Quarks : Three Family Mixing

Consider three doublets of left-handed fields.

$$q_{L1} = \begin{bmatrix} u_{L1} \\ d_{L1} \end{bmatrix} \qquad q_{L2} = \begin{bmatrix} u_{L2} \\ d_{L2} \end{bmatrix} \qquad q_{L3} = \begin{bmatrix} u_{L3} \\ d_{L3} \end{bmatrix}$$

and the corresponding six singlets $u_{R1}, d_{R1}, u_{R2}, d_{R2}, u_{R3}, d_{R3}$.

The Lagrangian is similar to the case for the leptons already considered. The difference is that there are three quark families and each $SU(2)_L$ scalar (such as $\bar{q}_{Li}\phi_c$) can be paired with any of the three corresponding right $SU(2)_L$ scalars (such as u_{Rj}). Hence, allowing "mixing" of the families (via weak charge-changing interaction) results in nine pairings where the lepton case had one. The nine couplings form the 3×3 CKM matrix.

To couple gauge fields to these quarks we replace derivative with covariant derivatives. This will not allow mixing between quark flavors, but coupling with the Higgs field does not follow from a gauge principle so will mix the flavors.

For any given possible pairing, recall we have terms

$$a\bar{q}_L\phi_c u_R \quad , \quad b\bar{q}_L\phi d_R$$

where the coefficients measure the strength of the couplings. Also, the Hermitian conjugates of these terms must be included.

$$
\begin{aligned}
(a\bar{q}_L \phi_c u_R)^\dagger &= (\phi_c u_R)^\dagger (a q_L^\dagger \gamma_0)^\dagger \\
&= u_R^\dagger \phi_c^\dagger (q_L^\dagger \gamma_0)^\dagger a^\dagger \\
&= u_R^\dagger \phi_c^\dagger \gamma_0^\dagger q_L a^\dagger \\
&= u_R^\dagger \phi_c^\dagger \gamma_0 q_L a^\dagger \\
&= u_R^\dagger \gamma_0 \phi_c^\dagger q_L a^\dagger \\
&= a^\dagger \bar{u}_R \phi_c^\dagger q_L
\end{aligned}
$$

where γ_0 is real and diagonal. So when we generalize to all such terms, this becomes

$$(a_{ij} \bar{q}_{Li} \phi_c u_{Rj})^\dagger = a_{ij}^\dagger \bar{u}_{Rj} \phi_c^\dagger q_{Li}$$

Hence, each of the nine possible pairings has ~~fewer~~ four terms in the Lagrangian.

$$\mathcal{L} = \sum_{i,j} \left[a_{ij} \bar{q}_{Li} \phi_c u_{Rj} + a_{ij}^\dagger \bar{u}_{Rj} \phi_c^\dagger q_{Li} + b_{ij} \bar{q}_{Li} \phi d_{Rj} + b_{ij}^\dagger \bar{d}_{Rj} \phi^\dagger q_{Li} \right]$$

So far the elements of a_{ij} and b_{ij} may be any $\mathbb{C}$ value. The consequence is a violation of any discrete or flavor conservation symmetry. However, it is possible to

simplify the form by making appropriate chiral transformations. To find the required transformations, diagonalize the Hermitian matrices by squaring a and b. Define unitary matrices U and W by

$$aa^\dagger = UD_a^2U^\dagger$$
$$a^\dagger a = WD_a^2W^\dagger$$

where D_a^2 is a diagonal matrix with positive eigenvalues. Recall from modern algebra (Dummit + Foote) the matrices may be diagonalized in such a way.

$$\begin{aligned} aa^\dagger &= UD_a^2U^\dagger \\ &= UD_aD_a^\dagger U^\dagger \\ &= UD_aW^\dagger WD_a^\dagger U^\dagger \\ &= UD_aW^\dagger(UD_aW^\dagger)^\dagger \end{aligned}$$

Hence, $a = UD_aW^\dagger$ and $a^\dagger = (UD_aW^\dagger)^\dagger = WD_a^\dagger U^\dagger$.

Similarly, define unitary matrices S and T by

$$bb^\dagger = SD_b^2S^\dagger$$
$$b^\dagger b = TD_b^2T^\dagger$$

Then $b = SD_bT^\dagger$ and $b^\dagger = (SD_bT^\dagger)^\dagger = TD_b^\dagger S^\dagger$.

