

Accelerator Simulations  
PHYS 498, Accelerators

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# Simulating Accelerators

## 1 Introduction

The aim of this project is to demonstrate the power of a simulation computer software in the design of accelerators. We will concentrate on the transverse-betatron motion of the beam particles. Our objective is to display some characteristics of the beam and to describe the basic concept of controlling the transverse size of it.

The software we will use (MATLAB) is a good engineering resource for mathematics, data manipulations and simulations of any system.

The basic building blocks of any accelerator, that affect the transverse motion of the particles, are the drift areas, the focusing and the defocusing quadrupoles. The dipoles and sextupoles can be replaced by drift areas without a major mistake at first approximation.

The variables that we try to control is the transverse position and transverse velocity of the particles. We will concentrate on the horizontal direction, since the treatment is equivalent for the vertical one. The drift area does not affect the velocity of the particles, but it just linearly changes the position, according to the relation

$$x_{out} = x_{in} + x'_{in} \Delta L / u \quad (1)$$

where the primes denote the time derivatives,  $\Delta L$  is the length of the drift area and  $u$  is the speed of the particle in the longitudinal (s) direction. <sup>1</sup>

The focusing (defocusing) quadruple magnets do change the position of the particles and their velocity, according to the equations:

$$x_{out} = Ax_{in} + Bx'_{in} \quad (2)$$

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<sup>1</sup>in accelerator physics the variable s is used in the place of time. In MATLAB, time is the used variable, so we have to translate the equations using the longitudinal speed of the particles.

$$x'_{out} = Cx_{in} + Dx'_{in} \quad (3)$$

where  $A, B, C, D$  are functions of the length of the quadrupoles, and their strength (sin, cos for focusing, cosh, sinh for defocusing). For simplicity we will assume that the quads are point like (equivalent to thin lenses). In that case they do not alter the position but they do change the velocity of the particle, according to the equation:

$$x'_{out} = -Qux_{in} + x'_{in} \quad (4)$$

where  $Q$  is the strength of the quad (negative for focusing and positive for defocusing quads)

The MATLAB block for a drift area and a point like quad can be seen on figures 1 and 2 respectively. The exact block diagram for a focusing quad can be seen in figure 3. Every block has 2 input and 2 outputs (positions and velocities). These are going to be the building blocks of our accelerator lattice.

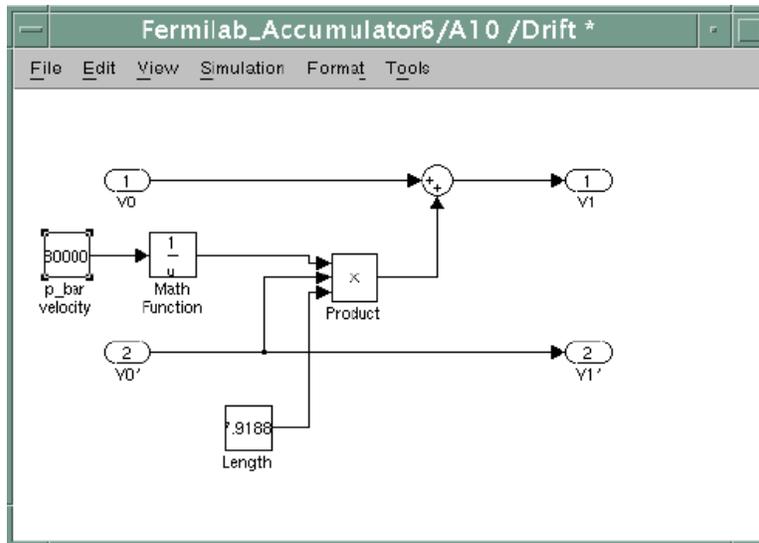


Figure 1: Drift area block diagram

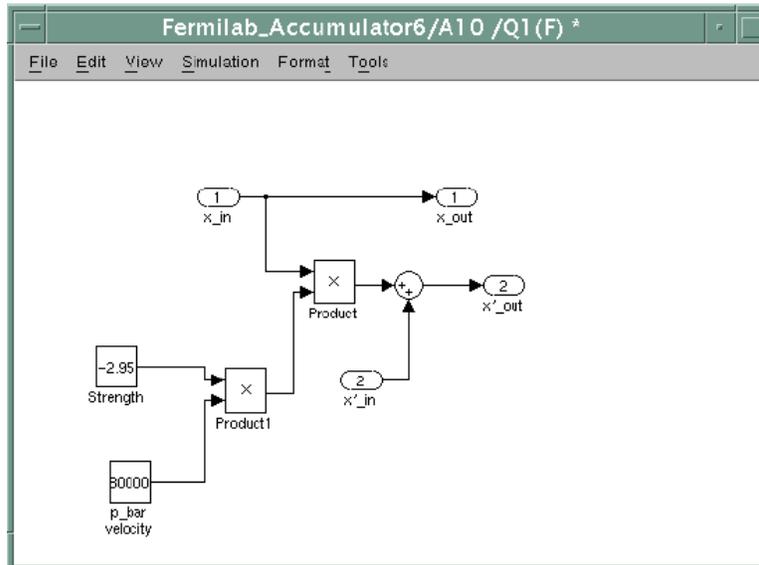


Figure 2: Point-like Quad block diagram

## 2 The FODO lattice

We first tried to simulate a general circular accelerator with a FODO lattice (Focusing Quad, Drift, Defocusing, Drift). The block diagram of the accelerator is shown in figure 4.

