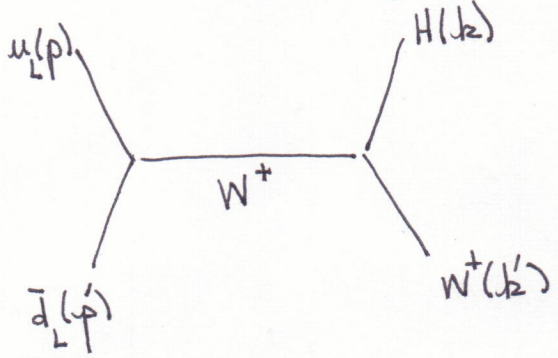
Higgs Production Cross Section: $q\bar{q} \to W^* \to WH \to \ell\nu b\bar{b}$

The Tevatron consists of a proton beam colliding with an antiproton beam. So let's consider the interaction a u-quark with an anti-d-quark. Recall that for weak intera these fermions are necessarily left-handed.



Redo w/ different coefficients

The full Lagrangian for the interaction is the sum of the Higgs Sector of the Standard Model and the Lagrangian for the quark doublet.

$$\mathcal{L}(x) = \frac{1}{2}(\partial_\mu H)(\partial^\mu H) + \frac{1}{2}\mu^2 H^2 + \frac{g^2 v^2}{4} W_\mu^\dagger W^\mu + \frac{g^2 v}{2} W_\mu^\dagger W^\mu H$$

$$- \frac{1}{4}\sum_{i=1,2}(\partial_\mu W_{i\nu} - \partial_\nu W_{i\mu})(\partial^\mu W_i^\nu - \partial^\nu W_i^\mu)$$

$$+ i\bar{u}\gamma^\rho\left(\frac{1-\gamma_5}{2}\right)\partial_\rho u + i\bar{d}\gamma^\rho\left(\frac{1-\gamma_5}{2}\right)\partial_\rho d + \frac{V_{ud}}{\sqrt{2}} g \bar{d}\gamma^\rho W_\rho^\dagger \left(\frac{1-\gamma_5}{2}\right) u$$

To calculate the cross section for this interaction, I want the interaction Lagrangian.

$$\mathcal{L}_I = \frac{g^2 v}{2} W_\mu^\dagger W^\mu H + \frac{g V_{ud}}{\sqrt{2}} \bar{d}\gamma^\rho W_\rho^\dagger \left(\frac{1-\gamma_5}{2}\right) u$$

I would like to change the form of the coefficients to be expressed in terms of the W mass and electric charge. Using $m_W = \frac{gv}{2}$ and $e_0 = g \sin\theta_W$.

$$\mathcal{L}_I = \frac{2m_W^2}{v} W_\mu^\dagger W^\mu H + \frac{e_0 V_{ud}}{\sqrt{2}\sin\theta_W} \bar{d}\gamma_\rho W^{\dagger\rho}\left(\frac{1-\gamma_5}{2}\right)u$$

Later, I'll re-express the leftover v in terms of the Fermi Coupling constant $G_F = \frac{\sqrt{2}}{2v^2}$. This way, I'll be able to express the cross section in terms of measured quantities.

From the interaction Lagrangian density I need the interaction Hamiltonian density.

$$\mathcal{H}_I(x) = \pi(x)\dot{\Phi}(x) - \mathcal{L}_I(x)$$

where $\Phi(x)$ is a position-space field and $\pi(x)$ is its conjugate momentum field. However, in this case there are no time-derivatives of fields in the interaction Lagrangian density. So it is simply

$$\mathcal{H}_I(x) = -\mathcal{L}(x)$$

$$= \underbrace{-\frac{2m_W^2}{v} W_\rho^\dagger W^\rho H}_{\mathcal{H}_{IH}(x)} \underbrace{- \frac{e_0 V_{ud}}{\sqrt{2}\sin\theta_W} \bar{d}\gamma_\rho W^{\dagger\rho}\left(\frac{1-\gamma_5}{2}\right)u}_{\mathcal{H}_{Iq}(x)}$$

$$\Rightarrow \mathcal{H}_I(x) = \mathcal{H}_{IH}(x) + \mathcal{H}_{Iq}(x)$$

The scattering matrix for this interaction is

$$\langle k_1', k_2' | S | p_1, p_2 \rangle = \langle k_1', k_2' | 1 | p_1, p_2 \rangle + i \langle k_1', k_2' | T | p_1, p_2 \rangle$$

where we recall that the scattering matrix is defined as the time-evolution operator as $t \to \infty$.

$$\langle k_1', k_2' | S | p_1, p_2 \rangle = \lim_{t \to \infty} \langle k_1, k_2 | e^{iH(2t)} | p_1, p_2 \rangle$$

The interaction component here is what I want to calculate. From (4.90) Peskin,

$$i \langle k_1', k_2' | T | p_1, p_2 \rangle = \lim_{t \to \infty (1 - i\varepsilon)} \langle k_1', k_2' | T \exp\left[-i \int_{-t}^{t} dt' H_I(t') \right] | p_1, p_2 \rangle$$

That exponential expands as (from (4.22) Peskin)

$$T \exp\left[-i \int_{-t}^{t} dt' H_I(t') \right] = 1 + (-i) \int_{-t}^{t} dt_1 H_I(t_1) + \frac{(-i)^2}{2!} \int_{-t}^{t} dt_1 dt_2 \, T\left[H_I(t_1) H_I(t_2) \right] + \cdots$$

As we are beginning and ending with two particle states, the 2nd order term is the first that can contribute to this interaction, and any higher-order terms contain loops. The interaction part of the scattering matrix element becomes,

$$i \langle k_1', k_2' | T | p_1, p_2 \rangle \cong \langle k_1', k_2' | \frac{(-i)^2}{2!} \int_{-t}^{t} dt_1 dt_2 \, T\left[H_I(x_1) H_I(x_2) \right] | p_1, p_2 \rangle$$

where

$$\begin{cases} H_I(x) = \int d^3x \, \mathcal{H}_I(x) \\ \qquad = \int d^3x \left[\mathcal{H}_{IH}(x) + \mathcal{H}_{Iq}(x) \right] \end{cases}$$

and in the Hamiltonian I replaced the variable t with full spacetime variable x because all components now come into play.

$$= \langle k_2' ; k_2 | \frac{(-i)^2}{2} \iint dt_1 dt_2 \, T\left[\int d^3x_1 \, \mathcal{H}_I(x_1) \int d^3x_2 \, \mathcal{H}_I(x_2) \right] | p' ; p \rangle$$

$$= \langle k_2' ; k_2 | \frac{(-i)^2}{2} T\left[\int d^4x_1 \, \mathcal{H}_I(x_1) \int d^4x_2 \, \mathcal{H}_I(x_2) \right] | p' ; p \rangle$$

$$= \frac{(-i)^2}{2} \iint d^4x_1 d^4x_2 \, \langle k_2' ; k_2 | T\left[\mathcal{H}_I(x_1) \mathcal{H}_I(x_2) \right] | p' ; p \rangle$$

$$= \frac{(-i)^2}{2} \iint d^4x_1 d^4x_2 \, \langle k_2' ; k_2 | T\Big[\mathcal{H}_{IH}(x_1) \mathcal{H}_{IH}(x_2) + \mathcal{H}_{IH}(x_1) \mathcal{H}_{Iq}(x_2) + \mathcal{H}_{Iq}(x_1) \mathcal{H}_{IH}(x_2) + \mathcal{H}_{Iq}(x_1) \mathcal{H}_{Iq}(x_2) \Big] | p' ; p \rangle$$

Since I have an interaction that involves both the quark doublet and the Higgs, the $\mathcal{H}_{IH}(x_1) \mathcal{H}_{IH}(x_2)$ and $\mathcal{H}_{Iq}(x_1) \mathcal{H}_{Iq}(x_2)$ terms do not contribute.

$$= \frac{(-i)^2}{2} \iint d^4x_1 d^4x_2 \, \langle k_2' ; k_2 | T\left[\mathcal{H}_{IH}(x_1) \mathcal{H}_{Iq}(x_2) + \mathcal{H}_{Iq}(x_1) \mathcal{H}_{IH}(x_2) \right] | p' ; p \rangle$$

Next, I have to calculate these two time-ordered products.