D_a and D_b are diagonal matrices whose elements are the positive square root of the elements of D_a^2 and

D_b^2, respectively.

Before putting this back into the Lagrangian we also want to do a simple change of variables on the right-handed singlets.

$$\begin{cases} u_{Rj} = W_{jk} u_{Rk} \\ d_{Rj} = T_{jk} d_{Rk} \end{cases}$$

This will eliminate the components of W and T from the Lagrangian.

$$\mathcal{L} = \sum_{i,j} \left[a_{ij} \bar{q}_{Li} \phi_c u_{Rj} + a^\dagger_{ij} \bar{u}_{Rj} \phi^\dagger_c q_{Li} + b_{ij} \bar{q}_{Li} \phi d_{Rj} + b^\dagger_{ij} \bar{d}_{Rj} \phi^\dagger q_{Li} \right]$$

$$= \sum_{i,j,k} \Big[(UD_a W^\dagger)_{ij} \bar{q}_{Li} \phi_c (W_{jk} u_{Rk}) + (UD_a W^\dagger)^\dagger_{ij} (W_{jk} u_{Rk})^\dagger \gamma_0 \phi^\dagger_c q_{Li}$$

$$+ (SD_b T^\dagger)_{ij} \bar{q}_{Li} \phi (T_{jk} d_{Rk}) + (SD_b T^\dagger)^\dagger_{ij} (T_{jk} d_{Rk})^\dagger \gamma_0 \phi^\dagger q_{Li} \Big]$$

$$= \sum_{i,j,k} \Big[(U_{ik} D_{a,kk} W^\dagger_{kj}) \bar{q}_{Li} \phi_c (W_{jk} u_{Rk}) + (U_{ik} D_{a,kk} W^\dagger_{kj})^\dagger (u^\dagger_{Rk} (W_{jk})^\dagger) \gamma_0 \phi^\dagger_c q_{Li}$$

$$\cancel{+ (SD_b T^\dagger)_{ij} \bar{q}_{Li} \phi}$$

$$+ (S_{ik} D_{b,kk} T^\dagger_{kj}) \bar{q}_{Li} \phi (T_{jk} d_{Rk}) + (S_{ik} D_{b,kk} T^\dagger_{kj})^\dagger (d^\dagger_{Rk} (T_{jk})^\dagger) \gamma_0 \phi^\dagger q_{Li} \Big]$$

$$= \sum_{i,j,k} \Big[\big(U_{ik} D_{a,kk} W^*_{kj}\big) \bar{q}_{Li} \phi_c \big(W_{jk} u_{Rk}\big) + \big((W^*_{kj})^\dagger (U_{ik} D_{a,kk})^\dagger\big)\big(u^\dagger_{Rk} W^*_{kj}\big) \gamma_0 \phi^\dagger_c q_{Li}$$

$$+ \big(S_{ik} D_{b,kk} T^*_{kj}\big) \bar{q}_{Li} \phi \big(T_{jk} d_{Rk}\big) + \big((T^*_{kj})^\dagger (S_{ik} D_{b,kk})^\dagger\big)\big(d^\dagger_{Rk} T^*_{kj}\big) \gamma_0 \phi^\dagger q_{Li} \Big]$$

$$= \sum_{i,j,k} \Big[\big(U_{ik} D_{a,kk} W^*_{kj}\big) \bar{q}_{Li} \phi_c \big(W_{jk} u_{Rk}\big) + \big(W_{jk} D_{a,kk} U^*_{ki}\big)\big(u^\dagger_{Rk} W^*_{kj}\big) \gamma_0 \phi^\dagger_c q_{Li}$$

$$+ \big(S_{ik} D_{b,kk} T^*_{kj}\big) \bar{q}_{Li} \phi \big(T_{jk} d_{Rk}\big) + \big(T_{jk} D_{b,kk} S^*_{ki}\big)\big(d^\dagger_{Rk} T^*_{kj}\big) \gamma_0 \phi^\dagger q_{Li} \Big]$$