The injection is a random number generator for the positions and the velocities of the particles, which last for a time period smaller than the period of rotation of the particles. (To avoid unwanted interference)

As soon as we start the simulation, the particles start oscillating covering ellipses in the phase space (velocity vs position) We pick up a point in the lattice and we probe it with a scope, to record the positions and velocities of the particles as they rotate around the ring. We also record the phase space this way. The initial phase space covered by the particles is shown in figure 5.

The positions and the velocities of particles as a function of time are shown in figures 6 and 7.

We see the strong focusing effects which keep the particles constraint and slightly

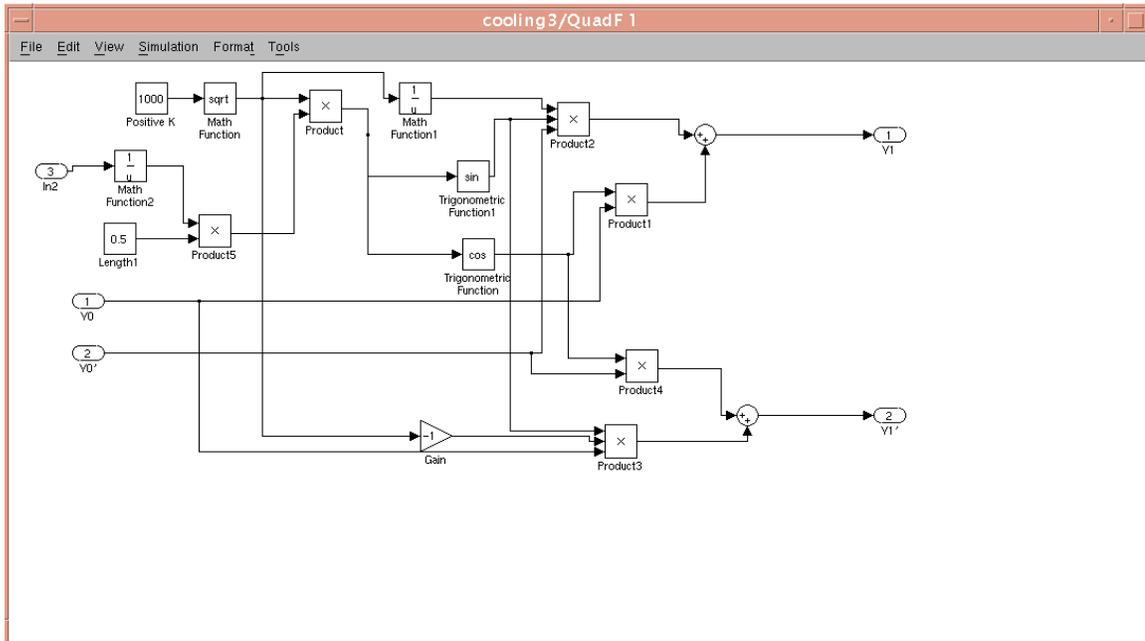


Figure 3: Exact Focusing Quad block diagram

reduces the size of the beam after some rotations. The phase difference between a beam entering the FODO and the same beam exiting it, is responsible for the observed oscillations, whose amplitude is proportional to  $\sqrt{\beta}$  at that point. The phase space scanned by the particles can be seen in figure 8.

### 3 Cooling

If we want to reduce the phase space area by increasing its density (cool the beam) we have to add a pickup-kicker control system, as it can be seen in figure 9.

The pickup is usually a probe of the position of the beam. We could in general pickup the position of the particles or their velocity at the end of the FODO and correct their velocity at the start of the next FODO. The possibility of the velocity pickup is included in figure 9, but we actually use only the position probe.

If the transverse position of the particles becomes greater than a threshold value, the switch closes and a correction signal (proportional to the error) is applied through