$$T\left[\mathcal{H}_{IH}(x_1) \mathcal{H}_{Iq}(x_2) \right]$$

$$= \frac{2 e_0 m_W^2 V_{ud}}{v\sqrt{2} \sin\theta_W} T\left[W_\mu^\dagger(x_1) W^\mu(x_1) H(x_1) \bar{d}(x_2) W^\dagger(x_2) \left(\frac{1-\gamma_5}{2} \right) u(x_2) \right]$$

$$= \frac{2 e_0 m_W^2 V_{ud}}{v\sqrt{2} \sin\theta_W} N\left[W_\mu^\dagger(x_1) W^\mu(x_1) H(x_1) \bar{d}(x_2) W^\dagger(x_2) \left(\frac{1-\gamma_5}{2} \right) u(x_2) \right]$$

$$= \frac{2e_o m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}$$

$$N\Big[W_\mu^\dagger(x_1) W^\nu(x_1) H(x_1) \bar{d}(x_2) \not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right) u(x_2)$$

$$+ \overbrace{W_\mu^\dagger(x_1} W^\nu(x_1) H(x_1) \bar{d}(x_2) \not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right) u(x_2)$$

$$+ \overbrace{W_\mu^\dagger(x_1) W^\nu(x_1)} H(x_1) \bar{d}(x_2) \not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right) u(x_2)$$

$$+ \overbrace{W_\mu^\dagger(x_1) W^\nu(x_1) H(x_1)} \bar{d}(x_2) \not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right) u(x_2)$$

$$+ \overbrace{W_\mu^\dagger(x_1) W^\nu(x_1) H(x_1) \bar{d}(x_2)} \not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right) u(x_2)$$

$$+ \overbrace{W_\mu^\dagger(x_1) W^\nu(x_1) H(x_1) \bar{d}(x_2) \not{W}^\dagger(x_2)} \left(\tfrac{1-\gamma_5}{2}\right) u(x_2)$$

$$+ W_\mu^\dagger(x_1) \overbrace{W^\nu(x_1)} H(x_1) \bar{d}(x_2) \not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right) u(x_2)$$

$$+ W_\mu^\dagger(x_1) \overbrace{W^\nu(x_1) H(x_1)} \bar{d}(x_2) \not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right) u(x_2)$$

$$+ W_\mu^\dagger(x_1) \overbrace{W^\nu(x_1) H(x_1) \bar{d}(x_2)} \not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right) u(x_2)$$

$$+ W_\mu^\dagger(x_1) \overbrace{W^\nu(x_1) H(x_1) \bar{d}(x_2) \not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right)} u(x_2)$$

$$+ W_\mu^\dagger(x_1) W^\nu(x_1) \overbrace{H(x_1) \bar{d}(x_2)} \not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right) u(x_2)$$

$$+ W_\mu^\dagger(x_1) W^\nu(x_1) \overbrace{H(x_1) \bar{d}(x_2) \not{W}^\dagger(x_2)} \left(\tfrac{1-\gamma_5}{2}\right) u(x_2)$$

$$+ W_\mu^\dagger(x_1) W^\nu(x_1) \overbrace{H(x_1) \bar{d}(x_2) \not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right)} u(x_2)$$

$$+ W_\mu^\dagger(x_1) W^\nu(x_1) H(x_1) \overbrace{\bar{d}(x_2) \not{W}^\dagger(x_2)} \left(\tfrac{1-\gamma_5}{2}\right) u(x_2)$$

$$+ W_\mu^\dagger(x_1) W^\nu(x_1) H(x_1) \overbrace{\bar{d}(x_2) \not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right)} u(x_2)$$

$$+ W_\mu^\dagger(x_1) W^\nu(x_1) H(x_1) \bar{d}(x_2) \overbrace{\not{W}^\dagger(x_2) \left(\tfrac{1-\gamma_5}{2}\right)} u(x_2)$$

$$+ \text{terms w/ more than one contraction} \Big]$$

- Contractions between fields of different types vanish.
- If there are no contractions of a particular field at x_1 with a field at x_2, then there is no interaction between the initial and final states.
- The initial and final particle fields must be uncontracted. They contract with the initial and final state vectors later.

Hence, the only term left is the one that contracts $W^\nu(x_1)$ to $W^{\dagger\rho}(x_2)$. Notice that this is the only transition from initial to final states possible at tree-level.

Taking a step back, here's where we are:

$$i\langle k_2', k_2 | T | p', p\rangle$$

$$= \frac{(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \int\int d^4x_1 d^4x_2 \langle k_2', k_2 | \Big[W^\dagger_\mu(x_1) W^\nu(x_1) H(x_1) \bar{d}(x_2) \gamma_\rho W^{\dagger\rho}(x_2) \left(\frac{1-\gamma_5}{2}\right) u(x_2)$$
$$+ \bar{d}(x_1)\gamma_\rho W^{\dagger\rho}(x_1)\left(\frac{1-\gamma_5}{2}\right) u(x_1) W^\dagger_\mu(x_2) W^\nu(x_2) H(x_2)\Big] | p', p\rangle$$

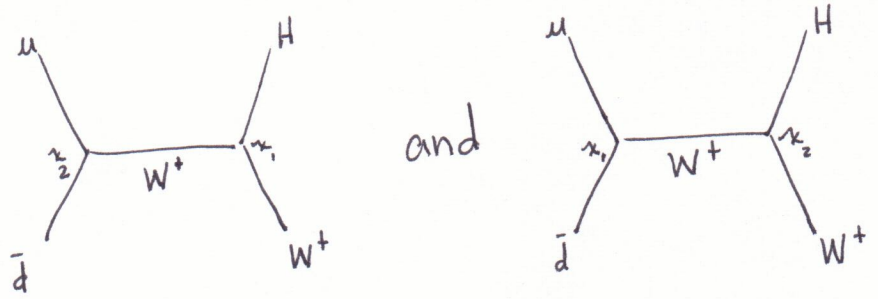
$$= \frac{(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \int\int d^4x_1 d^4x_2 \langle k_2', k_2 | W^\dagger_\mu(x_1) W^\nu(x_1) H(x_1) \bar{d}(x_2)\gamma_\rho W^{\dagger\rho}(x_2)\left(\frac{1-\gamma_5}{2}\right) u(x_2) | p', p\rangle$$

~~$+ \frac{(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \int\int d^4x_1 d^4x_2 \langle k_2', k_2 | \bar{d}(x_1)\gamma_\rho W^{\dagger\rho}(x_1)\left(\frac{1-\gamma_5}{2}\right) u(x_1) W^\dagger_\mu(x_2) H(x_2) | p', p\rangle$~~

$$+ \frac{(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \int\int d^4x_1 d^4x_2 \langle k_2', k_2 | \bar{d}(x_1)\gamma_\rho W^{\dagger\rho}(x_1)\left(\frac{1-\gamma_5}{2}\right) u(x_1) W^\dagger_\mu(x_2) H(x_2) | p', p\rangle$$

($W^\dagger_\mu(x_2) \to W^\nu(x_2)$)

These two terms cover



respectively.

So how do these contract?

$$\overbrace{\langle k_2'|}\, W_\mu^\dagger(x) = \langle 0|\epsilon_\mu^{a*}(k_2')\, e^{ik_2'\cdot x}$$

$$\overbrace{\langle k_2|}\, H(x) = \langle 0|\, e^{ik_2\cdot x_1}$$

$$\overbrace{\bar{d}(x)|p'\rangle} = e^{-ip'\cdot x}\,\bar{d}^{r_1}(p')|0\rangle$$

$$\overbrace{u(x)|p\rangle} = e^{-ip\cdot x}\,u^{r_2}(p)|0\rangle$$

In position space, the W propagator includes an integral over momentum q.

$$\overbrace{W^{\dagger\rho}(x_1)W^{\nu}(x_2)} = \int \frac{d^4q}{(2\pi)^4}\, i\left[\frac{-g^{\nu\rho} + \frac{q^\nu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon}\right] e^{-iq\cdot(x_1 - x_2)}$$

Let's make the replacements in the scattering matrix.

$$i\langle k_2', k_2 | T | p', p \rangle$$

$$= \frac{(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \int\int d^4x_1 d^4x_2 \Big[\underbrace{\langle 0 | \epsilon_\rho^{*}(k_2') e^{ik_2' \cdot x_1}}_{W^\dagger} \underbrace{\langle 0 | e^{ik_2 \cdot x_1}}_{H} \underbrace{\int \frac{d^4q}{(2\pi)^4} i \left[\frac{-g^{\nu\rho} + \frac{q^\nu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon} \right] e^{-iq\cdot(x_2 - x_1)}}_{W\text{- propagator}}$$

$$\cdot \underbrace{e^{-ip'\cdot x_2} \bar{d}(p') |0\rangle}_{\bar{d}} \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) \underbrace{e^{-ip\cdot x_2} u^{r_2}(p) |0\rangle}_{u} \Big]$$

$$+ \frac{(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \int\int d^4x_1 d^4x_2 \Big[\underbrace{e^{-ip'\cdot x_1} \bar{d}^{r_1}(p') |0\rangle}_{\bar{d}} \gamma_\rho \underbrace{\int \frac{d^4q}{(2\pi)^4} i \left[\frac{-g^{\nu\rho} + \frac{q^\nu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon} \right] e^{-iq\cdot(x_1 - x_2)}}_{W \text{ propagator}} \left(\frac{1-\gamma_5}{2}\right)$$

