structure of these terms is two $SU(2)_L$ doublets multiplied to form an $SU(2)_L$ scalar $(\bar{l}_L \phi, \phi^\dagger l)$, and that scalar multiplies the $SU(2)_L$ scalar R-component. Thus, this Lagrangian is $SU(2)_L$ invariant — the symmetry is preserved.

Recall that the vacuum expectation value of the Higgs doublet is assumed

$$\langle 0|\phi|0\rangle = \begin{bmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{bmatrix}$$

The consequence in the Lagrangian is

$$\mathcal{L}_e = -g_e \bar{l}_L \phi e_R - g_e \bar{e}_R \phi^\dagger l_L$$

$$= -g_e \overline{\begin{bmatrix} \nu_L \\ e_L \end{bmatrix}} \phi e_R - g_e \bar{e}_R \phi^\dagger \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}$$

$$= -g_e \begin{bmatrix} \bar{\nu}_L & \bar{e}_L \end{bmatrix} \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} e_R - g_e \bar{e}_R \begin{bmatrix} \phi^+ & \phi^0 \end{bmatrix} \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}$$

$$= -g_e (\bar{\nu}_L \phi^+ + \bar{e}_L \phi^0) e_R - g_e \bar{e}_R (\phi^+ \nu_L + \phi^0 e_L)$$

$$= -g_e \left[\bar{\nu}_L \phi^+ e_R + \bar{e}_L \phi^0 e_R + \bar{e}_R \phi^+ \nu_L + \bar{e}_R \phi^0 e_L \right]$$

$$\langle 0|\mathcal{L}_e|0\rangle = -g_e \left[\bar{\nu}_L \cancelto{0}{\langle 0|\phi^+|0\rangle} e_R + \bar{e}_L \langle 0|\phi^0|0\rangle e_R \right.$$
$$\left. + \bar{e}_R \cancelto{0}{\langle 0|\phi^+|0\rangle} \nu_L + \bar{e}_R \langle 0|\phi^0|0\rangle e_L \right]$$

$$= -\frac{g_e v}{\sqrt{2}} \left[\bar{e}_L e_R + \bar{e}_R e_L \right]$$

This is exactly a Dirac mass with $m_e = \frac{g_e v}{\sqrt{2}}$.

That was precisely the vacuum. Now let's see that if we consider also oscillations about the vacuum we generate a coupling between the electron and the Higgs field. In the last line, use $v+H$ instead of just v.

$$\langle 0|\mathcal{L}_e|0\rangle = -\frac{g_e}{\sqrt{2}}\left[\bar{e}_L(v+H)e_R + \bar{e}_R(v+H)e_L\right]$$

$$= -\frac{g_e}{\sqrt{2}}\left[v\bar{e}_L e_R + \bar{e}_L H e_R + v\bar{e}_R e_L + \bar{e}_R H e_L\right]$$

$$= -\frac{g_e}{\sqrt{2}}\left[v e^\dagger\left(\frac{1-\gamma_5}{2}\right)\gamma_0\left(\frac{1+\gamma_5}{2}\right)e + e^\dagger\left(\frac{1-\gamma_5}{2}\right)\gamma_0 H\left(\frac{1+\gamma_5}{2}\right)e\right.$$
$$\left. + v e^\dagger\left(\frac{1+\gamma_5}{2}\right)\gamma_0\left(\frac{1-\gamma_5}{2}\right)e + e^\dagger\left(\frac{1+\gamma_5}{2}\right)\gamma_0 H\left(\frac{1-\gamma_5}{2}\right)e\right]$$

$$= -\frac{g_e}{\sqrt{2}}\left[v\bar{e}\left(\frac{1+\gamma_5}{2}\right)e + \bar{e}He\left(\frac{1+\gamma_5}{2}\right) + v\bar{e}\left(\frac{1-\gamma_5}{2}\right)e + \bar{e}He\left(\frac{1-\gamma_5}{2}\right)\right]$$

but recall $\left(\frac{1+\gamma_5}{2}\right)+\left(\frac{1-\gamma_5}{2}\right) = \frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} + \frac{1}{2}\Big[$

$$= -\frac{g_e}{\sqrt{2}}\left[v\bar{e}e + \bar{e}He\right]$$

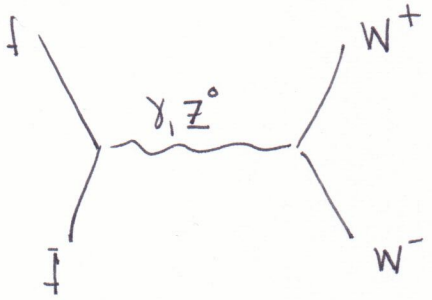
↑ Dirac electron mass

↑ Higgs Coupling

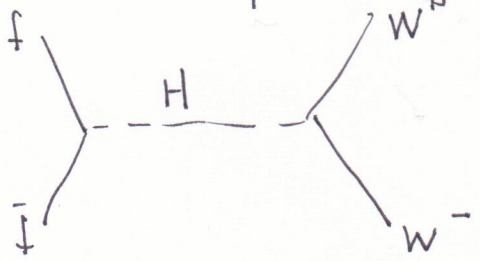
Notice for the coupling term

$$\left(\frac{-g_e}{\sqrt{2}}\right)\bar{e}He = \left(\frac{-m_e}{v}\right)\bar{e}He = \left(\frac{-gm_e}{2m_W}\right)\bar{e}He$$

So in addition to



we also have the possibility



The presence of the fermion mass in the coupling to the Higgs is important.