Recall again that U, W, S, T are unitary matrices, so their components are $\mathbb{C}$-valued, so they commute with everything. Also, by unitarity $\sum_j \big(W^*_{kj} W_{jk}\big) = 1$.

$$= \sum_{i,j,k} \Big[U_{ik} D_{a,kk} \bar{q}_{Li} \phi_c \big(W^*_{kj} W_{jk}\big) u_{Rk} + D_{a,kk} U^*_{ki} \big(W^*_{kj} W_{jk}\big) \bar{u}_{Rk} \phi^\dagger_c q_{Li}$$

$$+ S_{ik} D_{b,kk} \bar{q}_{Li} \phi \big(T^*_{kj} T_{jk}\big) d_{Rk} + D_{b,kk} S^*_{ki} \big(T^*_{kj} T_{jk}\big) \bar{d}_{Rk} \phi^\dagger q_{Li} \Big]$$

Notice that the j index only appears on the W and T components. Hence, we need only perform the sum over j and the unitarity of W and T causes them to disappear.

$$= \sum_{i,k} \Big[U_{ik} D_{a,kk} \bar{q}_{Li} \phi_c u_{Rk} + D_{a,kk} U^*_{ki} \bar{u}_{Rk} \phi^\dagger_c q_{Li}$$

$$+ S_{ik} D_{b,kk} \bar{q}_{Li} \phi d_{Rk} + D_{b,kk} S^*_{ki} \bar{d}_{Rk} \phi^\dagger q_{Li} \Big]$$

Similarly, the U and S unitary matrix elements have yet to disappear (they don't completely and, in fact, will compose the CKM matrix). To do this, we use the same 'change of variables' method as before, except on the left-handed quark fields now. Reintroducing the j index, let

$$\begin{cases} u_{Li} = U_{ij}\, u_{Lj} \\ d_{Li} = S_{ij}\, d_{Lj} \end{cases}$$

One difference is that we must now multiply out the doublets.

$$= \sum_{i,k} \Big[U_{ik} D_{a,kk} \begin{bmatrix} \bar{u}_{Li} & \bar{d}_{Li} \end{bmatrix} \begin{bmatrix} \phi^{0\dagger} \\ -\phi^- \end{bmatrix} u_{Rk} + D_{a,kk} U^*_{ki} \bar{u}_{Rk} \begin{bmatrix} \phi^{0\dagger} & -\phi^- \end{bmatrix} \begin{bmatrix} u_{Li} \\ d_{Li} \end{bmatrix}$$

$$+ S_{ik} D_{b,kk} \begin{bmatrix} \bar{u}_{Li} & \bar{d}_{Li} \end{bmatrix} \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} d_{Rk} + D_{b,kk} S^*_{ki} \bar{d}_{Rk} \begin{bmatrix} \phi^+ & \phi^0 \end{bmatrix} \begin{bmatrix} u_{Li} \\ d_{Li} \end{bmatrix} \Big]$$

$$= \sum_{i,k} \Big[U_{ik} D_{a,kk} \left(\bar{u}_{Li} \phi^+ - \bar{d}_{Li} \phi^- \right) u_{Rk} + D_{a,kk} U^*_{ki} \bar{u}_{Rk} \left(\phi^{0\dagger} u_{Li} - \phi^- d_{Li} \right)$$

$$+ S_{ik} D_{b,kk} \left(\bar{u}_{Li} \phi^+ + \bar{d}_{Li} \phi^0 \right) d_{Rk} + D_{b,kk} S^*_{ki} \bar{d}_{Rk} \left(\phi^+ u_{Li} + \phi^0 d_{Li} \right) \Big]$$

$$= \sum_{i,k} \Big[U_{ik} D_{a,kk} \bar{u}_{Li} \phi^{0\dagger} u_{Rk} - U_{ik} D_{a,kk} \bar{d}_{Li} \phi^- u_{Rk} + D_{a,kk} U^*_{ki} \bar{u}_{Rk} \phi^{0\dagger} u_{Li} - D_{a,kk} U^*_{ki} \bar{u}_{Rk} \phi^- d_{Li}$$