$$\cdot \underbrace{e^{-ip\cdot x_1} u^{r_2}(p) |0\rangle}_{u} \underbrace{\langle 0 | \epsilon_\mu^{*}(k_2') e^{ik_2' \cdot x_2}}_{W^\dagger} \underbrace{\langle 0 | e^{ik_2 \cdot x_2}}_{H} \Big]$$

$$= \frac{(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \int\int d^4x_1 d^4x_2 \Big[\epsilon_\mu^{*}(k_2') e^{-i(-k_2' - k_2 - q)\cdot x_1} \int \frac{d^4q}{(2\pi)^4} i \left[\frac{-g^{\nu\rho} + \frac{q^\nu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon} \right] \bar{d}^{r_1}(p') \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) u^{r_2}(p) e^{-i(p' + p + q)\cdot x_2}$$

$$+ \frac{(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \int\int d^4x_1 d^4x_2 \Big[\bar{d}^{r_1}(p') \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) u^{r_2}(p) e^{-i(p' + p + q)\cdot x_1} \int \frac{d^4q}{(2\pi)^4} i \left[\frac{-g^{\nu\rho} + \frac{q^\nu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon} \right] \epsilon_\mu^{*}(k_2') e^{-i(-k_2' - k_2 - q)\cdot x_2} \Big]$$

$$= \frac{(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \int\int d^4x_1 d^4x_2$$

$$= \frac{(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \epsilon_\mu^{*}(k_2)(2\pi)^4 \delta^4(-k_1'-k_2-q) \left[\int \frac{d^4q}{(2\pi)^4} i \left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon}\right] \bar{d}(p_1') \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) u(p_2) (2\pi)^4 \delta^4(p_1'+p_2+q)\right.$$

$$+ \frac{(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \bar{d}(p_1') \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) u(p_2) (2\pi)^4 \delta^4(p_1'+p_2+q) \left[\int \frac{d^4q}{(2\pi)^4} i \left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon}\right] \epsilon_\mu^{*}(k_2)(2\pi)^4 \delta^4(-k_1'-k_2-q)\right.$$

$$= \frac{i(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \int d^4q\, \epsilon_\mu^{*}(k_2') \left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon}\right] \bar{d}(p_1') \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) u(p_2) (2\pi)^4 \delta^4(-k_1'-k_2-q) \delta^4(p_1'+p_2+q)$$

$$+ \frac{i(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \int d^4q\, \bar{d}(p_1') \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) u(p_2) \left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon}\right] \epsilon_\mu^{*}(k_2') (2\pi)^4 \delta^4(-k_1'-k_2-q) \delta^4(p_1'+p_2+q)$$

$$= \frac{i(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} (2\pi)^4 \epsilon_\mu^{*}(k_2') \left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon}\right] \bar{d}(p_1') \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) u(p_2) \delta^4(p_1'+p_2-k_1'-k_2)$$

$$+ \frac{i(-i)^2 e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} (2\pi)^4 \bar{d}(p_1') \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) u(p_2) \left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon}\right] \epsilon_\mu^{*}(k_1') \delta^4(p_1'+p_2-k_1'-k_2)$$

Now that the integral over q has been carried out over the δ-functions, it is understood that $q = k_1' + k_2 = p_1' + p_2$ explicitly now.

$$= -\frac{2ie_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} (2\pi)^4 \epsilon_\mu^{*}(k_2') \left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon}\right] \bar{d}(p_1') \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) u(p_2) \delta^4(p_1'+p_2-k_1'-k_2)$$

Recall from Peskin (4.73)

$$i\langle k_1', k_2' | T | p_1', p_2' \rangle = (2\pi)^4 \delta^4(p_1'+p_2-k_1'-k_2) \cdot i\mathcal{M}(p_1', p_2 \to k_1', k_2')$$

$$\boxed{i\mathcal{M} = -\frac{2ie_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W} \epsilon_\mu^{*}(k_2') \left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}}{q^2 - m_W^2 + i\varepsilon}\right] \bar{d}(p_1') \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) u(p_2) \delta^4(p_1'+p_2-k_1'-k_2)}$$

The next step in finding the differential cross section is to compute $|M|^2$. This means finding the complex conjugate M^*.

$$M^* = -\frac{2e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\left\{\epsilon_\mu^*(k_2)\left[\frac{-g^{\mu\rho}+\frac{q^\mu q^\rho}{m_W^2}}{q^2-m_W^2}\right]\left[\bar{d}^{r_1}(p')\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)u^{r_2}(p)\right]\right\}^*$$

$$= -\frac{2e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\left[\bar{d}^{r_1}(p')\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)u^{r_2}(p)\right]^*\left[\epsilon_\mu^*(k_2)\left[\frac{-g^{\mu\rho}+\frac{q^\mu q^\rho}{m_W^2}}{q^2-m_W^2}\right]\right]^*$$

$$= -\frac{2e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\underbrace{\left[\bar{d}^{r_1}(p')\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)u^{r_2}(p)\right]^*}\left[\left[\frac{-g^{\mu\rho}+\frac{q^\mu q^\rho}{m_W^2}}{q^2-m_W^2}\right]\epsilon_\mu^2(k_2)\right]$$

$$\downarrow$$

$$\left(\bar{d}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)u\right)^* = \left(d^\dagger\gamma^0\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)u\right)^*$$

$$= \left(\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)u\right)^\dagger\left(d^\dagger\gamma_0\right)^\dagger$$

$$= u^\dagger\left(\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)\right)^\dagger\gamma_0^\dagger d$$

$$= u^\dagger\left(\frac{1-\gamma_5}{2}\right)\gamma_\rho^\dagger\gamma_0^\dagger d \qquad \text{b/c } \left(\frac{1-\gamma_5}{2}\right)^\dagger = \left(\frac{1-\gamma_5}{2}\right)$$

$$= u^\dagger\left(\frac{1-\gamma_5}{2}\right)\left(\gamma_0\gamma_\rho\right)^\dagger d$$

$$= u^\dagger\left(\frac{1-\gamma_5}{2}\right)\left(2g^{0\rho}-\gamma_\rho\gamma_0\right)^\dagger d \qquad \text{b/c } \{\gamma_\mu,\gamma_\nu\} = 2g_{\mu\nu}$$

$$= u^\dagger\left(\frac{1-\gamma_5}{2}\right)\gamma_\rho^\dagger\gamma_0 d \qquad \text{b/c } \gamma_0 = \gamma_0^\dagger$$

$$= u^\dagger\left(\frac{1-\gamma_5}{2}\right)\gamma_0\gamma_\rho d \qquad \text{b/c } \gamma_\rho^\dagger = \gamma_0\gamma_\rho\gamma_0$$

$$= u^\dagger\gamma_0\left(\frac{1+\gamma_5}{2}\right)\gamma_\rho d$$

$$= \bar{u}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)d$$

$$\Rightarrow M^* = -\frac{2e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\,\bar{u}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)d\left[\frac{-g^{\mu\rho}+\frac{q^\mu q^\rho}{m_W^2}}{q^2-m_W^2}\right]\epsilon_\mu^2(k')$$

$$\Rightarrow |M|^2 = +\left(\frac{2e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\right)^2 \epsilon_\mu^{1*}(k')\left[\frac{-g^{\mu\rho}+\frac{q^\mu q^\rho}{m_W^2}}{q^2-m_W^2}\right]\bar{d}^{r_1}(p')\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)u^{r_2}(p)\,\bar{u}^{r_2}(p)\gamma_\nu\left(\frac{1-\gamma_5}{2}\right)$$
$$\cdot d^{r_1}(p')\left[\frac{-g^{\sigma\nu}+\frac{q^\sigma q^\nu}{m_W^2}}{q^2-m_W^2}\right]\epsilon_\sigma^{1}(k')$$

The beam at the Tevatron is unpolarized, so average over spins r_1, r_2 of the quarks. The polarization of the end states is not measured, so the cross section is a sum of the possible polarization states of the W. As such, we want to compute

$$\frac{1}{2}\sum_{r_1}\frac{1}{2}\sum_{r_2}\sum_{\lambda}|M|^2$$

To do this, we use the spin sums (3.66) and (3.67) of Peskin

$$\frac{1}{4}\sum_{r_1,r_2,\lambda}|M|^2$$
$$= +\left(\frac{e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\right)^2\sum_{r_1,r_2,\lambda}\left\{\epsilon_\mu^{1*}(k')\left[\frac{-g^{\mu\rho}+\frac{q^\mu q^\rho}{m_W^2}}{q^2-m_W^2}\right]\bar{d}^{r_1}(p')\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)u^{r_2}(p)\right.$$
$$\left.\bar{u}^{r_2}(p)\gamma_\nu\left(\frac{1-\gamma_5}{2}\right)d^{r_1}(p')\left[\frac{-g^{\sigma\nu}+\frac{q^\sigma q^\nu}{m_W^2}}{q^2-m_W^2}\right]\epsilon_\sigma^{1}(k')\right\}$$