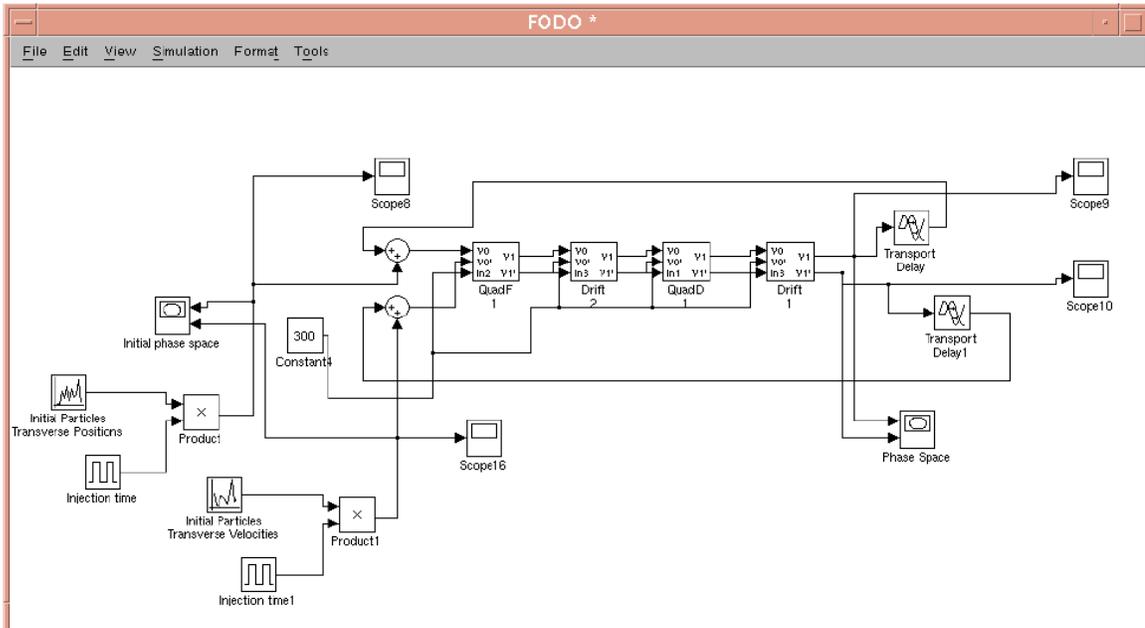


Figure 4: General FODO circular accelerator

the kicker, altering the velocity of the particles. The problems arising in real machines (phase agreement between the pickup and the kicker and synchronization between the beam and the kicker correction) are not simulated. The correction looks just like a feedback signal which is always on phase. The value of the feedback gain could be calculated theoretically, so that the feedback system is stable.

In figure 10, the phase space after the activation of the pickup-kicker system is shown. The position, velocity and correction signal can be seen in figures 11,12 and 13 respectively. We see that the beam indeed cools down. The phase space area is decreased considerably. Our threshold value for the size of the beam is 5 mm, while the initial spread of the particles was 50 mm (Figure 5). The achieved size of the beam can be reduced indefinitely in our simulation.

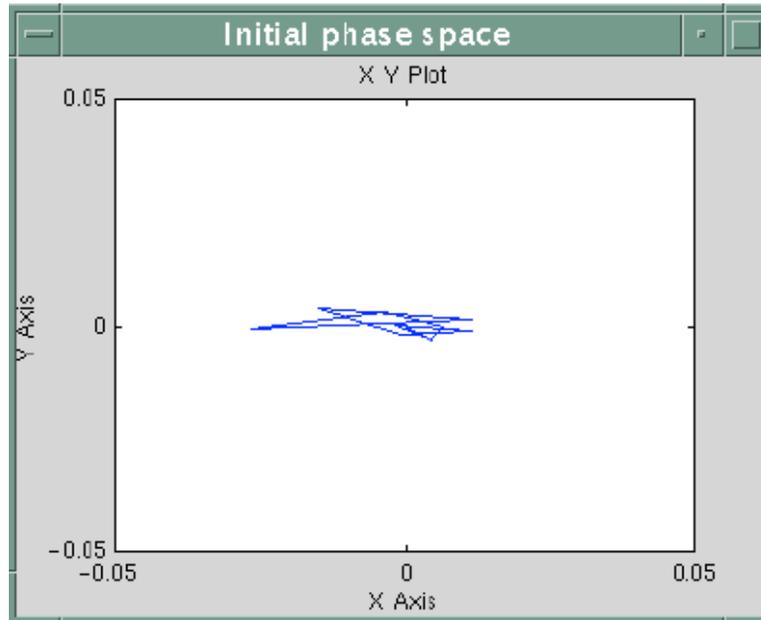


Figure 5: Initial phase space

## 4 The Fermilab Accumulator.

We decided to simulate an antiproton storage ring, since the cooling of the beam is its major function. The Fermilab Accumulator is going to be the accelerator we will describe (it is not really an accelerator in the sense of increasing the energy of the particles, but in the sense of keeping them confined and reducing their dimension).

The lattice of the Accumulator can be seen on figure 14. This configuration is repeated 6 times.

We have 6 parts (A10- A60) with the even numbered ones being the reflections of the odd ones. The schematic of the accelerator is shown in figure 15. Low and high dispersion areas are needed for cooling at the transverse and longitudinal direction respectively. The MATLAB realization of A10 and A20 can be seen on figures 16 and 17. The Quad strengths and elements length have been taken from the Fermilab antiproton group data sheet.

We added the delay time at the end of the lattice to make the design realistic.