$$+ S_{ik} D_{b,kk} \bar{u}_{Li} \phi^+ d_{Rk} + S_{ik} D_{b,kk} \bar{d}_{Li} \phi^0 d_{Rk} + D_{b,kk} S^*_{ki} \bar{d}_{Rk} \phi^+ u_{Li} + D_{b,kk} S^*_{ki} \bar{d}_{Rk} \phi^0 d_{Li} \Big]$$

$$= \sum_{i,k,j} \Big[U_{ik} D_{a,kk_2} (U_{ij} u_{Lj})^{\dagger} \gamma_0 \phi^{0\dagger} u_{Rk_2} - U_{ik} D_{a,kk_2} (S_{ij} d_{Lj})^{\dagger} \gamma_0 \phi^{-} u_{Rk_2}$$

$$+ D_{a,kk_2} U^{*}_{ki} \bar{u}_{Rk_2} \phi^{0\dagger} (U_{ij} u_{Lj}) - D_{a,kk_2} U^{*}_{ki} \bar{u}_{Rk_2} \phi^{-} (S_{ij} d_{Lj})$$

$$+ S_{ik} D_{b,kk_2} (U_{ij} u_{Lj})^{\dagger} \gamma_0 \phi^{+} d_{Rk_2} + S_{ik} D_{b,kk_2} (S_{ij} d_{Lj})^{\dagger} \gamma_0 \phi^{0} d_{Rk_2}$$

$$+ D_{b,kk_2} S^{*}_{ki} \bar{d}_{Rk_2} \phi^{+} (U_{ij} u_{Lj}) + D_{b,kk_2} S^{*}_{ki} \bar{d}_{Rk_2} \phi^{0} (S_{ij} d_{Lj}) \Big]$$

Now we have the peculiar situation where the U and S components do not drop out of every term. However, it is these very terms that do vanish when the vacuum expectation value is taken.

$$\langle 0 | \mathcal{L} | 0 \rangle = \sum_{i,k,j} \Big[\frac{1}{\sqrt{2}} U_{ik} D_{a,kk_2} (U_{ij} u_{Lj})^{\dagger} \gamma_0 u_{Rk_2} (v+H) + \frac{1}{\sqrt{2}} D_{a,kk_2} U^{*}_{ki} \bar{u}_{Rk_2} (U_{ij} u_{Lj}) (v+H)$$

$$+ \frac{1}{\sqrt{2}} S_{ik} D_{b,kk_2} (S_{ij} d_{Lj})^{\dagger} \gamma_0 d_{Rk_2} (v+H) + \frac{1}{\sqrt{2}} D_{b,kk_2} S^{*}_{ki} \bar{d}_{Rk_2} (S_{ij} d_{Lj}) (v+H) \Big]$$

$$= \frac{1}{\sqrt{2}} \sum_{i,k,j} \Big[U_{ik} D_{a,kk_2} u^{\dagger}_{Lj} U^{*}_{ji} \gamma_0 u_{Rk_2} (v+H) + D_{a,kk_2} U^{*}_{ki} \bar{u}_{Rk_2} U_{ij} u_{Lj} (v+H)$$

$$+ S_{ik} D_{b,kk_2} d^{\dagger}_{Lj} S^{*}_{ji} \gamma_0 d_{Rk_2} (v+H) + D_{b,kk_2} S^{*}_{ki} \bar{d}_{Rk_2} S_{ij} d_{Lj} (v+H)$$

$$= \frac{1}{\sqrt{2}} \sum_{i,k,j} \Big[(U^{*}_{ji} U_{ik}) D_{a,kk_2} \bar{u}_{Lj} u_{Rk_2} (v+H) + (U^{*}_{ki} U_{ij}) D_{a,kk_2} \bar{u}_{Rk_2} u_{Lj} (v+H)$$

$$+ (S^{*}_{ji} S_{ik}) D_{b,kk_2} \bar{d}_{Lj} d_{Rk_2} (v+H) + (S^{*}_{ki} S_{ij}) D_{b,kk_2} \bar{d}_{Rk_2} d_{Lj} (v+H)$$

$$= \frac{1}{\sqrt{2}} \sum_{k,j} \Big[D_{a,kk} \bar{u}_{Lj} u_{Rk} v + D_{a,kk} \bar{u}_{Lj} u_{Rk} H + D_{a,kk} \bar{u}_{Rk} u_{Lj} v + D_{a,kk} \bar{u}_{Rk} u_{Lj} H$$

$$+ D_{b,kk} \bar{d}_{Lj} d_{Rk} v + D_{b,kk} \bar{d}_{Lj} d_{Rk} H + D_{b,kk} \bar{d}_{Rk} d_{Lj} v + D_{b,kk} \bar{d}_{Rk} d_{Lj} H \Big]$$

Now it is apparent we have only Dirac mass terms for the quarks and interaction terms for those quarks with the Higgs field. The fact that $D_{a,kk}$ and $D_{b,kk}$ are elements on the diagonal ensures that the two-field terms are actually Dirac mass terms.

