Lum statistical uncertainty for zero counting method

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General infos

In Tevatron there are 159 buckets (= 1113 RF buckets /7).

36 buckets are "filled" with particles → bunches

Luminosity is measured for each bucket by measuring the average number of interaction per bucket μ_i, total luminosity is the sum over 36 buckets containing particles bunches.

$$\mu_i \cdot f_{bc} = \sigma \cdot L_i$$
$$L = \sum_i L_i$$

Zero counting algorithm measures **bin 0** of the Poisson distribution (=P_0) to get the average μ_i.

Some maths

$$P_0(operative) = \frac{N_{0C}}{N_{TOT}}$$

$$P_0(\mu) = e^{(\varepsilon_0 - 1)\mu}[e^{\varepsilon_e \mu} + e^{\varepsilon_w \mu} - 1]$$

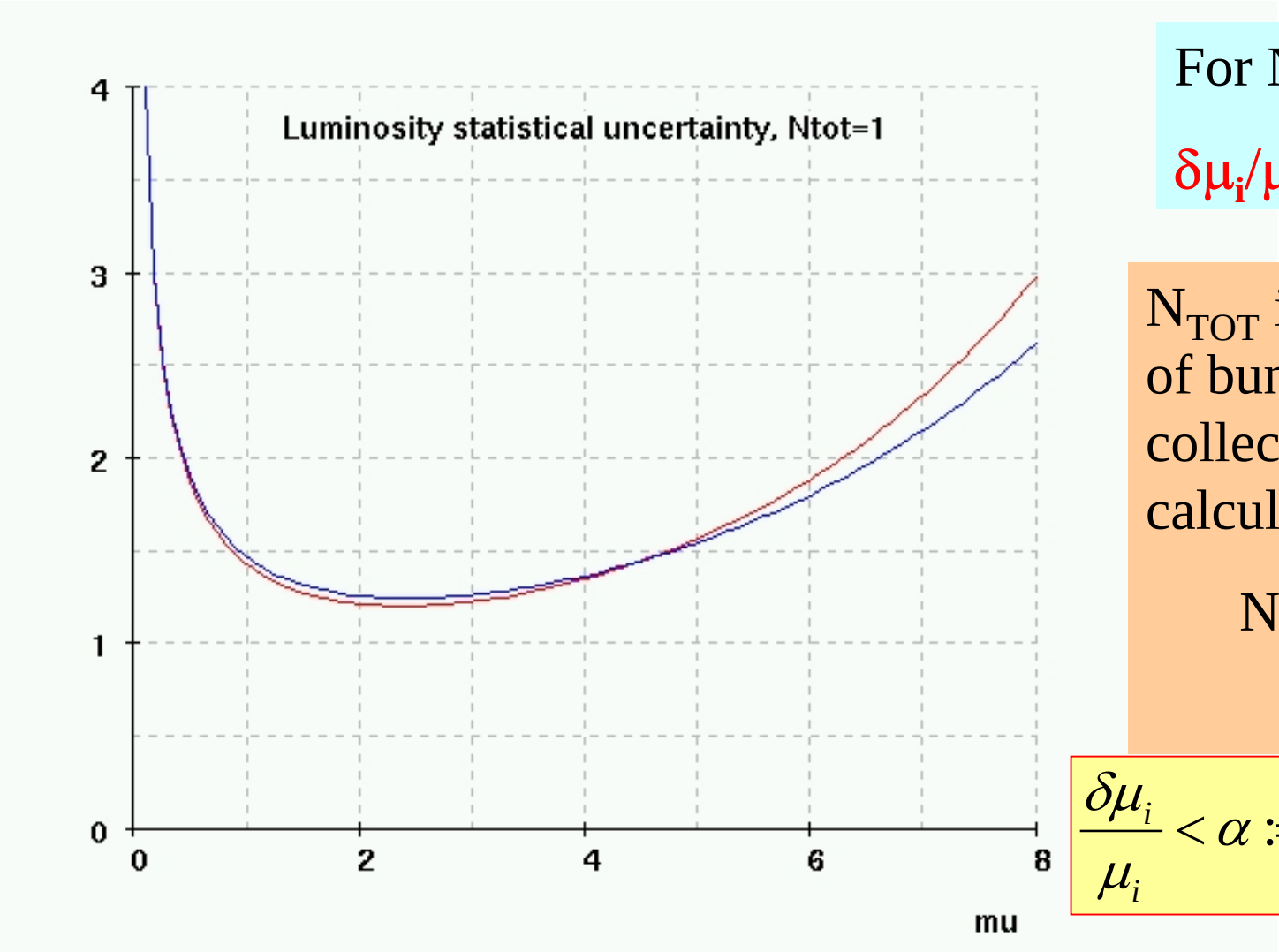
Empty crossing follows binomial distribution with P_0 probability.

$$\delta P_0 = \sqrt{\frac{P_0(1-P_0)}{N_{TOT}}}$$

$$\delta\mu = \left|\frac{\partial P_0}{\partial \mu}\right|^{-1} \delta P_0$$

$$\Rightarrow \quad \frac{\delta\mu}{\mu} = \frac{1}{\mu\sqrt{N_{TOT}}}\left|\frac{\partial P_0(\mu)}{\partial \mu}\right|^{-1} \sqrt{P_0(\mu)[1-P_0(\mu)]}$$

Stat uncertainty per BC



For $N_{TOT}=1$

$\delta\mu_i/\mu_i < 4$

N_{TOT} is the number of bunch crossing collected for calculation.