To use the spin sums, commute $\bar{d}$ to the right of d. Notice how $\bar{d}\gamma^\rho\left(\frac{1-\gamma_5}{2}\right)u$ is a row vector times a 4×4 matrix (yielding another row vector) times a column vector (yielding a scalar). To proceed, I'll write the same thing in terms of individual matrix elements rather than matrices. Assume repeated indices are summed over.

$$= +\left(\frac{e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\right)^2 \sum_{r_1,r_2,s} \left\{ \epsilon^{s*}_\mu(k_2)\left[-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}\right]\frac{1}{q^2 - m_W^2}\bar{d}^{r_1}(p')_a\left[\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)\right]_{ab} u^{r_2}(p)_b \right.$$
$$\left. \bar{u}^{r_2}(p)_c\left[\gamma_\nu\left(\frac{1-\gamma_5}{2}\right)\right]_{cd} d^{r_1}(p')_d\left[\frac{-g^{\sigma\nu} + \frac{q^\sigma q^\nu}{m_W^2}}{q^2 - m_W^2}\right]\epsilon^s_\sigma(k_1)\right\}$$

$$= +\left(\frac{e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\right)^2 \sum_{r_1,r_2,s} \left\{ \epsilon^{s*}_\mu(k_2)\left[-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}\right]\frac{1}{q^2 - m_W^2}\left[\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)\right]_{ab} u^{r_2}(p)_b \bar{u}^{r_2}(p)_c \left[\gamma_\nu\left(\frac{1-\gamma_5}{2}\right)\right]_{cd} \right.$$
$$\left. \underbrace{\bar{d}^{r_1}(p')_g d^{r_1}(p')_d}\left[\frac{-g^{\sigma\nu} + \frac{q^\sigma q^\nu}{m_W^2}}{q^2 - m_W^2}\right]\epsilon^s_\sigma(k_1)\right\}$$

$$= +\left(\frac{e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\right)^2 \sum_{s} \left\{ \epsilon^{s*}_\mu(k_2)\left[-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}\right]\frac{1}{q^2 - m_W^2}\left[\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)\right]_{ab}\left(\not{p} + m_u\right)_{bc}\left[\gamma_\nu\left(\frac{1-\gamma_5}{2}\right)\right]_{cd} \right.$$
$$\left. \left(\not{p}' - m_d\right)_{da}\left[\frac{-g^{\sigma\nu} + \frac{q^\sigma q^\nu}{m_W^2}}{q^2 - m_W^2}\right]\epsilon^s_\sigma(k_1)\right\}$$

where I used the fermion spin sums:

$$\begin{cases} \sum_s u^s(p)\bar{u}^s(p) = \gamma\cdot p + m_u \\ \sum_s v^s(p')\bar{v}^s(p') = \gamma\cdot p' - m_d \end{cases}$$

Easily commute $\epsilon^2(k_2)$ to the left of $\epsilon^{2*}(k_2')$ and use the vector particle spin sum:

$$\sum_{\sigma} \epsilon^{1}_{\sigma}(k_1)\epsilon^{2*}_{\mu}(k_2') = -g_{\sigma\mu} + \frac{k_{2\sigma}' k_{2\mu}'}{m_W^2}$$

$$= +\left(\frac{e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\right)^2 \sum_{s_1,s_2,s_3} \left\{ \left[-g_{\sigma\mu} + \frac{k_{2\sigma}' k_{2\mu}'}{m_W^2}\right] \left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}}{q^2 - m_W^2}\right] \left[\gamma_\rho \left(\frac{1-\gamma_5}{2}\right)\right]_{ab} (\not{p} + m_u)_{bc} \left[\gamma_\nu \left(\frac{1-\gamma_5}{2}\right)\right]_{cd} \right.$$
$$\left. (\not{p}' - m_d)_{da} \left[\frac{-g^{\sigma\nu} + \frac{q^\sigma q^\nu}{m_W^2}}{q^2 - m_W^2}\right] \right\}$$

$$= +\left(\frac{e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\right)^2 \sum_{s_1,s_2,s_3} \left\{ \left[-g_{\sigma\mu} + \frac{k_{2\sigma}' k_{2\mu}'}{m_W^2}\right] \left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}}{q^2 - m_W^2}\right] \left[\gamma_\rho \left(\frac{1-\gamma_5}{2}\right)(\not{p} + m_u)\right]_{ac} \right.$$
$$\left. \left[\gamma_\nu \left(\frac{1-\gamma_5}{2}\right)(\not{p}' - m_d)\right]_{ca} \left[\frac{-g^{\sigma\nu} + \frac{q^\sigma q^\nu}{m_W^2}}{q^2 - m_W^2}\right] \right\}$$

$$= +\left(\frac{e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\right)^2 \sum_{s_1,s_2,s_3} \left\{ \left[-g_{\sigma\mu} + \frac{k_{2\sigma}' k_{2\mu}'}{m_W^2}\right] \left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}}{q^2 - m_W^2}\right] \left[\gamma_\rho \left(\frac{1-\gamma_5}{2}\right)(\not{p} + m_u)\gamma_\nu \left(\frac{1-\gamma_5}{2}\right)(\not{p}' - m_d)\right] \right.$$
$$\left. \left[\frac{-g^{\sigma\nu} + \frac{q^\sigma q^\nu}{m_W^2}}{q^2 - m_W^2}\right] \right\}$$

$$= +\left(\frac{e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\right)^2 \sum_{s_1,s_2,s_3} \left\{ \left[-g_{\sigma\mu} + \frac{k_{2\sigma}' k_{2\mu}'}{m_W^2}\right] \left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}}{q^2 - m_W^2}\right] \left[\frac{-g^{\sigma\nu} + \frac{q^\sigma q^\nu}{m_W^2}}{q^2 - m_W^2}\right] \right.$$
$$\left. \mathrm{Tr}\left[\gamma_\rho \left(\frac{1-\gamma_5}{2}\right)(\not{p} + m_u)\gamma_\nu \left(\frac{1-\gamma_5}{2}\right)(\not{p}' - m_d)\right] \right\}$$

Evaluate the trace.

$$\mathrm{Tr}\left[\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)(\not{p}+m_u)\gamma_\nu\left(\frac{1-\gamma_5}{2}\right)(\not{p}'-m_d)\right]$$

$$=\frac{1}{4}\mathrm{Tr}\left[(\gamma_\rho-\gamma_\rho\gamma_5)(\not{p}+m_u)(\gamma_\nu-\gamma_\nu\gamma_5)(\not{p}'-m_d)\right]$$

$$=\frac{1}{4}\mathrm{Tr}\Big[(\gamma_\rho\not{p}+m_u\gamma_\rho-\gamma_\rho\gamma_5\not{p}-m_u\gamma_\rho\gamma_5)(\gamma_\nu\not{p}'-m_d\gamma_\nu-\gamma_\nu\gamma_5\not{p}'+m_d\gamma_\nu\gamma_5)\Big]$$

$$=\frac{1}{4}\mathrm{Tr}\Big[\gamma_\rho\not{p}\gamma_\nu\not{p}'-m_d\gamma_\rho\not{p}\gamma_\nu-\gamma_\rho\not{p}\gamma_\nu\gamma_5\not{p}'+m_d\gamma_\rho\not{p}\gamma_\nu\gamma_5$$
$$+m_u\gamma_\rho\gamma_\nu\not{p}'-m_um_d\gamma_\rho\gamma_\nu-m_u\gamma_\rho\gamma_\nu\gamma_5\not{p}'+m_um_d\gamma_\rho\gamma_\nu\gamma_5$$
$$-\gamma_\rho\gamma_5\not{p}\gamma_\nu\not{p}'+m_d\gamma_\rho\gamma_5\not{p}\gamma_\nu+\gamma_\rho\gamma_5\not{p}\gamma_\nu\gamma_5\not{p}'-m_d\gamma_\rho\gamma_5\not{p}\gamma_\nu\gamma_5$$
$$-m_u\gamma_\rho\gamma_5\gamma_\nu\not{p}'+m_um_d\gamma_\rho\gamma_5\gamma_\nu+m_u\gamma_\rho\gamma_5\gamma_\nu\gamma_5\not{p}'-m_um_d\gamma_\rho\gamma_5\gamma_\nu\gamma_5\Big]$$

$$=\frac{1}{4}\mathrm{Tr}\Big[\not{p}^\alpha\not{p}'^\beta\gamma_\rho\gamma_\alpha\gamma_\nu\gamma_\beta-\not{p}^\alpha\not{p}'^\beta\gamma_\rho\gamma_\alpha\gamma_\nu\gamma_5\gamma_\beta-m_um_d\gamma_\rho\gamma_\nu$$
$$-\not{p}^\alpha\not{p}'^\beta\gamma_\rho\gamma_5\gamma_\alpha\gamma_\nu\gamma_\beta+\not{p}^\alpha\not{p}'^\beta\gamma_\rho\gamma_5\gamma_\alpha\gamma_\nu\gamma_5\gamma_\beta-m_d\not{p}^\alpha\gamma_\rho\gamma_5\gamma_\alpha\gamma_\nu\gamma_5$$
$$+m_u\not{p}^\beta\gamma_\rho\gamma_5\gamma_\nu\gamma_5\gamma_\beta+m_um_d\gamma_\rho\gamma_\nu\Big]$$