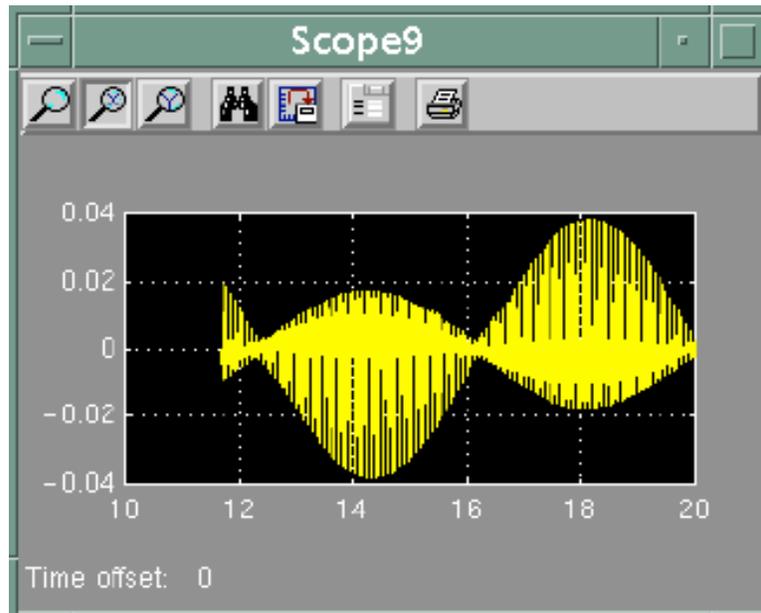


Figure 6: Transverse position of the particles

The lengths of the drifts and the strengths of the quads are taken from the fermilab division of beams web page. We use the point-like quad approximation.

The pickups are in the low dispersion area A10 and the kickers in A30.

The results of the simulation look similar to the FODO results and can be similarly arbitrarily manipulated, thus cannot correspond to the real operation of the machine. The fact that practical cooling difficulties and the interaction between the particles of the beam, which competes against the cooling, have not been simulated, does not lead to a realistic simulation, in the sense that special characteristics of the machine are not revealed. It would be an interesting goal to parametrize all the phenomena related to the accelerator and the electronics of phase space control, in order to include them in the simulation. Also the components of the lattice that affect even slightly the transverse motion and were replaced by drifts, should be parametrized properly. Still, the design of figures 15 to 17, is a good first order approximation of the machine and the first step to the real simulation.

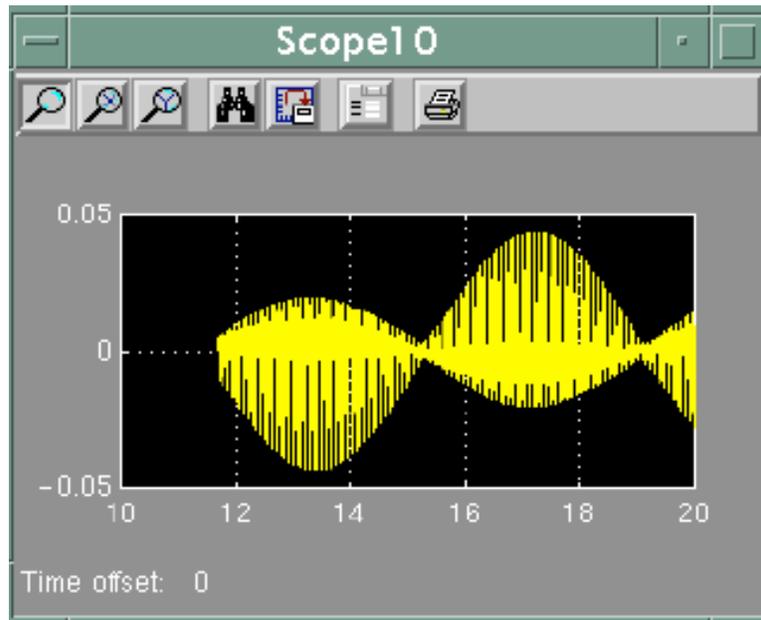


Figure 7: Transverse velocity of particles

## 5 Discussion

In this report we described the basic steps for simulating an accelerator using MATLAB. We used quad and drift building blocks and elementary injection and controlling systems for the simulation of the transverse motion of the particles. Many of the real-world problems related to the beam characteristics, signals timing etc. have not been simulated. The design is just an approximation that needs to be improved by additional simulations of critical phenomena.