So define

$$\begin{cases} m_{a,k} \equiv \frac{1}{\sqrt{2}} D_{a,kk} v \\ m_{b,k} \equiv \frac{1}{\sqrt{2}} D_{b,kk} v \end{cases}$$

$$= \sum_{k,j} \Big[m_{a,k} \Big(\bar{u}_{Lj} u_{Rk} + \bar{u}_{Lj} u_{Rk} \frac{H}{v} + \bar{u}_{Rk} u_{Lj} + \bar{u}_{Rk} u_{Lj} \frac{H}{v} \Big)$$

$$+ m_{b,k} \Big(\bar{d}_{Lj} d_{Rk} + \bar{d}_{Lj} d_{Rk} \frac{H}{v} + \bar{d}_{Rk} d_{Lj} + \bar{d}_{Rk} d_{Lj} \frac{H}{v} \Big) \Big]$$

Summing over j at this point just picks out the kth value because $D_{k,k}$ is on the diagonal.

$$\boxed{\mathcal{L} = \sum_{k} \Big[m_{a,k} \Big(\bar{u}_{Lk} u_{Rk} \Big(1 + \frac{H}{v}\Big) + \bar{u}_{Rk} u_{Lk} \Big(1 + \frac{H}{v}\Big) \Big) + m_{b,k} \Big(\bar{d}_{Lk} d_{Rk} \Big(1 + \frac{H}{v}\Big) + \bar{d}_{Rk} d_{Lk} \Big(1 + \frac{H}{v}\Big) \Big) \Big]}$$

This is the standard form of quark mass terms and Higgs couplings.

It is important to note that the mass and Higgs interaction terms are not the only place the quark fields appear in the standard model Lagrangian, so the aforementioned change of variables must be propagated into the other places the quark fields appear.

We shall see that elements of W and T completely disappear from the theory, but U and S do not disappear from the charge-changing weak interactions.

For example,

$$\begin{cases} u_{Rj} = W_{jk}\, u_{Rk} \\ d_{Rj} = T_{jk}\, d_{Rk} \end{cases}$$

Then

$$\sum_j \left[\bar{u}_{Rj} (i\not{D}) u_{Rj} + \bar{d}_{Rj} (i\not{D}) d_{Rj} \right]$$

$$= \sum_{j,k} \left[(W_{jk}\, u_{Rk})^\dagger \gamma_0 (i\not{D}) (W_{jk}\, u_{Rk}) + (T_{jk}\, d_{Rk})^\dagger \gamma_0 (i\not{D}) (T_{jk}\, d_{Rk}) \right]$$

$$= \sum_{j,k} \left[u^\dagger_{Rk} W^*_{kj} \gamma_0 (i\not{D}) W_{jk}\, u_{Rk} + d^\dagger_{Rk} T^*_{kj} \gamma_0 (i\not{D}) T_{jk}\, d_{Rk} \right]$$

The components of W and T have no problem commuting past the covariant derivative, so when we sum over j:

$$= \sum_k \left[\bar{u}_{Rk} (i\not{D}) u_{Rk} + \bar{d}_{Rk} (i\not{D}) d_{Rk} \right]$$

The case is much different for the change of variables in the left-handed fields. Begin with the entire electroweak Lagrangian for quarks and leptons.

