TRIUMF Summer Institute, July-August 2009, Statistics : Thomas Junk

Homework Assignment, Day 3

1) Suppose a beam of charged particles consists of 90% pions and 10% kaons. We have a particle identification device which has the following performance. If a charged kaon traverses it, 82% of the time it is identified as a kaon, 5% of the time it is identified as a pion, and the remainder of the time no decision is made. If a pion flies through it, 70% of the time it is identified as a pion, 10% of the time it is identified as a kaon, and the remainder of the time no decision is reported.
Give the posterior probabilities that a particle is a pion and that it is a kaon (two probabilties per case below) given:

a) It is identified as a pion

b) It is identified as a kaon

c) The particle identification device yields no decision.

2) Suppose a Monte Carlo event generator and simulation and reconstruction packge is run to generate a finite Monte Carlo sample. In a particular bin of a histogram of a reconstructed variable, n Monte Carlo events fall in that bin.

Calculate the posterior density in the true rate r of events in that histogram bin, assuming a flat prior in r. What is the mean expected value of r?

3) Suppose a model of new physics predicts the production of a new particle with a precisely predicted production cross section, and which decays to a particular final state which can be reconstructed in a detector, but with an unknown branching fraction B, which can be between 0 and 1. The other decays, which happen $1 - B$ of the time are not reconstructed. An experiment excludes $0 < B < 0.95$.

Explain what is wrong with the following argument: If we take B as an unknown nuisance parameter, we can average the posterior over it as we do any other nuisance parameter, and exclude the entire model of new physics for any branching ratio.

4) In a search for new physics, the SM predicts a background yield of b events, and a test hypothesis predicts a signal in addition to the background, for a total of $s + b$ events. Zero events are observed.

Show that the 95% credibility upper limit on s is independent of the prediction of b, for zero observed events. What is the upper limit on s, in events? Does the median expected upper limit depend on b? You need not calculate it but rather explain why it does or doesn't depend on b.