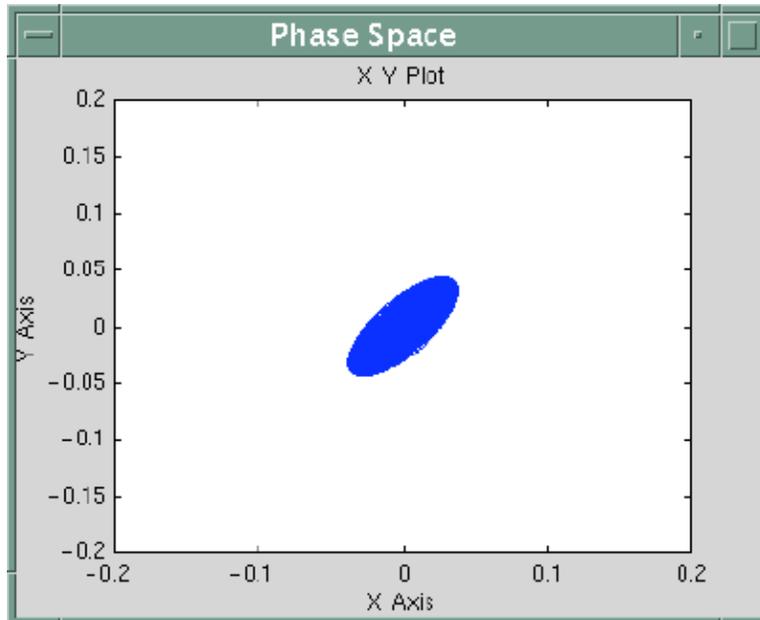


Figure 8: Phase space scanned by the particles

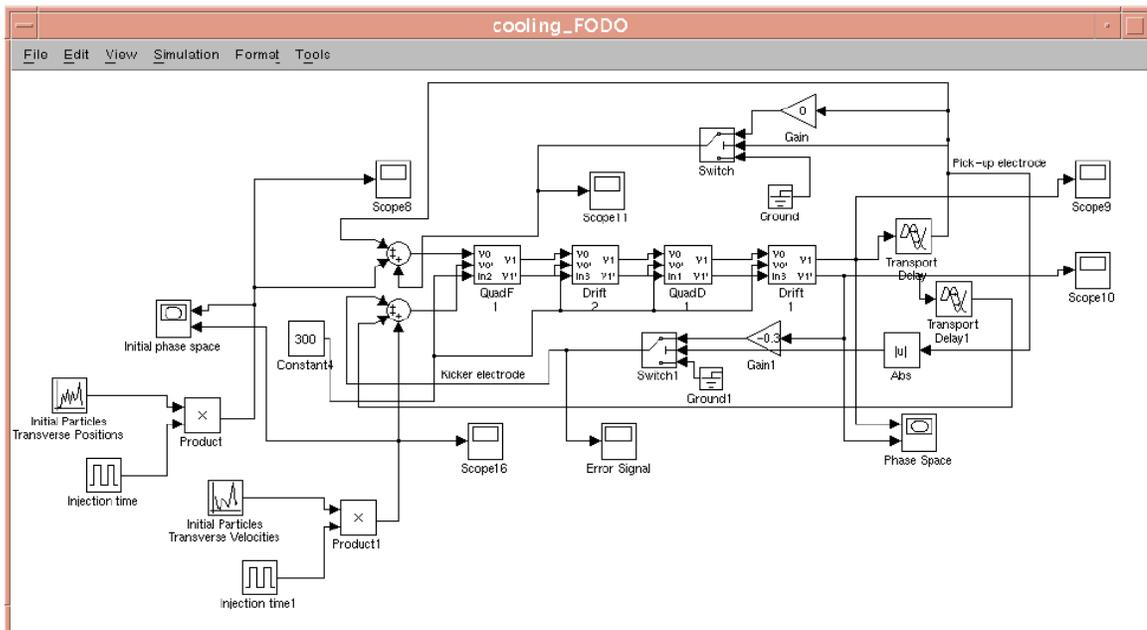


Figure 9: Beam Control System

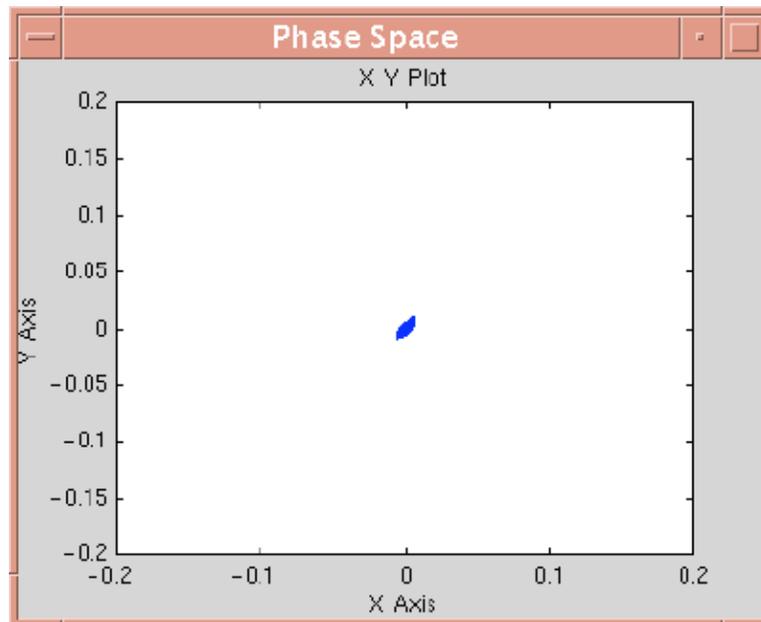


