TRIUMF Summer Institute, July-August 2009, Statistics : Thomas Junk

Homework Assignment, Day 1

1) Two random numbers n_1 and n_2 are drawn from Poisson distributions with means μ_1 and μ_2, respectively. What is the variance of the distribuiton of n_1? Of n_2? Of $n_1 + n_2$? Of $n_1 - n_2$? Of $(n_1 - n_2)/(n_1 + n_2)$? You may use Gaussian approximations.

2) A common technique to estimate the robustness of an analysis is to tighten or loosen a selection requirement and to redo the measurement to see if the result is significantly different. Here we will address what "significantly different" means.

A measurement m_1 is made with a sample of n_1 events and has a statistical uncertainty of $\sigma_1 = A/\sqrt{n_1}$. A measurement m_2 is made on a subset of the events $n_2 < n_1$ and has a statistical uncertainty $\sigma_2 = A/\sqrt{n_2}$. What is the statistical uncertainty on $m_1 - m_2$ in terms of σ_1 and σ_2? You may assume Gaussian uncertainties throughout.

Hint which may or may not be useful to you: treat m_1 as a weighted average of m_2 and m_d, where m_d is the measured value on the difference sample (of size $n_1 - n_2$). m_2 and m_d are therefore statistically independent. Express $m_1 - m_2$ in terms of m_2 and m_d and propagate uncertainties.

3) An easier way to look at BLUE. An experiment is measuring the mass of the top quark, m_t. It makes two measurements in different decay modes, with non-overlapping data samples. For simplicity, let's consider a case with just data statistical errors and one source of systematic uncertainty, the jet energy scale (JES). The two measurements are

$$m_1 = 173.5 \pm 3.0(\text{stat}) \pm 2.0(\text{JES})$$

and

$$m_2 = 168.2 \pm 4.0(\text{stat}) \pm 2.0(\text{JES})$$

Treat the statistical uncertainties as uncorrelated and the JES uncertainties as correlated. Combine them using BLUE. Give the average value of m_t and the total uncertainty. The formula given in lecture is a little cumbersome; here's an easier way out.

$$m_{\text{avg}} = w_1 m_1 + w_2 m_2 = w_1 m_1 + (1 - w_1) m_2$$

Find the total uncertainty on m_{avg} as a function of w_1, and find the value of w_1 which minimizes the total uncertainty.

What is the correlation ρ between the two measurements?

4) A pathological BLUE combination. This is a similar problem, but now treat the two statistical uncertainties as completely correlated and the JES uncertainties as completely

correlated. Combine the two measurements with BLUE. What are the two weights? Use the measurements below for this problem.

$$m_1 = 173.5 \pm 3.0(\text{stat}) \pm 2.0(\text{JES})$$

and

$$m_2 = 173.0 \pm 3.0(\text{stat}) \pm 2.1(\text{JES})$$