$$\mathcal{L}_{EW} = \mathcal{L}_{Q} + \mathcal{L}_{Lep}$$

$$\mathcal{L}_{Lep} = i\bar{e}\gamma_\rho\left(\frac{1+\gamma_5}{2}\right)\partial^\rho e + i\bar{e}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)\partial^\rho e + i\bar{\nu}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)\partial^\rho \nu$$

$$- \frac{g}{\sqrt{2}}\bar{\nu}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)eW^\rho - \frac{g}{\sqrt{2}}\bar{e}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)\nu W^{\dagger\rho}$$

$$+ \frac{g}{2\cos\theta_W} Z^\rho\left[-\bar{\nu}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)\nu + \bar{e}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)e\left(1-2\sin^2\theta_W\right) - \bar{e}\gamma_\rho\left(\frac{1+\gamma_5}{2}\right)e\left(2\sin^2\theta_W\right)\right]$$

$$+ \bar{e}\gamma_\rho e A^\rho e_0$$

$$\mathcal{L}_{Q} = i\bar{u}\gamma_\rho\left(\frac{1+\gamma_5}{2}\right)\partial^\rho u + i\bar{d}\gamma_\rho\left(\frac{1+\gamma_5}{2}\right)\partial^\rho d + i\bar{u}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)\partial^\rho u + i\bar{d}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)\partial^\rho d$$

$$+ \frac{1}{\sqrt{2}} g\bar{u}\gamma_\rho W^\rho\left(\frac{1-\gamma_5}{2}\right)d + \frac{1}{\sqrt{2}} g\bar{d}\gamma_\rho W^{\dagger\rho}\left(\frac{1-\gamma_5}{2}\right)u$$

$$+ \frac{g}{2\cos\theta_W} Z^\rho\left[\bar{u}\gamma_\rho\left(\frac{1+\gamma_5}{2}\right)u\left(\frac{4}{3}\sin^2\theta_W\right) - \bar{d}\gamma_\rho\left(\frac{1+\gamma_5}{2}\right)d\left(\frac{2}{3}\sin^2\theta_W\right)\right.$$

$$\left. + \bar{u}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)u\left(-1+\frac{4}{3}\sin^2\theta_W\right) - \bar{d}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)d\left(-1+\frac{2}{3}\sin^2\theta_W\right)\right]$$

$$- \frac{2e_0}{3}\bar{u}\gamma_\rho u A^\rho + \frac{e_0}{3}\bar{d}\gamma_\rho d A^\rho$$

Just as for the right-handed terms, the U and S components disappear in the kinetic terms. They also easily disappear in the Z^0 and EM interactions. However, notice that the W terms link different types of fields. Because of this, for $k=1,2,3$ (three quark generations)

$$\sum_k \frac{1}{\sqrt{2}} g \bar{u}_k \gamma_\rho W^\rho \left(\frac{1-\gamma_5}{2}\right) d_k + \frac{1}{\sqrt{2}} g \bar{d}_k \gamma_\rho W^{\dagger\rho} \left(\frac{1-\gamma_5}{2}\right) u_k$$

$$= \frac{g}{\sqrt{2}} \sum_{k,i,j} \left[\left(U_{ki} u_i\right)^\dagger \gamma_0 W \left(S_{kj} d_j\right)\left(\frac{1-\gamma_5}{2}\right) + \left(S_{kj} d_j\right)^\dagger \gamma_0 W^\dagger \left(U_{ki} u_i\right)\left(\frac{1-\gamma_5}{2}\right)\right]$$

$$= \frac{g}{\sqrt{2}} \sum_{k,i,j} \left[u_i^\dagger U^*_{ik} \gamma_0 W S_{kj} d_j \left(\frac{1-\gamma_5}{2}\right) + d_j^\dagger S^*_{jk} \gamma_0 W^\dagger U_{ki} u_i \left(\frac{1-\gamma_5}{2}\right)\right]$$

$$= \frac{g}{\sqrt{2}} \sum_{k,i,j} \left[\bar{u}_i W d_j \left(U^*_{ik} S_{kj}\right)\left(\frac{1-\gamma_5}{2}\right) + \bar{d}_j W^\dagger u_i \left(S^*_{jk} U_{ki}\right)\left(\frac{1-\gamma_5}{2}\right)\right]$$

That is, the charge changing weak interactions link the three u_i quarks with a unitary rotation of the triplet of d_i quarks, with this rotation given by the unitary matrix

$$V \equiv U^\dagger S$$

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

The quark interaction vertices with the W must now include these factors.