$N_{TOT}=8243$

$\Downarrow$

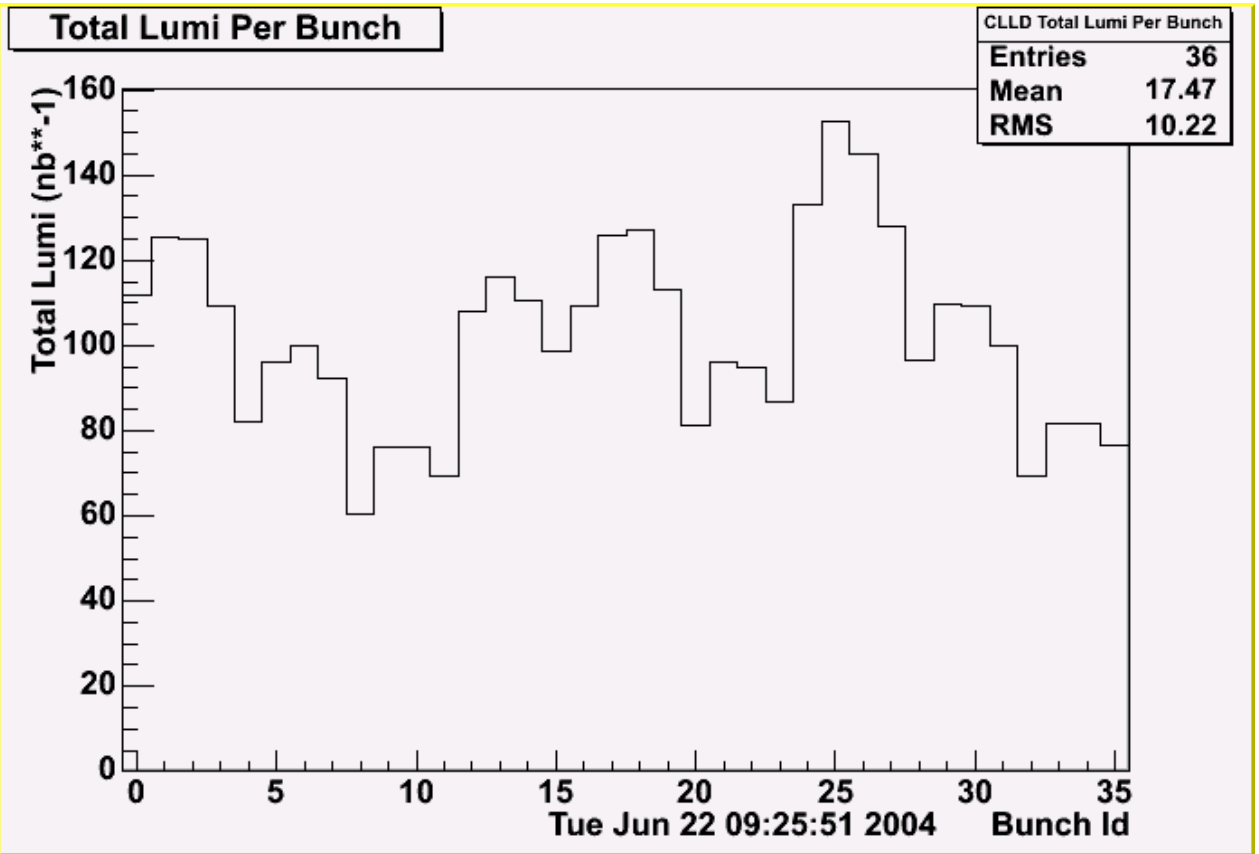
$$\frac{\delta\mu_i}{\mu_i} < \alpha := \frac{4}{\sqrt{8243}} \cong 0.044$$

Stat uncertainty on L

$$L = \sum_{i=1}^{M} L_i \qquad L_i = k\mu_i \Rightarrow \frac{\delta L_i}{L_i} < \alpha$$

$$\frac{\delta L^2}{L^2} < \alpha^2 \frac{\sum L_i^2}{\left(\sum L_i\right)^2} = \frac{\alpha^2}{M}\left(1 + \frac{\sigma_L^2}{\overline{L}^2}\right)$$

M=36



$$\frac{\sigma_L^2}{\overline{L}^2} = 0.04$$

Conclusions

$$\frac{\delta L^2}{L^2} < \frac{\alpha^2}{M} \Rightarrow \frac{\delta L}{L} < \frac{\alpha}{\sqrt{M}} = \frac{0.044}{6} = 0.007$$

Statistical uncertainty is less than 1% per single measurement! (~7 seconds)

Statistical uncertainty on integrated luminosity is neglegible.