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# Extracting the helicity coefficients of the $W$ boson produced in $t\bar{t}$ events

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**Abstract**

MENTION IN THE TEXT THE SM PREDICTION We describe a method of extracting the helicity coefficients $F_0$, $F_+$, and $F_-$ of the $W$ boson resulting from the decay of a top quark using the method of moments. This method allows the simultaneous measurement of the coefficients, without the need for fixing any of them at the Standard Model values. In addition, no binning of the data is necessary. This method is ideal for extracting of the coefficients that correspond to a particular MC simulation, and also important for experimental unbiased measurements.

## I. EXTRACTING THE COEFFICIENTS USING THE METHOD OF MOMENTS

The theoretical distribution of the angle $\theta^*$ between the charged lepton direction in the $W$ rest-frame and the $W$ direction in the top rest-frame is:

$$\frac{dN}{d\cos\theta^*} \equiv D(\theta^*) = N[\frac{3}{4}F_0(1-\cos^2\theta^*)+\frac{3}{8}(1-F_0-F_+)(1-\cos\theta^*)^2+\frac{3}{8}F_+(1+\cos\theta^*)^2], \quad (1)$$

where $N$ is the total number of events, and we assumed that $F_0+F_-+F_+=1$. The helicity coefficients $F_i$ are correlated with the helicity of the produced $W$ and can be extracted at both MC and data level, using the method of moments described below.

Define a moment $\langle m(\theta^*)\rangle$ of a function $m(\theta^*)$ as

$$\langle m(\theta^*)\rangle = \frac{\int D(\theta^*)m(\theta^*)d\cos\theta^*}{\int D(\theta^*)d\cos\theta^*}. \quad (2)$$

Now, choose to use the following $m_i$ functions:

$$\begin{aligned} m_0(\theta^*) &= \cos\theta^*, \\ m_1(\theta^*) &= \sin^2\theta^*. \end{aligned} \quad (3)$$

Using Equations (1), (2), and (3), we conclude that:

$$\begin{aligned} \langle m_0\rangle &= \frac{2F_+ + F_0 - 1}{2}, \\ \langle m_1\rangle &= \frac{F_0+3}{5}. \end{aligned} \quad (4)$$

At our discrete environment, the integrals of Equation (2) are replaced by sums, so the final extracted values of the helicity coefficients, using Equation (4), are:

$$\begin{aligned} F_0 &= 5\langle m_1\rangle - 3 = 5\frac{\sum_i \sin^2\theta_i^*}{N} - 3, \\ F_+ &= 2 + \langle m_0\rangle - \frac{5\langle m_1\rangle}{2} = 2 + \frac{\sum_i \cos\theta_i^*}{N} - \frac{5}{2}(\frac{\sum_i \sin^2\theta_i^*}{N}), \end{aligned} \quad (5)$$

where $\theta_i^*$ is the $\theta^*$ value of event $\#i$ and $N$ is the total number of events.

If the method is used on experimental data, the data should be unfolded to really be described by Equation (1), if accurate transfer functions from observed to real angles are available. If not, then the (acceptance)$\times$(efficiency) will be calculated as a function of $\theta^*$, and be part of the integrals of Equation (2) and the subsequent equations. Let us assume that $\mathcal{A}(\theta^*)$ is the overall geometrical and kinematic acceptance multiplied with the overall $\theta^*$

reconstruction efficiency (in the rest of the note, we will refer to $\mathcal{A}(\theta^*)$ as simply acceptance). Then, the expected distribution of our experimental data will be:

$$\frac{dN_{\rm exp}}{d\cos\theta^*} \equiv D_{\rm exp}(\theta^*) = D(\theta^*)\mathcal{A}(\theta^*) + B(\theta^*), \tag{6}$$

where $D(\theta^*)$ is given by Equation (1) and $B(\theta^*)$ is the expected distribution of the lepton that comes from any background processes. If we use the same $m_i$ given by Equations (3), we have:

$$\begin{aligned} \langle m_0 \rangle &= (\alpha_0 - \beta_0)F_0 + (\gamma_0 - \beta_0)F_+ + \beta_0 + \delta_0, \\ \langle m_1 \rangle &= (\alpha_1 - \beta_1)F_0 + (\gamma_1 - \beta_1)F_+ + \beta_1 + \delta_1, \end{aligned} \tag{7}$$

where

$$\begin{aligned} \alpha_i &= \frac{3}{4}N\int(1-\cos^2\theta^*)m_i(\theta^*)\mathcal{A}(\theta^*)d\cos\theta^*/N_{\rm exp}, \\ \beta_i &= \frac{3}{8}N\int(1-\cos\theta^*)^2 m_i(\theta^*)\mathcal{A}(\theta^*)d\cos\theta^*/N_{\rm exp}, \\ \gamma_i &= \frac{3}{8}N\int(1+\cos\theta^*)^2 m_i(\theta^*)\mathcal{A}(\theta^*)d\cos\theta^*/N_{\rm exp}, \\ \delta_i &= \int B(\theta^*)m_i(\theta^*)d\cos\theta^*/N_{\rm exp}, \\ N_{\rm exp} &= \int D(\theta^*)\mathcal{A}(\theta^*)d\cos\theta^* + \int B(\theta^*)d\cos\theta^*. \end{aligned} \tag{8}$$

In the above, $N$ is the number of events for perfect acceptance ($N = \sigma_{\rm top} \times \mathcal{B}(l+\text{jets}) \times \mathcal{L}$), whereas $N_{\rm exp}$ depends on the $F_0$ and $F_+$. If the acceptance and the backgrounds are parametrized as a continuous functions, the integrals of Equations (7) are analytically calculated. If, on the other hand, the acceptance and the backgrounds are determined in a histogram form $\mathcal{A}_|$ and $B_j$ respectively, where $j$ is the number of histogram bin, then the following estimation is used instead:

$$\begin{aligned}
\alpha_i &= \frac{3}{4} N \sum_j \mathcal{A}_j \int_{\theta^* \in \text{bin } j} (1 - \cos^2\theta^*) m_i(\theta^*) d\cos\theta^* / N_{\text{exp}}, \\
\beta_i &= \frac{3}{8} N \sum_j \mathcal{A}_j \int_{\theta^* \in \text{bin } j} (1 - \cos\theta^*)^2 m_i(\theta^*) d\cos\theta^* / N_{\text{exp}}, \\
\gamma_i &= \frac{3}{8} N \sum_j \mathcal{A}_j \int_{\theta^* \in \text{bin } j} (1 + \cos\theta^*)^2 m_i(\theta^*) d\cos\theta^* / N_{\text{exp}}, \\
\delta_i &= \sum_j B_j \int_{\theta^* \in \text{bin } j} m_i(\theta^*) d\cos\theta^* / N_{\text{exp}}, \\
N_{\text{exp}} &= \sum_j \mathcal{A}_j \int_{\theta^* \in \text{bin } j} D(\theta^*) d\cos\theta^* + \sum_j B_j.
\end{aligned} \tag{9}$$

In any case the moments are again given by

$$\begin{aligned}
\langle m_0 \rangle &= \frac{\sum_i \cos\theta_i^*}{N_{\text{data}}}, \\
\langle m_1 \rangle &= \frac{\sum_i \sin^2\theta_i^*}{N_{\text{data}}},
\end{aligned} \tag{10}$$

where $\theta_i^*$ is the $\theta^*$ value of experimental event $\#i$, and $N_{\text{data}}$ is the total number of observed events. Using equations (7),(10) and [(8) or (9)] we will solve for $F_0$ and $F_+$.

The advantage of the method of moments is that both helicity coefficients can be extracted at the same time, without the need of fixing any of them at a particular value. This is important, as we do not know what the true values are, both at the generator and the experimental level. The extraction of the coefficients is more general and model independent, if the moments are used. The fact that the extraction is based on calculation of sums without the need of binning or fitting, makes the method more robust.