$$=\frac{1}{4}\mathrm{Tr}\Big[\not{p}^\alpha\not{p}'^\beta\gamma_\rho\gamma_\alpha\gamma_\nu\gamma_\beta+\not{p}^\alpha\not{p}'^\beta\gamma_\rho\gamma_\alpha\gamma_\nu\gamma_\beta\gamma_5-m_um_d\gamma_\rho\gamma_\nu+\not{p}^\alpha\not{p}'^\beta\gamma_\rho\gamma_\alpha\gamma_\nu\gamma_\beta\gamma_5$$
$$+\not{p}^\alpha\not{p}'^\beta\gamma_\rho\gamma_\alpha\gamma_\nu\gamma_\beta-m_u\not{p}^\alpha\gamma_\rho\gamma_\alpha\gamma_\nu+m_um_d\gamma_\rho\gamma_\nu\Big]$$

$$=\frac{1}{4}\mathrm{Tr}\Big[2\not{p}^\alpha\not{p}'^\beta\gamma_\rho\gamma_\alpha\gamma_\nu\gamma_\beta+2\not{p}^\alpha\not{p}'^\beta\gamma_\rho\gamma_\alpha\gamma_\nu\gamma_\beta\gamma_5\Big]-2m_um_d\mathrm{Tr}[\gamma_\rho\gamma_\nu]$$

$$=\frac{1}{2}\mathrm{Tr}\left[\not{p}^\alpha\not{p}'^\beta\gamma_\rho\gamma_\alpha\gamma_\nu\gamma_\beta+\not{p}^\alpha\not{p}'^\beta\gamma_\rho\gamma_\alpha\gamma_\nu\gamma_\beta\gamma_5\right]$$

$$= \frac{1}{2} p^{\alpha} p'^{\beta} \mathrm{Tr}\left[\gamma_\rho \gamma_\alpha \gamma_\nu \gamma_\beta\right] + \frac{1}{2} p^{\alpha} p'^{\beta} \mathrm{Tr}\left[\gamma_\rho \gamma_\alpha \gamma_\nu \gamma_\beta \gamma_5\right]$$

$$= \frac{1}{2} p^{\alpha} p'^{\beta} 4\left(g_{\rho\alpha} g_{\nu\beta} - g_{\rho\nu} g_{\alpha\beta} + g_{\rho\beta} g_{\alpha\nu}\right) + \frac{1}{2} p^{\alpha} p'^{\beta} \left(-4i\epsilon_{\rho\alpha\nu\beta}\right)$$

$$= 2\left(p_\rho p'_\nu - g_{\rho\nu}(p\cdot p') + p_\nu p'_\rho\right) - 2i p^{\alpha} p'^{\beta} \epsilon_{\rho\alpha\nu\beta}$$

Put this back into the spin averaged amplitude squared.

$$\frac{1}{4}\sum |M|^2 = +\left(\frac{e_0 m_W^2 V_{ud}}{v\sqrt{2}\sin\theta_W}\right)^2 \left[-g_{\sigma\rho} + \frac{k'_\sigma k'_\nu}{m_W^2}\right]\left[\frac{-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W}}{q^2 - m_W^2}\right]\left[\frac{-g^{\sigma\nu} + \frac{q^\sigma q^\nu}{m_W}}{q^2 - m_W^2}\right]$$
$$\cdot 2\left(p_\rho p'_\nu - g_{\rho\nu}(p\cdot p') + p_\nu p'_\rho - i p^{\alpha} p'^{\beta} \epsilon_{\rho\alpha\nu\beta}\right)$$

$$= +\left(\frac{e_0 m_W^2 V_{ud}}{v\sin\theta_W}\right)\frac{1}{(q^2-m_W^2)^2}\left(-g_{\sigma\rho} + \frac{k'_\sigma k'_\nu}{m_W^2}\right)\left(-g^{\mu\rho} + \frac{q^\mu q^\rho}{m_W^2}\right)\left(-g^{\sigma\nu} + \frac{q^\sigma q^\nu}{m_W}\right)$$
$$\cdot\left(p_\rho p'_\nu + p_\nu p'_\rho - g_{\rho\nu}(p\cdot p') - i p^{\alpha} p'^{\beta} \epsilon_{\rho\alpha\nu\beta}\right)$$

$$= \left(\frac{e_0 m_W^2 V_{ud}}{v\sin\theta_W}\right)\frac{1}{(q^2-m_W^2)^2}\left[g_\sigma^{\ \rho} - \frac{q_\sigma q^\rho}{m_W^2} - \frac{k'_\sigma k'^\rho}{m_W^2} + \frac{k'_\sigma q^\rho (k'\cdot q)}{m_W^4}\right]\left(-g^{\sigma\nu} + \frac{q^\sigma q^\nu}{m_W}\right)$$
$$\left(p_\rho p'_\nu + p_\nu p'_\rho - g_{\rho\nu}(p\cdot p') - i p^{\alpha} p'^{\beta} \epsilon_{\rho\alpha\nu\beta}\right)$$

$$= \left(\frac{e_0 m_W^2 V_{ud}}{v\sin\theta_W}\right)\frac{1}{(q^2-m_W^2)^2}\left[-g^{\nu\rho} + \frac{q^\rho q^\nu}{m_W^2} + \frac{q^\nu q^\rho}{m_W^2} - \frac{q^2 q^\rho q^\nu}{m_W^4} + \frac{k'^\nu k'^\rho}{m_W^2} - \frac{(k'\cdot q) k'^\rho q^\nu}{m_W^4}\right.$$
$$\left. - \frac{k'^\nu q^\rho (k'\cdot q)}{m_W^4} + \frac{(k'\cdot q)^2 q^\rho q^\nu}{m_W^6}\right]\left(p_\rho p'_\nu + p_\nu p'_\rho - g_{\rho\nu}(p\cdot p') - i p^{\alpha} p'^{\beta} \epsilon_{\rho\alpha\nu\beta}\right)$$

$$= \left(\frac{e_0 m_W^2 V_{ud}}{v \sin\theta_W}\right)^2 \frac{1}{(q^2-m_W^2)^2}\Big[-(p\cdot p') - (p\cdot p') + 4(p\cdot p') + \cancel{i p^\alpha p^\beta \varepsilon^\nu{}_{\alpha\nu\beta}} \to 0$$

repeated index

$$+ \frac{(q\cdot p)(q\cdot p')}{m_W^2} + \frac{(q\cdot p)(q\cdot p')}{m_W^2} - \frac{q^2(p\cdot p')}{m_W^2} - \cancel{\frac{i}{m_W^2} q^\rho q^\nu p^\alpha p'^\beta \varepsilon_{\rho\alpha\nu\beta}} \to 0$$

repeated momentum

$$+ \frac{(q\cdot p)(q\cdot p')}{m_W^2} + \frac{(q\cdot p)(q\cdot p')}{m_W^2} - \frac{q^2(p\cdot p')}{m_W^2} - \cancel{\frac{i}{m_W^2} q^\rho q^\nu p^\alpha p'^\beta \varepsilon_{\rho\alpha\nu\beta}} \to 0$$

$$- \frac{q^2(q\cdot p)(q\cdot p')}{m_W^4} - \frac{q^2(q\cdot p)(q\cdot p')}{m_W^4} + \frac{q^4(p\cdot p')}{m_W^4} - \cancel{\frac{i q^2}{m_W^4} q^\rho q^\nu p^\alpha p'^\beta \varepsilon_{\rho\alpha\nu\beta}} \to 0$$

$$+ \frac{(k'\cdot p')(k'\cdot p)}{m_W^2} + \frac{(k'\cdot p')(k'\cdot p)}{m_W^2} - \frac{k'^2(p\cdot p')}{m_W^2} - \cancel{\frac{i}{m_W^2} k'^\nu k'^\rho p^\alpha p'^\beta \varepsilon_{\rho\alpha\nu\beta}} \to 0$$

$$- \frac{(k'\cdot q)(k'\cdot p)(q\cdot p')}{m_W^4} - \frac{(k'\cdot q)(k'\cdot p')(q\cdot p)}{m_W^4} + \frac{(k'\cdot q)^2(p\cdot p')}{m_W^4} + \cancel{\frac{i(k'\cdot q)}{m_W^4} k'^\rho q^\nu p^\alpha p'^\beta \varepsilon_{\rho\alpha\nu\beta}} \to 0$$

q is not indep. momentum

$$- \frac{(k'\cdot p')(q\cdot p)(k'\cdot q)}{m_W^4} - \frac{(k'\cdot p)(q\cdot p')(k'\cdot q)}{m_W^4} + \frac{(k'\cdot q)^2(p\cdot p')}{m_W^4} + \cancel{\frac{i(k'\cdot q)}{m_W^4} k'^\nu q^\rho p^\alpha p'^\beta \varepsilon_{\rho\alpha\nu\beta}}$$

$$+ \frac{(k'\cdot q)^2(q\cdot p)(q\cdot p')}{m_W^6} + \frac{(k'\cdot q)^2(q\cdot p)(q\cdot p')}{m_W^6} - \frac{q^2(k'\cdot q)^2(p\cdot p')}{m_W^6} - \cancel{\frac{i(k'\cdot q)^2}{m_W^6} q^\rho q^\nu p^\alpha p^\beta \varepsilon_{\rho\alpha\nu\beta}}$$