Figure 10: Phase space after cooling

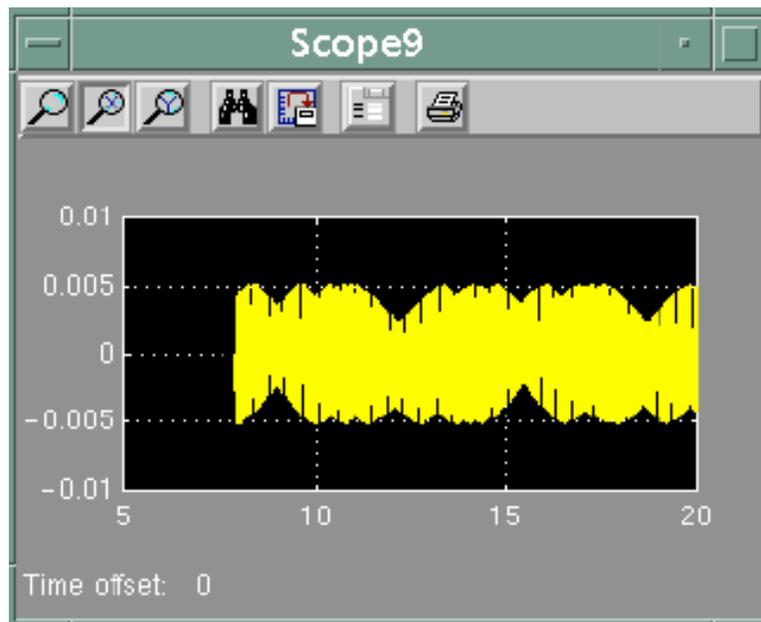


Figure 11: Transverse position of particles

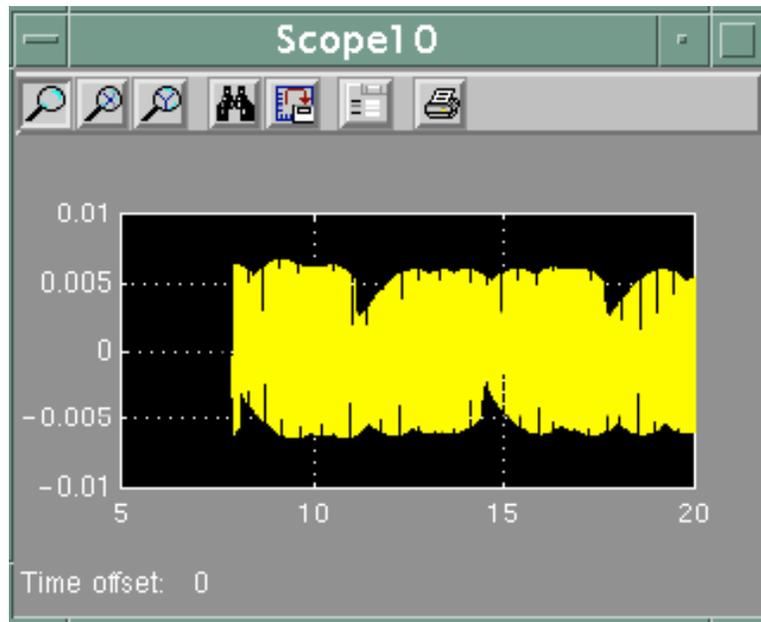


Figure 12: Transverse velocity of particles