We must now recall that if an SU(2) doublet transforms as

$$l' = e^{\frac{i}{2}\vec{\alpha}\cdot\vec{\tau}}\, l$$

then the charge conjugate states

$$i\tau_2 \begin{bmatrix} u^* \\ d^* \end{bmatrix}$$

transform the same way. So then the charge conjugate of the Higgs field is

$$\phi_c = i\tau_2 \phi^*$$

$$= i \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix}^*$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2)^* \\ \frac{1}{\sqrt{2}}(\phi_3 - i\phi_4)^* \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \\ \frac{-1}{\sqrt{2}}(\phi_1 + i\phi_2) \end{bmatrix}$$

$$\equiv \begin{bmatrix} \phi^{0\uparrow} \\ -\phi^- \end{bmatrix}$$

ϕ_c is also an SU(2) doublet which transforms the same way as ϕ.

Note that in the use of the ϕ Higgs doublet we could not use the terms

$$\bar{l}_L \phi \nu_R \quad , \quad \bar{\nu}_R \phi^\dagger l_L$$

in the Lagrangian (ν_R has replaced e_R) because it leads to unphysical terms

$$\bar{e}_L \nu_R \quad , \quad \bar{\nu}_R e_L \quad .$$

With the Higgs conjugate field doublet ϕ_c we may include

$$\bar{l}_L \phi_c \nu_R \;, \quad \bar{\nu}_R \phi_c^\dagger l_L$$

terms (but not $\bar{l}_L \phi_c e_R$, $\bar{e}_R \phi_c^\dagger l_L$ terms for the same reason just discussed) which yield Dirac masses for the neutrinos as well as Higgs interactions. Observe,

$$\mathcal{L}_\nu = -g_\nu \left[\bar{l}_L \phi_c \nu_R + \bar{\nu}_R \phi_c^\dagger l_L \right]$$

$$= -g_\nu \left[\begin{bmatrix} \bar{\nu}_L & \bar{e}_L \end{bmatrix} \begin{bmatrix} \phi^{0\dagger} \\ -\phi^- \end{bmatrix} \nu_R + \bar{\nu}_R \begin{bmatrix} \phi^0 & -\phi^{-\dagger} \end{bmatrix} \begin{bmatrix} \nu_L \\ e_L \end{bmatrix} \right]$$

$$= -g_\nu \left[(\bar{\nu}_L \phi^{0\dagger} - \bar{e}_L \phi^-) \nu_R + \bar{\nu}_R (\phi^0 \nu_L - \phi^{-\dagger} e_L) \right]$$

$$= -g_\nu \left[\bar{\nu}_L \phi^{0\dagger} \nu_R - \bar{e}_L \phi^- \nu_R + \bar{\nu}_R \phi^0 \nu_L - \bar{\nu}_R \phi^{-\dagger} e_L \right]$$

Take the vacuum expectation value and all ϕ^- terms vanish. The ϕ^0 factors become $\frac{1}{\sqrt{2}}(v+H)$ again.

$$\langle 0 | \mathcal{L}_\nu | 0 \rangle = \frac{-g_\nu}{\sqrt{2}} \left[\bar{\nu}_L (v+H) \nu_R + \bar{\nu}_R (v+H) \nu_L \right]$$

$$= \frac{-g_\nu}{\sqrt{2}} \left[\underbrace{v \bar{\nu}_L \nu_R + v \bar{\nu}_R \nu_L}_{\text{Dirac Mass}} + \underbrace{\bar{\nu}_L H \nu_R + \bar{\nu}_R H \nu_L}_{\substack{\text{Neutrino - Higgs} \\ \text{interaction}}} \right]$$

$$= -\frac{g_\nu}{\sqrt{2}} \left[v \bar{\nu} \nu + \bar{\nu} H \nu \right]$$

Summarily, to give the electron-neutrino SU(2) (as well as the other leptons and quarks) mass, we are adding more terms to "SU(2) × U(1) Gauge Invariant Lagrangian (massless)" of the form

$$\mathcal{L}_{\text{Higgs,e}} = -g_e\left[\bar{l}_L \phi e_R + \bar{e}_R \phi^\dagger l_L\right] - g_\nu\left[\bar{l}_L \phi_c \nu_R + \bar{\nu}_R \phi_c^\dagger l_L\right]$$

$$= -\frac{g_e}{\sqrt{2}}\left[v\bar{e}e + \bar{e}He\right] - \frac{g_\nu}{\sqrt{2}}\left[v\bar{\nu}\nu + \bar{\nu}H\nu\right]$$