$q = p + p'$, so its not an independent momentum

$\varepsilon(k', q, p, p') = \varepsilon(k', p+p', p, p')$

$= \varepsilon(k', p, p, p') + \varepsilon(k', p', p, p')$

$= 0$

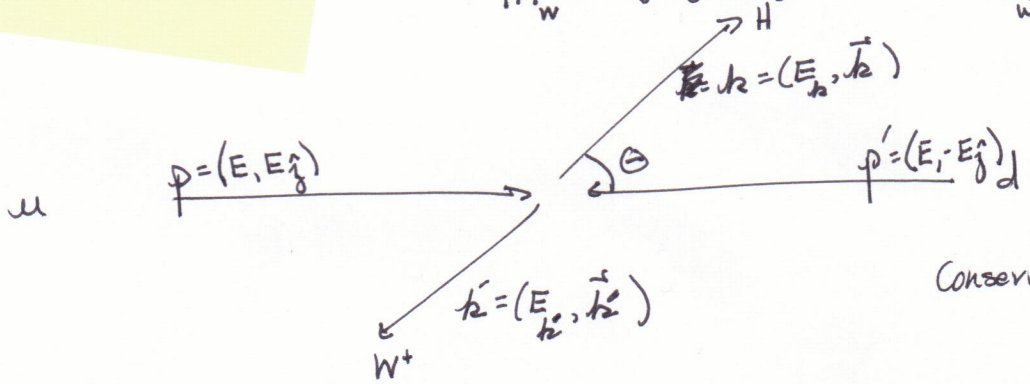
$$= \left(\frac{e_0 m_w^2 V_{ud}}{v \sin\theta_w}\right)^2 \frac{1}{(q^2 - m_w^2)^2} \Big[2(p\cdot p') + \frac{4}{m_w^2}(q\cdot p)(q\cdot p') - \frac{2}{m_w^2} q^2 (p'\cdot p) - \frac{2}{m_w^4} q^2 (q\cdot p)(q\cdot p')$$

$$+ \frac{2}{m_w^2}(k_2\cdot p')(k_2'\cdot p) - \frac{k_2'^2}{m_w^2}(p\cdot p') - \frac{2}{m_w^4}(k_2'\cdot q)(k_2'\cdot p)(q\cdot p')$$

$$- \frac{1}{m_w^4}(k_2'\cdot q)(k_2'\cdot p')(q\cdot p) + \frac{2}{m_w^4}(k_2'\cdot q)^2 (p\cdot p')$$

$$+ \frac{2}{m_w^6}(k_2'\cdot q)^2 (q\cdot p)(q\cdot p') - \frac{1}{m_w^6} q^2 (k_2'\cdot q)^2 (p\cdot p') \Big]$$

Do I just consider everything $\frac{1}{m^4}, \frac{1}{m^6}$ to vanish? Keep $\frac{1}{m^2}$, it cancels the one in the coef.



Conservation: $E_{k_2} + E_{k_2'} = 2E$

Kinematically, $q = p + p' = (E,0,0,E) + (E,0,0,-E) = (2E, \vec{0})$

$$k_2'\cdot q = (E_{k_2'}, \vec{k}_2')\cdot(2E, \vec{0}) = 2E_{k_2'}E$$

$$k_2\cdot q = (E_{k_2}, \vec{k}_2)\cdot(2E, \vec{0}) = 2E_{k_2}E$$

$$q^2 = 4E^2$$

$$p\cdot p' = (E,0,0,E)\cdot(E,0,0,-E) = E^2 - (-E^2) = 2E^2$$

$$p\cdot k_2 = (E, E\hat{z})\cdot(E_{k_2}, \vec{k}_2) = EE_{k_2} - E|\vec{k}_2|\cos\theta = E(E_{k_2} - |\vec{k}_2|\cos\theta)$$

$$p\cdot k_2' = (E, E\hat{z})\cdot(E_{k_2'}, \vec{k}_2') = EE_{k_2'} - E|\vec{k}_2'|\cos(\pi + \theta) = E(E_{k_2'} + |\vec{k}_2'|\cos\theta)$$

$$p'\cdot k_2 = (E, -E\hat{z})\cdot(E_{k_2}, \vec{k}_2) = EE_{k_2} + E|\vec{k}_2|\cos\theta = E(E_{k_2} + |\vec{k}_2|\cos\theta)$$

$$p'\cdot k_2' = (E, -E\hat{z})\cdot(E_{k_2'}, \vec{k}_2') = EE_{k_2'} + E|\vec{k}_2'|\cos(\pi+\theta) = E(E_{k_2'} - |\vec{k}_2'|\cos\theta)$$

$$q\cdot p' = q\cdot p = (2E, \vec{0})\cdot(E, E\hat{z}) = 2E^2$$

$$= \left(\frac{e_0 m_W^2 V_{ud}}{v \sin\theta_W}\right)^2 \frac{1}{(q^2 - m_W^2)^2} \Bigg[4E^2 + \frac{4}{m_W^2}(2E^2)(2E^2) - \frac{2}{m_W^2}(4E^2)(2E^2) - \frac{2}{m_W^4}(4E^2)(2E^2)(2E^2)$$

$$+ \frac{2}{m_W^2} E\left(E_{k_2} - |\vec{k}_2'|\cos\theta\right) E\left(E_{k_2'} + |\vec{k}_2'|\cos\theta\right) - \frac{1}{m_W^2}\left(E_{k_2'}^2 - |\vec{k}_2'|^2\right)(2E^2)$$

$$- \frac{2}{m_W^4}(2E\, E_{k_2})\, E\left(E_{k_2'} + |\vec{k}_2'|\cos\theta\right)(2E^2) - \frac{1}{m_W^4}(2E\, E_{k_2'})\, E\left(E_{k_2'} - |\vec{k}_2'|\cos\theta\right)(2E^2)$$

$$+ \frac{2}{m_W^4}(2E\, E_{k_2})^2 (2E^2) + \frac{2}{m_W^6}(2E\, E_{k_2'})^2 (2E^2)(2E^2)$$

$$- \frac{1}{m_W^6}(4E^2)(2E\, E_{k_2'})^2 (2E^2) \Bigg]$$

$$= \left(\frac{e_0 m_W^2 V_{ud}}{v \sin\theta_W}\right)^2 \frac{1}{(4E^2 - m_W^2)^2} \Bigg[4E^2 + \frac{1}{m_W^2} 16E^4 - \frac{1}{m_W^2} 16E^4 - \frac{1}{m_W^4} 32E^6 + \frac{1}{m_W^2} 2E^2\left(E_{k_2'}^2 - |\vec{k}_2'|^2\cos^2\theta\right)$$

$$- \frac{1}{m_W^2} 2E^2\left(E_{k_2'}^2 - |\vec{k}_2'|^2\right) - \frac{1}{m_W^4} 8E^4 E_{k_2}\left(E_{k_2'} + |\vec{k}_2'|\cos\theta\right)$$

$$- \frac{1}{m_W^4} 4E^4 E_{k_2'}\left(E_{k_2'} - |\vec{k}_2'|\cos\theta\right) + \frac{1}{m_W^4} 16E^4 E_{k_2'}^2$$

$$+ \frac{1}{m_W^6} 32 E^6 E_{k_2'}^2 - \frac{1}{m_W^6} 32 E^6 E_{k_2'}^2 \Bigg]$$

$$= \left(\frac{e_0 m_W^2 V_{ud}}{v \sin\theta_W}\right)^2 \frac{1}{(4E^2 - m_W^2)^2} \Bigg[4E^2 + \frac{1}{m_W^2}\left(2E^2\left(E_{k_2'}^2 - |\vec{k}_2'|^2\cos^2\theta\right) - 2E^2\left(E_{k_2'}^2 - |\vec{k}_2'|^2\right)\right)$$

$$+ \frac{1}{m_W^4}\left(-32E^6 - 8E^4 E_{k_2}\left(E_{k_2'} + |\vec{k}_2'|\cos\theta\right) - 4E^4 E_{k_2'}\left(E_{k_2'} - |\vec{k}_2'|\cos\theta\right) + 16E^4 E_{k_2'}^2\right)$$

$$+ \frac{1}{m_W^6}\left(-16E^6 E_{k_2'}^2\right) \Bigg]$$

$$= \left(\frac{e_0 m_W^2 V_{ud}}{v \sin\theta_W}\right)^2 \frac{1}{(4E^2 - m_W^2)^2} \left[4E^2 + \frac{1}{m_W^2}\left(2E^2|\vec{k}|^2 - 2E^2|\vec{k}'|^2\cos^2\theta\right) \right.$$

$$+ \frac{1}{m_W^4}\left(16E^4E^2_{k_2} - 32E^6 - 8E^4E^2_{k_2} - 8E^4E_{k_2}|\vec{k}'_2|\cos\theta - 4E^4E^2_{k_2} + 4E^4E_{k_2}|\vec{k}'_2|\cos\theta\right.$$

$$\left. \left. - \frac{1}{m_W^6} 16E^6E^2_{k_2} \right) \right]$$

Do not cancel!