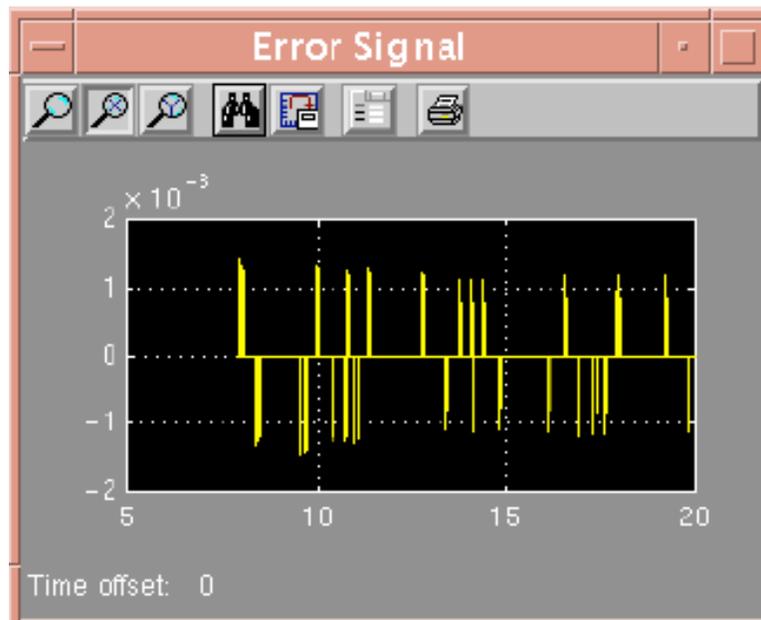


Figure 13: Error signal

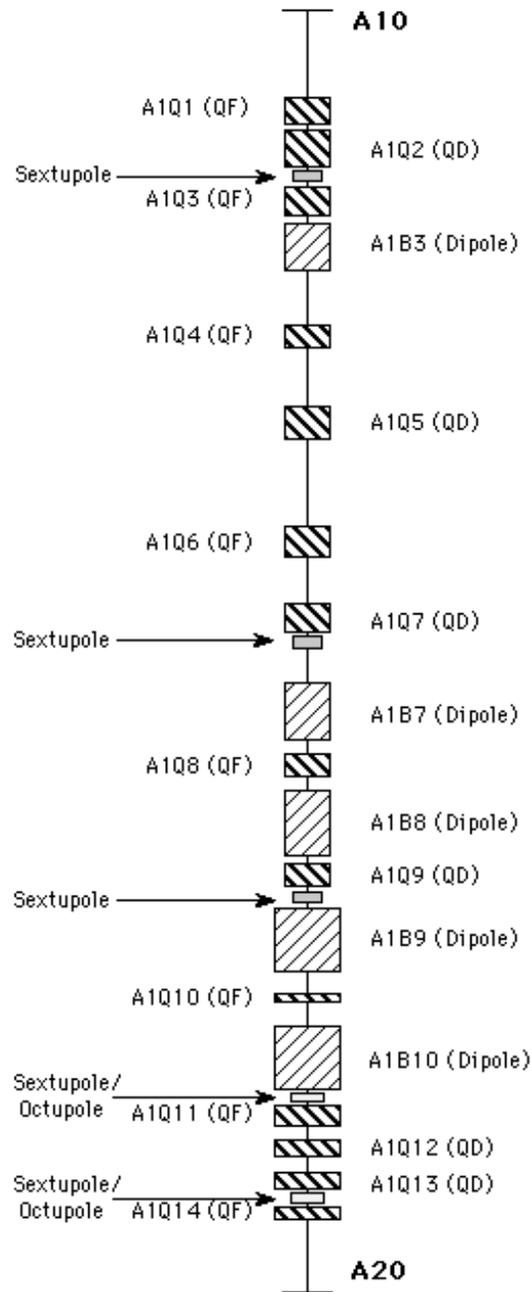


Figure 14: The Accumulator lattice

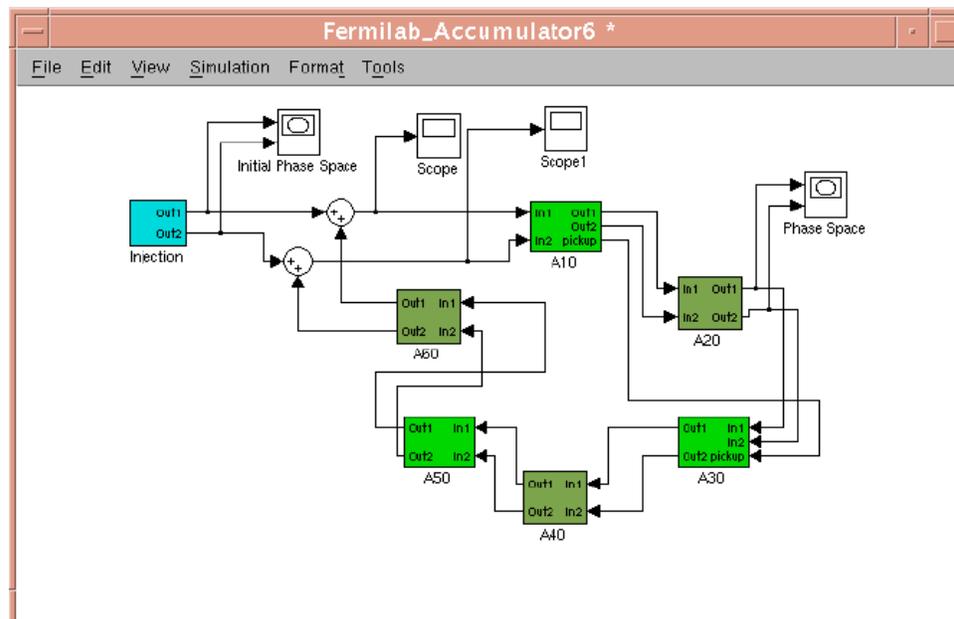