$$= \left(\frac{e_0 m_W^2 V_{ud}}{v \sin\theta_W}\right)^2 \frac{1}{(4E^2 - m_W^2)^2} \left[4E^2 + \frac{2E^2|\vec{k}'_2|^2}{m_W^2}(1 - \cos^2\theta) + \frac{1}{m_W^4}\left(-32E^6 + 4EE^4_{k_2} - 4E^4E_{k_2}|\vec{k}'_2|\cos\theta\right) \right]$$

$$= \left(\frac{e_0 m_W^2 V_{ud}}{v \sin\theta_W}\right)^2 \frac{1}{(4E^2 - m_W^2)^2} \left[4E^2 + \frac{2E^2|\vec{k}'_2|^2}{m_W^2}\sin^2\theta + \frac{4E^4}{m_W^4}\left(-8E^2 + E^2_{k_2} - E_{k_2}|\vec{k}'_2|\cos\theta\right) \right]$$

Proceed to the cross section from (4.79) Peskin

$$d\sigma = \frac{1}{2E_A 2E_B |v_A - v_B|} \left[\frac{d^3k_2}{2E_{k_2}(2\pi)^3} \frac{d^3k'_2}{2E_{k'_2}(2\pi)^3} \right] \frac{1}{4} \sum_{A,r_1,r_2} |M|^2 (2\pi)^4 \delta^{(4)}(k + k' - p - p')$$

where $|v_A - v_B|$ is the relative velocity difference in the lab frame, so $|v_A - v_B| \simeq 2$ ($c=1$). Also, $E_A = E_B = E' = E$.

$$d\sigma = \frac{1}{128E^2(2\pi)^2} \left[\frac{d^3k_2}{E_{k_2}} \frac{d^3k'_2}{E_{k'_2}} \right] \left(\frac{e_0 m_W^2 V_{ud}}{v \sin\theta_W}\right)^2 \frac{1}{(4E^2 - m_W^2)^2} \left[4E^2 + \frac{2E^2|\vec{k}'_2|^2}{m_W^2}\sin^2\theta \right.$$

$$\left. + \frac{4E^4}{m_W^4}\left(-8E^2 + E^2_{k_2} - E_{k'_2}|\vec{k}'_2|\cos\theta\right) \right] \delta^4(k + k' - p - p')$$

Assuming an initial state in CM frame: $\vec{p} = -\vec{p}'$

$$\sigma = \frac{1}{128E^2(2\pi)^2(4E^2-m_W^2)^2}\left(\frac{e_0 m_W^2 V_{ud}}{v\sin\theta_W}\right)^2 \int \frac{d^3k_2}{\sqrt{m_H^2+|\vec{k}_2|^2}} \frac{d^3k_2'}{\sqrt{m_W^2+|\vec{k}_2'|^2}} \Big[4E^2 + \frac{2E^2|\vec{k}_2'|^2}{m_W^2}\sin^2\theta$$

$$+ \frac{4E^4}{m_W^4}\left(-8E^2 + E_{\vec{k}_2}^2 - E_{\vec{k}_2}|\vec{k}_2'|\cos\theta\right)\Big]\delta(E_{\vec{k}}+E_{\vec{k}'}-2E)\delta^3(\vec{k}+\vec{k}')$$

Integrate k_2 with the δ^3.

$$\sigma = \frac{1\,(16eV)}{128E^2(2\pi)^2(4E^2-m_W^2)^2}\left(\frac{e_0 m_W^2 V_{ud}}{v\sin\theta_W}\right)^2 \int \frac{d^3k_2'}{\sqrt{m_H^2+|\vec{k}_2'|^2}\sqrt{m_W^2+|\vec{k}_2'|^2}} \Big[4E^2 + \frac{2E^2|\vec{k}_2'|^2}{m_W^2}\sin^2\theta$$

$$+ \frac{4E^4}{m_W^4}\left(-8E^2 + E_{\vec{k}_2}^2 - E_{\vec{k}_2}|\vec{k}_2'|\cos\theta\right)\Big]\delta(\cancel{E_{\vec{k}_2}+E_{\vec{k}_2'}-2E})$$

$$\delta\left(\sqrt{m_H^2+|\vec{k}_2'|^2}+\sqrt{m_W^2+|\vec{k}_2'|^2}-2E\right)$$

Express the remaining differential in spherical coordinates

$$d^3k_2' = |\vec{k}_2'|^2 d|\vec{k}_2'|\sin\theta\, d\theta\, d\phi.$$

$$\sigma = \frac{(1\,GeV)}{128E^2(2\pi)^2(4E^2-m_W^2)^2}\left(\frac{e_0 m_W^2 V_{ud}}{v\sin\theta_W}\right)^2 \int \frac{d|\vec{k}_2'|\,d\theta\,d\phi}{\sqrt{m_H^2+|\vec{k}_2'|^2}\sqrt{m_W^2+|\vec{k}_2'|^2}} \Big[4E^2|\vec{k}_2'|^2\sin\theta$$

$$+ \frac{2E^2|\vec{k}_2'|^4}{m_W^2}\sin^3\theta - \frac{32E^6|\vec{k}_2'|^2\sin\theta}{m_W^4} + \frac{4E^4E_{\vec{k}_2}^2|\vec{k}_2'|^2\sin\theta}{m_W^4}$$

$$- \frac{4E^4E_{\vec{k}_2'}|\vec{k}_2'|^3\sin\theta\cos\theta}{m_W^4}\Big]\delta\left(\sqrt{m_H^2+|\vec{k}_2'|^2}+\sqrt{m_W^2+|\vec{k}_2'|^2}-2E\right)$$

The integral over ϕ is just $\int_0^{2\pi} d\phi = 2\pi$. Next, do all the θ integrals.

$$\int_0^{\pi} d\theta \sin\theta = -\cos\theta \Big|_0^{\pi} = -(-1-1) = 2$$

$$\int_0^{\pi} d\theta \sin^3\theta = -\frac{1}{3}(2+\sin^2\theta)\cos\theta \Big|_0^{\pi} = -\frac{1}{3}\left[(2+0)(-1) - (2+0)(1)\right] = -\frac{1}{3}(-2-2) = \frac{4}{3}$$

$$\int_0^{\pi} d\theta \sin\theta\cos\theta = \int x\, dx \qquad x = \sin\theta, \quad dx = \cos\theta\, d\theta$$

$$= \frac{x^2}{2}\Big|$$

$$= \frac{1}{2}\sin^2\theta \Big|_0^{\pi} = 0$$

$$\sigma = \frac{(1\text{ GeV})^2}{128E^2(2\pi)^3} \left(\frac{e_0 m_W^2 V_{ud}}{v \sin\theta_W}\right)^2 \int \frac{d|\vec{p}_2'|}{\sqrt{m_H^2 + |\vec{p}_2'|^2}\sqrt{m_W^2 + |\vec{p}_2'|^2}} \left[8E^2|\vec{p}_2'|^2 + \frac{8E^2|\vec{p}_2'|^4}{3m_W^2}\right.$$

$$\left. - \frac{64E^6|\vec{p}_2'|^2}{m_W^4} + \frac{8E^4 E_{\vec{p}_2'}^2 |\vec{p}_2'|^2}{m_W^4}\right] \delta\left(\sqrt{m_H + |\vec{p}_2'|^2} + \sqrt{m_W + |\vec{p}_2'|^2} - 2E\right)$$

So where is the zero of the δ-function? This can be tricky b/c the integration variable is squared, implying more than one solution in the argument of the δ-function.

Expand the δ-function with

$$\delta(f(x)) = \sum_j \frac{1}{f'(x_0^j)} \delta(x - x_0^j) \quad (x \to k_z)$$

Let $f(k_z) = \sqrt{m_H^2 + k_z^2} + \sqrt{m_W^2 + k_z^2} - 2E = 0$

$$\left(\sqrt{m_H^2 + k_z^2} + \sqrt{m_W^2 + k_z^2}\right)^2 = (2E)^2$$

$$m_H^2 + k_z^2 + m_W^2 + k_z^2 + 2\sqrt{(m_H^2 + k_z^2)(m_W^2 + k_z^2)} = 4E^2$$

$$2\sqrt{(m_H^2 + k_z^2)(m_W^2 + k_z^2)} = 4E^2 - 2k_z^2 - m_H^2 - m_W^2$$

$$4(m_H^2 + k_z^2)(m_W^2 + k_z^2) = \left(4E^2 - 2k_z^2 - m_H^2 - m_W^2\right)^2$$

$$4(m_H^2 m_W^2 + m_H^2 k_z^2 + m_W^2 k_z^2 + k_z^4) = 16E^4 - 8E^2 k_z^2 - 4E^2 m_H^2 - 4E^2 m_W^2$$
$$- 8E^2 k_z^2 + 4k_z^4 + 2k_z^2 m_H^2 + 2k_z^2 m_W^2$$
$$- 4E^2 m_H^2 + 2k_z^2 m_H^2 + m_H^4 + m_H^2 m_W^2$$
$$- 4E^2 m_W^2 + 2k_z^2 m_W^2 + m_H^2 m_W^2 + m_W^4$$

$$2m_H^2 m_W^2 = 16E^4 - 16E^2 k_z^2 - 8E^2 m_H^2 - 8E^2 m_W^2 + m_H^4 + m_W^4$$

$$16E^2 k_z^2 = 16E^4 - 8E^2 m_H^2 - 8E^2 m_W^2 - 2m_H^2 m_W^2 + m_H^4 + m_W^4$$

$$k_{z_0} = \pm \frac{1}{4E^2}\sqrt{16E^4 - (8m_H^2 + 8m_W^2)E^2 - 2m_H^2 m_W^2 + m_H^4 + m_W^4}$$

$$+ (m_H^2 - m_W^2)^2$$

$$\left(4E^2 - (m_H^2 - m_W^2)^2\right)^2$$

$$(m_H^2 - m_W^2)^2 = m_H^4 - 2m_H^2 m_W^2 + m_W^4$$

$$\left(4E^2 - (m_H^2 - m_W^2)\right)^2$$

$$= 16E^4 + (m_H^2 - m_W^2)^2 - 8E^2(m_H^2 - m_W^2)$$

$$k_{2_0} = \pm\frac{1}{4E^2}\left(4E^2 - (m_H^2 - m_W^2)^2\right)$$

$$k_{2_0} = \pm\left(1 - \frac{1}{4E^2}(m_H^2 - m_W^2)^2\right)$$

We only want the solution with positive momentum; the other is nonsense.

$$k_{2_0} = 1 - \left(\frac{m_H^2 - m_W^2}{2E}\right)^2$$

Find the derivative $f'(k)$.

$$f'(k_2) = \frac{1}{2}(m_H^2 + k_2^2)^{-\frac{1}{2}}(2k_2) + \frac{1}{2}(m_W^2 + k_2^2)^{-\frac{1}{2}}(2k_2)$$

$$= \frac{k_2}{\sqrt{m_H^2 + k_2^2}} + \frac{k_2}{\sqrt{m_W^2 + k_2^2}}$$

$$f'(k_{2_0}) = \frac{1 - \left(\frac{m_H^2 - m_W^2}{2E}\right)^2}{\sqrt{m_H^2 + \left(1 - \frac{(m_H^2 - m_W^2)^2}{4E^2}\right)^2}} + \frac{1 - \left(\frac{m_H^2 - m_W^2}{2E}\right)^2}{\sqrt{m_W^2 + \left(1 - \frac{(m_H^2 - m_W^2)^2}{4E^2}\right)^2}}$$

$$f'(k_0) = \frac{4E^2 - (m_H^2 - m_W^2)^2}{\sqrt{(4E^2)^2 m_H^2 + \left(4E^2 - (m_H^2 - m_W^2)^2\right)^2}} + \frac{4E^2 - (m_H^2 - m_W^2)^2}{\sqrt{(4E^2)^2 m_W^2 + \left(4E^2 - (m_H^2 - m_W^2)^2\right)^2}}$$

$$\delta(f(k_z)) = \frac{1}{f'(k_{z_0})}\,\delta\left(k_z - \left(\text{[illegible]} - \left(\frac{m_H^2 - m_W^2}{2E}\right)^2\right)\right)$$

$$\sigma = \frac{(1\ \text{GeV}^2)}{256\pi E^2 (4E^2 - m_W^2)^2}\left(\frac{e_0 m_W^2 V_{ud}}{r \sin\theta_W}\right)^2 \frac{1}{f'(k_{z_0})}\left[\frac{1}{\sqrt{m_H^2 + k_{z_0}^2}}\,\frac{1}{\sqrt{m_W^2 + k_{z_0}^2}}\left[8E^2 k_{z_0}^2 + \frac{8E^2 k_{z_0}^4}{3m_W^2} - \frac{64E^6 k_{z_0}^2}{m_W^4} + \frac{8E^4 (m_W^2 + k_{z_0}^2) k_{z_0}^2}{m_W^4}\right]\right]$$

This can probably be simplified.

From earlier: $\sqrt{(m_H^2 + k_{z_0}^2)(m_W^2 + k_{z_0}^2)} = \frac{1}{2}(4E^2 - 2k_{z_0}^2 - m_H^2 - m_W^2)$

$$\sigma = \frac{(1\ \text{GeV}^2)}{256\pi E^2 (4E^2 - m_W^2)^2}\left(\frac{e_0 m_W^2 V_{ud}}{r \sin\theta_W}\right)^2 \frac{1}{f'(k_{z_0})}\,\frac{2}{4E^2 - 2k_{z_0}^2 - m_H^2 - m_W^2} \cdot \left[8E^2 k_{z_0}^2 + \frac{8E^2 k_{z_0}^4}{3m_W^2} - \frac{64E^6 k_{z_0}^2}{m_W^4} + \frac{8E^4 k_{z_0}^2 (m_W^2 + k_{z_0}^2)}{m_W^4}\right]$$

Use these here: $\alpha = \frac{e_0^2}{\hbar c} = \frac{1}{137}$, $G_F = \frac{\sqrt{2}}{2V^2}$

$$f'(k_0) = \frac{4E^2 - (m_H^2 - m_W^2)^2}{\sqrt{(4E^2)^2 m_H^2 + (4E^2 - (m_H^2 - m_W^2))^2}} + \frac{4E^2 - (m_H^2 - m_W^2)^2}{\sqrt{(4E^2)^2 m_W^2 + (4E^2 - (m_H^2 - m_W^2)^2)^2}}$$

$$= \frac{-364798400}{714758610} + \frac{-364798400}{476999341}$$

$$= -0.510 - 0.765$$

$$= -1.275$$

$$\begin{cases} E = 980\ \text{GeV} \\ m_W = 80\ \text{GeV} \\ m_H = 160\ \text{GeV} \end{cases}$$

$$G_F = \frac{\sqrt{2}}{2v^2} = 1.166 \times 10^{-5}\ \text{GeV}^{-2}$$

$$k_0 = \left(1 - \frac{1}{4E^2}(m_H^2 - m_W^2)^2\right)$$

$$|k_0| = 94.96$$

$$k_0^2 = 9017$$

$$\sigma = (8.80 \times 10^{-23}\ \text{GeV}^{-6})\left(\frac{2G_F}{\sqrt{2}}\right)\left(\frac{e_0 m_W V_{ud}}{\sin\theta_W}\right)^2 \underbrace{(0.784)}_{f'(k_0)^{-1}} \cdot (5.27 \times 10^{-7}\ \text{GeV}^{-2}) \left| \frac{6.93 \times 10^{10}\ \text{GeV}^2 + 3.25 \times 10^{10}\ \text{GeV}^2 - 1.24 \times 10^{16}\ \text{GeV}^2 + 2.5 \times 10^{13}\ \text{GeV}^2}{-1.24 \times 10^{16}\ \text{GeV}^2} \right|$$

($1\ \text{GeV}^2$)

$$\sigma = (8.80 \times 10^{-23}\ \text{GeV}^{-6})(1.65 \times 10^{-5}\ \text{GeV}^{-2})\frac{1}{137^2}\cdot(80\ \text{GeV})^4 \cdot \frac{0.97417}{0.231}(0.784)(5.27 \times 10^{-7}\ \text{GeV}^{-2})(1.24 \times 10^{16}\ \text{GeV}^2)$$

$$V_{ud} = 0.97419$$

$$\alpha = \frac{e_0^2}{4\pi\epsilon_0} = \frac{1}{137}$$

$$\sin^2\theta_W = 0.231$$

$$\sigma = 6.90 \times 10^{-11}\ \text{GeV}^{-2} \cdot \left(\frac{0.3894\ \text{mb}}{\text{GeV}^2}\right)$$

$$= 2.69 \times 10^{-14}\ \text{mb}$$

$$= 2.69 \times 10^{-17}\ \text{b}$$

$$= 0.026\ \text{fb}$$

~~About half the actual answer, not bad for a very rough estimate.~~

3 orders of magnitude off