Figure 15: The schematic of the Accumulator

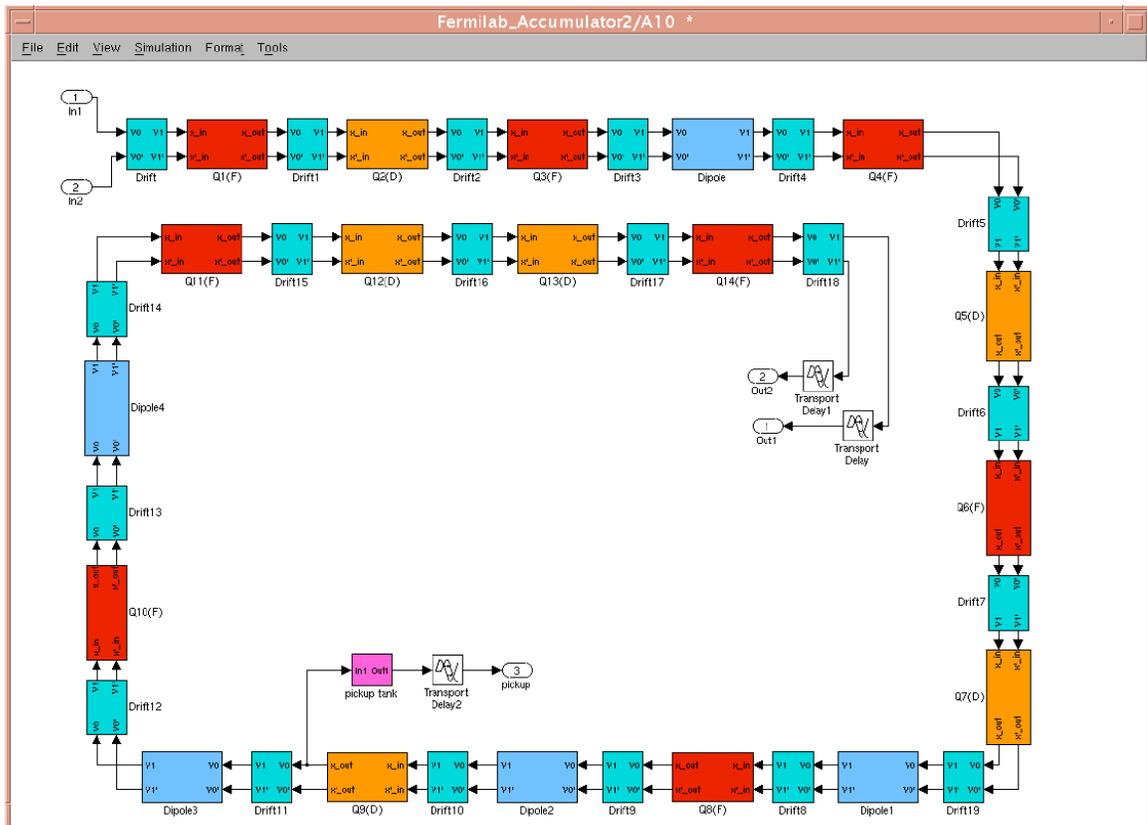


Figure 16: The A10 (odd) part of the Accumulator

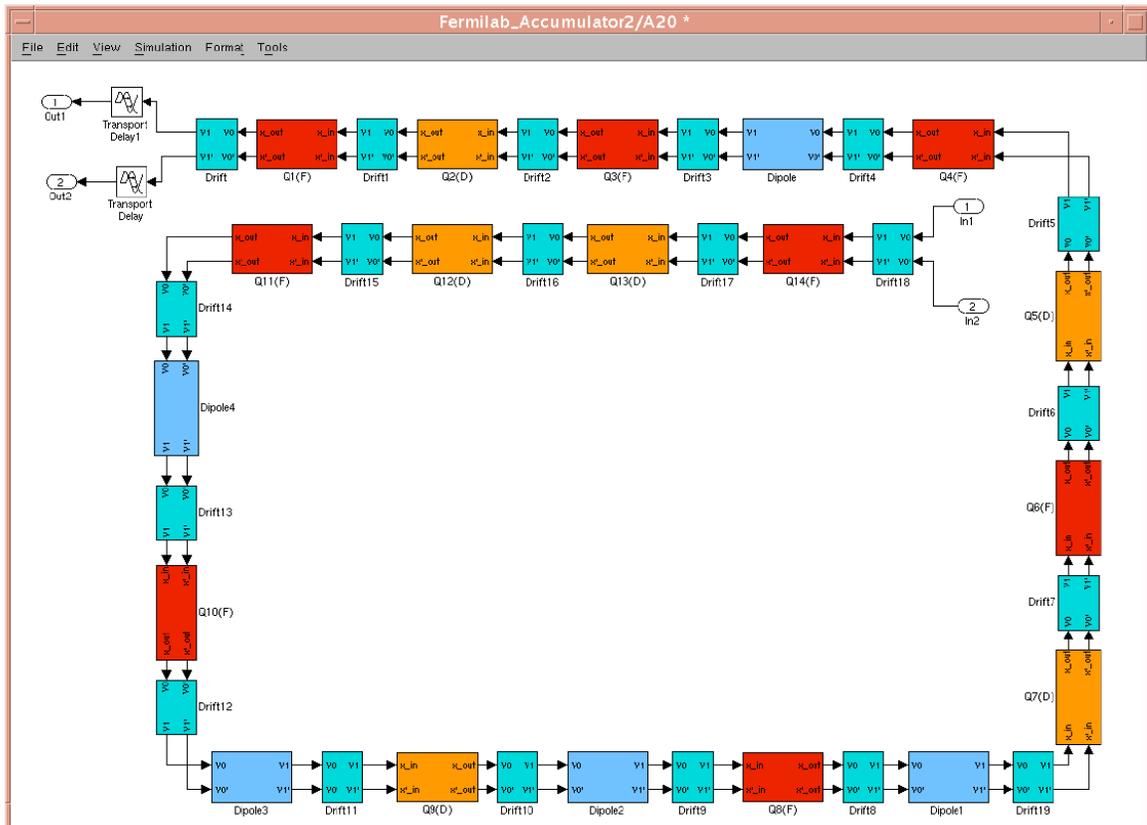


Figure 17: The A20 (even) part of the Accumulator